Chapter 5
Exponential and Logarithmic Functions

Section 5.1

1. \( f(3) = -4(3)^2 + 5(3) \)
   \( = -4(9) + 15 \)
   \( = -36 + 15 \)
   \( = -21 \)

2. \( f(3x) = 4 - 2(3x)^2 \)
   \( = 4 - 2(9x^2) \)
   \( = 4 - 18x^2 \)

3. \( f(x) = \frac{x^2 - 1}{x^2 - 25} \)
   \( x^2 - 25 \neq 0 \)
   \( (x + 5)(x - 5) \neq 0 \)
   \( x \neq -5, \ x \neq 5 \)
   Domain: \( \{ x | x \neq -5, x \neq 5 \} \)

4. composite function; \( f(g(x)) \)

5. False: \( f(g(x)) = (f \circ g)(x) \)

6. False. The domain of \( (f \circ g)(x) \) is a subset of the domain of \( g(x) \).

7. a. \( (f \circ g)(1) = f(g(1)) = f(0) = -1 \)
    b. \( (f \circ g)(-1) = f(g(-1)) = f(0) = -1 \)
    c. \( (g \circ f)(-1) = g(f(-1)) = g(-3) = 8 \)
    d. \( (g \circ f)(0) = g(f(0)) = g(1) = 0 \)
    e. \( (g \circ g)(-2) = g(g(-2)) = g(3) = 8 \)
    f. \( (f \circ f)(-1) = f(f(-1)) = f(-3) = -7 \)

8. a. \( (f \circ g)(1) = f(g(1)) = f(0) = 5 \)
    b. \( (f \circ g)(2) = f(g(2)) = f(-3) = 11 \)
    c. \( (g \circ f)(2) = g(f(2)) = g(1) = 0 \)
    d. \( (g \circ f)(3) = g(f(3)) = g(-1) = 0 \)

9. a. \( (g \circ f)(-1) = g(f(-1)) = g(1) = 4 \)
    b. \( (g \circ f)(0) = g(f(0)) = g(0) = 5 \)
    c. \( (f \circ g)(-1) = f(g(-1)) = f(3) = -1 \)
    d. \( (f \circ g)(4) = f(g(4)) = f(2) = -2 \)

10. a. \( (g \circ f)(1) = g(f(1)) = g(-1) = 3 \)
    b. \( (g \circ f)(5) = g(f(5)) = g(1) = 4 \)
    c. \( (f \circ g)(0) = f(g(0)) = f(5) = 1 \)
    d. \( (f \circ g)(2) = f(g(2)) = f(2) = -2 \)

11. \( f(x) = 2x \)
    \( g(x) = 3x^2 + 1 \)
    a. \( (f \circ g)(4) = f(g(4)) \)
        \( = f(3(4)^2 + 1) \)
        \( = f(49) \)
        \( = 98 \)
    b. \( (g \circ f)(2) = g(f(2)) \)
        \( = g(2 \cdot 2) \)
        \( = g(4) \)
        \( = 3(4)^2 + 1 \)
        \( = 48 + 1 \)
        \( = 49 \)
    c. \( (f \circ f)(1) = f(f(1)) \)
        \( = f(2(1)) \)
        \( = f(2) \)
        \( = 2(2) \)
        \( = 4 \)
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12. \( f(x) = 3x + 2 \) \( g(x) = 2x^2 - 1 \)

a. \((f \circ g)(4) = f(g(4))\)
   \[= f \left( 2(4)^2 - 1 \right)\]
   \[= f(31)\]
   \[= 3(31) + 2\]
   \[= 95\]

b. \((g \circ f)(2) = g(f(2))\)
   \[= g(3(2) + 2)\]
   \[= g(8)\]
   \[= 2(8)^2 - 1\]
   \[= 128 - 1\]
   \[= 127\]

c. \((f \circ f)(1) = f(f(1))\)
   \[= f(3(1) + 2)\]
   \[= f(5)\]
   \[= 3(5) + 2\]
   \[= 17\]

d. \((g \circ g)(0) = g(g(0))\)
   \[= g \left( 2(0)^2 - 1 \right)\]
   \[= g(-1)\]
   \[= 2(-1)^2 - 1\]
   \[= 1\]

13. \( f(x) = 4x^2 - 3 \) \( g(x) = 3 - \frac{1}{2}x^2 \)

a. \((f \circ g)(4) = f(g(4))\)
   \[= f \left( 3 - \frac{1}{2}(4)^2 \right)\]
   \[= f(-5)\]
   \[= 4(-5)^2 - 3\]
   \[= 97\]

14. \( f(x) = 2x^2 \) \( g(x) = 1 - 3x^2 \)

a. \((f \circ g)(4) = f(g(4))\)
   \[= f \left( 1 - 3(4)^2 \right)\]
   \[= f(-47)\]
   \[= 2(-47)^2\]
   \[= 4418\]

b. \((g \circ f)(2) = g(f(2))\)
   \[= g(2(2)^2)\]
   \[= g(8)\]
   \[= 1 - 3(8)^2\]
   \[= 1 - 192\]
   \[= -191\]

c. \((f \circ f)(1) = f(f(1))\)
   \[= f \left( 2(1)^2 \right)\]
   \[= f(2)\]
   \[= 2(2)^2\]
   \[= 8\]
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d. \((g \circ g)(0) = g(g(0))\)
   \[= g(1-3(0)^2)\]
   \[= g(1)\]
   \[= 1-3(1)^2\]
   \[= 1-3\]
   \[= -2\]

15. \(f(x) = \sqrt{x}\) \(g(x) = 2x\)
   a. \((f \circ g)(4) = f(g(4))\)
      \[= f(2(4))\]
      \[= f(8)\]
      \[= \sqrt{8}\]
      \[= 2\sqrt{2}\]
   b. \((g \circ f)(2) = g(f(2))\)
      \[= g(\sqrt{2})\]
      \[= 2\sqrt{2}\]
   c. \((f \circ f)(1) = f(f(1))\)
      \[= f(\sqrt{1})\]
      \[= f(1)\]
      \[= \sqrt{1}\]
      \[= 1\]
   d. \((g \circ g)(0) = g(g(0))\)
      \[= g(2(0))\]
      \[= g(0)\]
      \[= 2(0)\]
      \[= 0\]

16. \(f(x) = \sqrt{x+1}\) \(g(x) = 3x\)
   a. \((f \circ g)(4) = f(g(4))\)
      \[= f(3(4))\]
      \[= f(12)\]
      \[= \sqrt{12+1}\]
      \[= \sqrt{13}\]
   b. \((g \circ f)(2) = g(f(2))\)
      \[= g(\sqrt{2}+1)\]
      \[= g(\sqrt{3})\]
      \[= 3\sqrt{3}\]
   c. \((f \circ f)(1) = f(f(1))\)
      \[= f(\sqrt{1})\]
      \[= f(1)\]
      \[= \sqrt{1}\]
      \[= 1\]
   d. \((g \circ g)(0) = g(g(0))\)
      \[= g(\frac{1}{0^2+1})\]
      \[= g(1)\]
      \[= \frac{1}{1^2+1}\]
      \[= \frac{1}{2}\]
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18. \( f(x) = |x - 2| \quad g(x) = \frac{3}{x^2 + 2} \)
   
a. \((f \circ g)(4) = f(g(4))\)
   \begin{align*}
   & = f\left(\frac{3}{4^2 + 2}\right) \\
   & = f\left(\frac{3}{18}\right) \\
   & = f\left(\frac{1}{6}\right) \\
   & = \frac{1}{6} - 2 \\
   & = \frac{-11}{6} \\
   & = \frac{-11}{6} \\
   & = \frac{11}{6}
   \end{align*}

   b. \((g \circ f)(2) = g(f(2))\)
   \begin{align*}
   & = g\left(\left|2 - 2\right|\right) \\
   & = g(0) \\
   & = \frac{3}{0^2 + 2} \\
   & = \frac{3}{2}
   \end{align*}

   c. \((f \circ f)(1) = f(f(1))\)
   \begin{align*}
   & = f\left(\left|1 - 2\right|\right) \\
   & = f(1) \\
   & = \left|1 - 2\right| \\
   & = 1
   \end{align*}

   d. \((g \circ g)(0) = g(g(0))\)
   \begin{align*}
   & = g\left(\frac{3}{0^2 + 2}\right) \\
   & = g\left(\frac{3}{2}\right) \\
   & = \frac{3}{\left(\frac{3}{2}\right)^2 + 2} \\
   & = \frac{3}{\frac{17}{4}} \\
   & = \frac{12}{17}
   \end{align*}

19. \( f(x) = \frac{3}{x+1} \quad g(x) = \sqrt{x} \)
   
a. \((f \circ g)(4) = f(g(4))\)
   \begin{align*}
   & = f\left(\sqrt{4}\right) \\
   & = \frac{3}{\sqrt{4} + 1}
   \end{align*}

   b. \((g \circ f)(2) = g(f(2))\)
   \begin{align*}
   & = g\left(\frac{3}{2 + 1}\right) \\
   & = g\left(\frac{3}{3}\right) \\
   & = g(1) \\
   & = \sqrt{1} \\
   & = 1
   \end{align*}

   c. \((f \circ f)(1) = f(f(1))\)
   \begin{align*}
   & = f\left(\frac{3}{1 + 1}\right) \\
   & = f\left(\frac{3}{2}\right) \\
   & = \frac{3}{2 + 1} \\
   & = \frac{3}{5} \\
   & = \frac{6}{5}
   \end{align*}

   d. \((g \circ g)(0) = g(g(0))\)
   \begin{align*}
   & = g\left(\sqrt{0}\right) \\
   & = g(0) \\
   & = \sqrt{0} \\
   & = 0
   \end{align*}
20. \( f(x) = x^{3/2} \quad g(x) = \frac{2}{x + 1} \)

a. \((f \circ g)(4) = f(g(4)) = f\left(\frac{2}{4+1}\right) = f\left(\frac{2}{5}\right) = \left(\frac{2}{5}\right)^{3/2} = \sqrt[3]{\frac{2}{5}} = \frac{\sqrt[3]{2}}{\sqrt[3]{5}} = 2\sqrt{\frac{2}{5}} = 2\sqrt{\frac{\sqrt[3]{2}}{5}} = 2\sqrt[3]{2} \) or \( \frac{4\sqrt{2} - 2}{7} \)

b. \((g \circ f)(2) = g(f(2)) = g\left(\frac{2^{3/2}}{2}\right) = g\left(\sqrt[3]{4}\right) = g\left(2\sqrt{2}\right) = \frac{2}{2\sqrt{2} + 1} = \frac{4\sqrt{2} - 2}{7} \)

c. \((f \circ f)(1) = f(f(1)) = f\left(1^{3/2}\right) = f(1) = 1^{3/2} = 1 \)

d. \((g \circ g)(0) = g(g(0)) = g\left(\frac{2}{0 + 1}\right) = g(2) = 2 \) or \( \frac{4\sqrt{2} - 2}{7} \)

21. The domain of \( g \) is \( \{x \mid x \neq 0\} \). The domain of \( f \) is \( \{x \mid x \neq 1\} \). Thus, \( g(x) \neq 1 \), so we solve:

\[
g(x) = 1 \implies \frac{2}{x} = 1 \implies x = 2
\]

Thus, \( x \neq 2 \); so the domain of \( f \circ g \) is \( \{x \mid x \neq 0, x \neq 2\} \).

22. The domain of \( g \) is \( \{x \mid x \neq 0\} \). The domain of \( f \) is \( \{x \mid x \neq -3\} \). Thus, \( g(x) \neq -3 \), so we solve:

\[
g(x) = -3 \implies -\frac{2}{x} = -3 \implies x = \frac{2}{3}
\]

Thus, \( x \neq \frac{2}{3} \); so the domain of \( f \circ g \) is \( \{x \mid x \neq 0, x \neq \frac{2}{3}\} \).

23. The domain of \( g \) is \( \{x \mid x \neq 0\} \). The domain of \( f \) is \( \{x \mid x \neq 1\} \). Thus, \( g(x) \neq 1 \), so we solve:

\[
g(x) = 1 \implies -\frac{4}{x} = 1 \implies x = -4
\]

Thus, \( x \neq -4 \); so the domain of \( f \circ g \) is \( \{x \mid x \neq -4, x \neq 0\} \).

24. The domain of \( g \) is \( \{x \mid x \neq 0\} \). The domain of \( f \) is \( \{x \mid x \neq -3\} \). Thus, \( g(x) \neq -3 \), so we solve:

\[
g(x) = -3 \implies \frac{2}{x} = -3 \implies x = \frac{2}{3}
\]

Thus, \( x \neq \frac{2}{3} \); so the domain of \( f \circ g \) is \( \{x \mid x \neq \frac{2}{3}, x \neq 0\} \).
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25. The domain of $g$ is $\{x \mid x \text{ is any real number}\}$. The domain of $f$ is $\{x \mid x \geq 0\}$. Thus, $g(x) \geq 0$, so we solve:
   
   $2x + 3 \geq 0$
   
   $x \geq -\frac{3}{2}$

   Thus, the domain of $f \circ g$ is $\{x \mid x \geq -\frac{3}{2}\}$.

26. The domain of $g$ is $\{x \mid x \leq 1\}$. The domain of $f$ is $\{x \mid x \text{ is any real number}\}$. Thus, the domain of $f \circ g$ is $\{x \mid x \leq 1\}$.

27. The domain of $g$ is $\{x \mid x \geq 1\}$. The domain of $f$ is $\{x \mid x \text{ is any real number}\}$. Thus, the domain of $f \circ g$ is $\{x \mid x \geq 1\}$.

28. The domain of $g$ is $\{x \mid x \geq 2\}$. The domain of $f$ is $\{x \mid x \text{ is any real number}\}$. Thus, the domain of $f \circ g$ is $\{x \mid x \geq 2\}$.

29. $f(x) = 2x + 3 \quad g(x) = 3x$

   The domain of $f$ is $\{x \mid x \text{ is any real number}\}$.

   The domain of $g$ is $\{x \mid x \text{ is any real number}\}$.

   a. $(f \circ g)(x) = f(g(x))$
      
      $= f(3x)$
      
      $= 2(3x) + 3$
      
      $= 6x + 3$

      Domain: $\{x \mid x \text{ is any real number}\}$.

   b. $(g \circ f)(x) = g(f(x))$
      
      $= g(2x + 3)$
      
      $= 3(2x + 3)$
      
      $= 6x + 9$

      Domain: $\{x \mid x \text{ is any real number}\}$.

30. $f(x) = -x \quad g(x) = 2x - 4$

   The domain of $f$ is $\{x \mid x \text{ is any real number}\}$.

   The domain of $g$ is $\{x \mid x \text{ is any real number}\}$.

   a. $(f \circ g)(x) = f(g(x))$
      
      $= f(2x - 4)$
      
      $= -(2x - 4)$
      
      $= -2x + 4$

      Domain: $\{x \mid x \text{ is any real number}\}$.

   b. $(g \circ f)(x) = g(f(x))$
      
      $= g(-x)$
      
      $= 2(-x) - 4$
      
      $= -2x - 4$

      Domain: $\{x \mid x \text{ is any real number}\}$.

   c. $(f \circ f)(x) = f(f(x))$
      
      $= f(-x)$
      
      $= -(-x)$
      
      $= x$

      Domain: $\{x \mid x \text{ is any real number}\}$.

   d. $(g \circ g)(x) = g(g(x))$
      
      $= g(2x - 4)$
      
      $= 2(2x - 4) - 4$
      
      $= 4x - 8 - 4$
      
      $= 4x - 12$

      Domain: $\{x \mid x \text{ is any real number}\}$.
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31. \( f(x) = 3x+1 \quad g(x) = x^2 \)

   The domain of \( f \) is \( \{x \mid x \text{ is any real number}\} \).
   The domain of \( g \) is \( \{x \mid x \text{ is any real number}\} \).

   a. \((f \circ g)(x) = f(g(x))
       = f(x^2)
       = 3x^2 + 1
   \)
   Domain: \( \{x \mid x \text{ is any real number}\} \).

   b. \((g \circ f)(x) = g(f(x))
       = g(3x+1)
       = (3x+1)^2
       = 9x^2 + 6x + 1
   \)
   Domain: \( \{x \mid x \text{ is any real number}\} \).

   c. \((f \circ f)(x) = f(f(x))
       = f(3x+1)
       = 3(3x+1) + 1
       = 9x + 3 + 1
       = 9x + 4
   \)
   Domain: \( \{x \mid x \text{ is any real number}\} \).

   d. \((g \circ g)(x) = g(g(x))
       = g(x^2)
       = (x^2)^2
       = x^4
   \)
   Domain: \( \{x \mid x \text{ is any real number}\} \).

32. \( f(x) = x + 1 \quad g(x) = x^2 + 4 \)

   The domain of \( f \) is \( \{x \mid x \text{ is any real number}\} \).
   The domain of \( g \) is \( \{x \mid x \text{ is any real number}\} \).

   a. \((f \circ g)(x) = f(g(x))
       = f(x^2 + 4)
       = x^2 + 4 + 1
       = x^2 + 5
   \)
   Domain: \( \{x \mid x \text{ is any real number}\} \).

   b. \((g \circ f)(x) = g(f(x))
       = g(x+1)
       = (x+1)^2 + 4
       = x^2 + 2x + 1 + 4
       = x^2 + 2x + 5
   \)
   Domain: \( \{x \mid x \text{ is any real number}\} \).

33. \( f(x) = x^2 \quad g(x) = x^2 + 4 \)

   The domain of \( f \) is \( \{x \mid x \text{ is any real number}\} \).
   The domain of \( g \) is \( \{x \mid x \text{ is any real number}\} \).

   a. \((f \circ g)(x) = f(g(x))
       = f(x^2 + 4)
       = (x^2 + 4) + 1
       = x^2 + 5
   \)
   Domain: \( \{x \mid x \text{ is any real number}\} \).

   b. \((g \circ f)(x) = g(f(x))
       = g(x^2)
       = (x^2)^2
       = x^4
   \)
   Domain: \( \{x \mid x \text{ is any real number}\} \).

   c. \((f \circ f)(x) = f(f(x))
       = f(x^2)
       = (x^2)^2
       = x^4
   \)
   Domain: \( \{x \mid x \text{ is any real number}\} \).

   d. \((g \circ g)(x) = g(g(x))
       = g(x^2 + 4)
       = (x^2 + 4)^2 + 4
       = x^4 + 8x^2 + 16 + 4
       = x^4 + 8x^2 + 20
   \)
   Domain: \( \{x \mid x \text{ is any real number}\} \).

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34. \( f(x) = x^2 + 1 \quad g(x) = 2x^2 + 3 \)
   The domain of \( f \) is \( \{ x \mid x \text{ is any real number} \} \).
   The domain of \( g \) is \( \{ x \mid x \text{ is any real number} \} \).
   a. \((f \circ g)(x) = f(g(x)) \)
      \[= f(2x^2 + 3)\]
      \[= (2x^2 + 3)^2 + 1\]
      \[= 4x^4 + 12x^2 + 9 + 1\]
      \[= 4x^4 + 12x^2 + 10\]
      Domain: \( \{ x \mid x \text{ is any real number} \} \).
   b. \((g \circ f)(x) = g(f(x)) \)
      \[= g(x^2 + 1)\]
      \[= 2(x^2 + 1)^2 + 3\]
      \[= 2(x^4 + 2x^2 + 1) + 3\]
      \[= 2x^4 + 4x^2 + 9 + 3\]
      \[= 2x^4 + 4x^2 + 5\]
      Domain: \( \{ x \mid x \text{ is any real number} \} \).
   c. \((f \circ f)(x) = f(f(x)) \)
      \[= f(x^2 + 1)\]
      \[= (x^2 + 1)^2 + 1\]
      \[= x^2 + 2x^2 + 2 + 1\]
      \[= x^4 + 2x^2 + 2\]
      Domain: \( \{ x \mid x \text{ is any real number} \} \).
   d. \((g \circ g)(x) = g(g(x)) \)
      \[= g(2x^2 + 3)\]
      \[= 2(2x^2 + 3)^2 + 3\]
      \[= 4(4x^4 + 12x^2 + 9) + 3\]
      \[= 8x^4 + 24x^2 + 18 + 3\]
      \[= 8x^4 + 24x^2 + 21\]
      Domain: \( \{ x \mid x \text{ is any real number} \} \).

35. \( f(x) = \frac{3}{x-1} \quad g(x) = \frac{2}{x} \)
   The domain of \( f \) is \( \{ x \mid x \neq 1 \} \). The domain of \( g \) is \( \{ x \mid x \neq 0 \} \).
   a. \((f \circ g)(x) = f(g(x)) \)
      \[= f\left(\frac{2}{x}\right)\]
      \[= \frac{3}{\frac{2}{x} - 1}\]
      \[= \frac{3x}{2 - x}\]
      \[= \frac{3}{2 - x}\]
      Domain \( \{ x \mid x \neq 0, x \neq 2 \} \).
   b. \((g \circ f)(x) = g(f(x)) \)
      \[= g\left(\frac{3}{x-1}\right)\]
      \[= \frac{2}{\frac{3}{x-1}}\]
      \[= \frac{2(x-1)}{3}\]
      \[= \frac{3}{2(x-1)}\]
      Domain \( \{ x \mid x \neq 1 \} \).
   c. \((f \circ f)(x) = f(f(x)) \)
      \[= f\left(\frac{3}{x-1}\right)\]
      \[= \frac{3}{\frac{3}{x-1} - 1}\]
      \[= \frac{3}{\frac{3 - (x-1)}{x-1}}\]
      \[= \frac{3(x-1)}{4 - x}\]
      Domain \( \{ x \mid x \neq 1, x \neq 4 \} \).
   d. \((g \circ g)(x) = g(g(x)) = g\left(\frac{2}{x}\right) = \frac{2}{\frac{2}{x}} = \frac{2x}{2} = x \)
      Domain \( \{ x \mid x \neq 0 \} \).
36. \( f(x) = \frac{1}{x+3} \quad g(x) = \frac{2}{x} \)

The domain of \( f \) is \( \{ x \mid x \neq -3 \} \). The domain of \( g \) is \( \{ x \mid x \neq 0 \} \).

a. \((f \circ g)(x) = f(g(x)) = f\left(\frac{2}{x}\right)\)
\[
= \frac{1}{\frac{2}{x} + 3} = \frac{1}{\frac{2 + 3x}{x}}
\]
\[
= \frac{x}{-2 + 3x} \quad \text{or} \quad \frac{x}{3x - 2}
\]
Domain \( \{ x \mid x \neq 0, x \neq \frac{2}{3} \} \).

b. \((g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x+3}\right)\)
\[
= \frac{2}{1 + x + 3} = \frac{2}{x + 4}
\]
\[
= \frac{2}{x + 4} \quad \text{or} \quad \frac{2}{3x - 2}
\]
Domain \( \{ x \mid x \neq -3 \} \).

c. \((f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x+3}\right)\)
\[
= \frac{1}{\frac{1}{x+3} + 3} = \frac{1}{\frac{1 + 3x + 9}{x+3}}
\]
\[
= \frac{x+3}{3x+10}
\]
Domain \( \{ x \mid x \neq -\frac{10}{3}, x \neq -3 \} \).

d. \((g \circ g)(x) = g(g(x)) = g\left(\frac{-2}{x}\right)\)
\[
= \frac{-2^2}{x} = \frac{2x}{-2}
\]
\[
= x
\]
Domain \( \{ x \mid x \neq 0 \} \).

37. \( f(x) = \frac{x}{x-1} \quad g(x) = -\frac{4}{x} \)

The domain of \( f \) is \( \{ x \mid x \neq 1 \} \). The domain of \( g \) is \( \{ x \mid x \neq 0 \} \).

a. \((f \circ g)(x) = f(g(x)) = f\left(\frac{-4}{x}\right)\)
\[
= \frac{-4}{x - 1} = \frac{-4}{x - 1} = \frac{-4}{x - 1}
\]
\[
= \frac{-4}{x - 1}
\]
Domain \( \{ x \mid x \neq -4, x \neq 0 \} \).

b. \((g \circ f)(x) = g(f(x)) = g\left(\frac{x}{x-1}\right)\)
\[
= \frac{x}{x - 1} = \frac{x}{x - 1} = \frac{x}{x - 1}
\]
\[
= \frac{x}{x - 1}
\]
Domain \( \{ x \mid x \neq 0, x \neq 1 \} \).

c. \((f \circ f)(x) = f(f(x)) = f\left(\frac{x}{x-1}\right)\)
\[
= \frac{x}{x - 1} = \frac{x}{x - 1} = \frac{x}{x - 1}
\]
\[
= \frac{x}{x - 1}
\]
Domain \( \{ x \mid x \neq 1 \} \).

d. \((g \circ g)(x) = g(g(x)) = g\left(\frac{-4}{x}\right)\)
\[
= \frac{-4}{x} = \frac{-4}{x} = \frac{-4}{x}
\]
\[
= \frac{-4}{x}
\]
Domain \( \{ x \mid x \neq 0 \} \).
38. \( f(x) = \frac{x}{x+3} \quad g(x) = \frac{2}{x} \)

The domain of \( f \) is \( \{ x \mid x \neq -3 \} \). The domain of \( g \) is \( \{ x \mid x \neq 0 \} \).

a. \((f \circ g)(x) = f(g(x)) = f\left(\frac{2}{x}\right) = \frac{2}{x} = \frac{2}{x+3} = \frac{2}{\frac{2+3x}{x}} = \frac{2}{2+3x}\)

Domain \( \{ x \mid x \neq -\frac{2}{3}, x \neq 0 \} \).

b. \((g \circ f)(x) = g(f(x)) = g\left(\frac{x}{x+3}\right) = \frac{2}{x+3} = \frac{2}{\frac{2x+3}{x}} = \frac{2}{2x+3}\)

Domain \( \{ x \mid x \neq -3, x \neq 0 \} \).

c. \((f \circ f)(x) = f(f(x)) = f\left(\frac{x}{x+3}\right) = \frac{x}{x+3} = \frac{x}{\frac{x+3}{x}} = \frac{4x+9}{x+3}\)

Domain \( \{ x \mid x \neq -3, x \neq -\frac{9}{4} \} \).

d. \((g \circ g)(x) = g(g(x)) = g\left(\frac{2}{x}\right) = \frac{2}{\frac{2}{x}} = \frac{2x}{2} = x\)

Domain \( \{ x \mid x \neq 0 \} \).

39. \( f(x) = \sqrt{x} \quad g(x) = 2x + 3 \)

The domain of \( f \) is \( \{ x \mid x \geq 0 \} \). The domain of \( g \) is \( \{ x \mid x \text{ is any real number} \} \).

a. \((f \circ g)(x) = f(g(x)) = f(2x+3) = \sqrt{2x+3}\)

Domain \( \{ x \mid x \geq -\frac{3}{2} \} \).

b. \((g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = 2\sqrt{x} + 3\)

Domain \( \{ x \mid x \geq 0 \} \).

c. \((f \circ f)(x) = f(f(x)) = f(\sqrt{x}) = x\)

Domain \( \{ x \mid x \geq 0 \} \).

d. \((g \circ g)(x) = g(g(x)) = g(2x + 3) = 2(2x + 3) + 3 = 4x + 6 + 3 = 4x + 9\)

Domain \( \{ x \mid x \text{ is any real number} \} \).

40. \( f(x) = \sqrt{x-2} \quad g(x) = 1 - 2x \)

The domain of \( f \) is \( \{ x \mid x \geq 2 \} \). The domain of \( g \) is \( \{ x \mid x \text{ is any real number} \} \).

a. \((f \circ g)(x) = f(g(x)) = f(1 - 2x) = \sqrt{1 - 2x - 2} = \sqrt{-2x - 1}\)

Domain \( \{ x \mid x \leq -\frac{1}{2} \} \).

b. \((g \circ f)(x) = g(f(x)) = g(\sqrt{x-2}) = 1 - 2\sqrt{x-2}\)

Domain \( \{ x \mid x \geq 2 \} \).
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c. \((f \circ f)(x) = f(f(x)) = f(\sqrt{x-2}) = \sqrt{\sqrt{x-2} - 2}\)

Now, \(\sqrt{x-2} - 2 \geq 0\)
\(\sqrt{x-2} \geq 2\)
\(x - 2 \geq 4\)
\(x \geq 6\)
Domain \(\{x | x \geq 6\}\).

d. \((g \circ g)(x) = g(g(x)) = g(\sqrt{x-1}) = \sqrt{\sqrt{x-1} - 1}\)

Now, \(\sqrt{x-1} - 1 \geq 0\)
\(\sqrt{x-1} \geq 1\)
\(x - 1 \geq 1\)
\(x \geq 2\)
Domain \(\{x | x \geq 2\}\).

42. \(f(x) = x^2 + 4\quad g(x) = \sqrt{x - 2}\)

The domain of \(f\) is \(\{x | x \text{ is any real number}\}\).

The domain of \(g\) is \(\{x | x \geq 2\}\).

a. \((f \circ g)(x) = f(g(x)) = f(\sqrt{x-2}) = \left(\sqrt{x-2}\right)^2 + 4 = x + 2\)

Domain \(\{x | x \geq 2\}\).

b. \((g \circ f)(x) = g(f(x)) = g(x^2 + 1) = \sqrt{x^2 + 4 - 2} = \sqrt{x^2 + 2}\)

Domain \(\{x | x \text{ is any real number}\}\).

c. \((f \circ f)(x) = f(f(x)) = f(x^2 + 1) = (x^2 + 1)^2 + 1 = x^4 + 2x^2 + 1 + 1 = x^4 + 2x^2 + 2\)

Domain \(\{x | x \text{ is any real number}\}\).

d. \((g \circ g)(x) = g(g(x)) = g(\sqrt{x-2}) = \sqrt{x-2 - 2}\)

Now, \(\sqrt{x-2} - 2 \geq 0\)
\(\sqrt{x-2} \geq 2\)
\(x - 2 \geq 4\)
\(x \geq 6\)
Domain \(\{x | x \geq 6\}\).
43. \( f(x) = \frac{x-5}{x+1} \quad g(x) = \frac{x+2}{x-3} \)

The domain of \( f \) is \( \{ x \mid x \neq -1 \} \). The domain of \( g \) is \( \{ x \mid x \neq 3 \} \).

a. \( (f \circ g)(x) = f(g(x)) = f\left(\frac{x+2}{x-3}\right) \)

\[
\begin{align*}
\frac{x+2}{x-3} - 5 &= \frac{x+2}{x-3} - 5(x-3) \\
&= \frac{x+2 + 1}{x-3} \\
&= \frac{x+2 - 5(x-3)}{x+2 + l(x-3)} \\
&= \frac{x+2-5x+15}{x+2+x-3} \\
&= \frac{-4x+17}{2x-1} \quad \text{or} \quad \frac{-4x+17}{2x-1}
\end{align*}
\]

Now, \( 2x-1 \neq 0 \), so \( x \neq \frac{1}{2} \). Also, from the domain of \( g \), we know \( x \neq 3 \).

Domain of \( f \circ g : \{ x \mid x \neq \frac{1}{2}, x \neq 3 \} \).

b. \( (g \circ f)(x) = g(f(x)) = g\left(\frac{x-5}{x+1}\right) \)

\[
\begin{align*}
\frac{x-5}{x+1} + 2 &= \frac{x-5}{x+1} + 2(x+1) \\
&= \frac{x-5 + 3}{x+1} \\
&= \frac{x-5 + 2(x+1)}{x-5 - 3(x+1)} \\
&= \frac{3x-3}{-2x-8} \quad \text{or} \quad \frac{3x-3}{2x+8}
\end{align*}
\]

Now, \( -2x-8 \neq 0 \), so \( x \neq -4 \). Also, from the domain of \( f \), we know \( x \neq -1 \).

Domain of \( g \circ f : \{ x \mid x \neq -4, x \neq -1 \} \).

c. \( (f \circ f)(x) = f(f(x)) = f\left(\frac{x-5}{x+1}\right) \)

\[
\begin{align*}
\frac{x-5}{x+1} - 5 &= \frac{x-5}{x+1} - 5(x+1) \\
&= \frac{x-5 + 1}{x+1} \\
&= \frac{x-5 - 5(x+1)}{x-5 + l(x+1)} \\
&= \frac{-4x-10}{2x-4} = \frac{-2(2x+5)}{2(x-2)} = \frac{-2x+5}{x-2}
\end{align*}
\]

Now, \( x-2 \neq 0 \), so \( x \neq 2 \). Also, from the domain of \( f \), we know \( x \neq -1 \).

Domain of \( f \circ f : \{ x \mid x \neq -1, x \neq 2 \} \).

d. \( (g \circ g)(x) = g(g(x)) = g\left(\frac{x+2}{x-3}\right) \)

\[
\begin{align*}
\frac{x+2}{x-3} + 2 &= \frac{x+2}{x-3} + 2(x-3) \\
&= \frac{x+2 - 3}{x-3} = \frac{x+2}{x-3} - 3(x-3) \\
&= \frac{x+2 + 2(x-3)}{x+2 - 3(x-3)} = \frac{x+2 + 2x-6}{x+2 - 3x+9} \\
&= \frac{3x-4}{-2x+11} \quad \text{or} \quad \frac{3x-4}{2x-11}
\end{align*}
\]

Now, \(-2x+11 \neq 0\), so \( x \neq \frac{11}{2} \). Also, from the domain of \( g \), we know \( x \neq 3 \).

Domain of \( g \circ g : \{ x \mid x \neq \frac{11}{2}, x \neq 3 \} \).

44. \( f(x) = \frac{2x-1}{x-2} \quad g(x) = \frac{x+4}{2x-5} \)

The domain of \( f \) is \( \{ x \mid x \neq 2 \} \). The domain of \( g \) is \( \{ x \mid x \neq \frac{5}{2} \} \).

a. \( (f \circ g)(x) = f(g(x)) = f\left(\frac{x+4}{2x-5}\right) \)

\[
\begin{align*}
2\left(\frac{x+4}{2x-5}\right) - 1 &= \frac{2(x+4)}{2x-5} - 1 \\
&= \frac{2(x+4) - (2x-5)}{(2x-5)} \\
&= \frac{2(x+4) - 2(2x-5)}{(2x-5)} \\
&= \frac{2(x+4) - 2(2x-5)}{2x-5} \\
&= \frac{2x+8 - 2x+10}{2x-5} \\
&= \frac{13}{-3x+14} \quad \text{or} \quad \frac{13}{3x-14}
\end{align*}
\]

Now, \(-3x+14 \neq 0\), so \( x \neq \frac{14}{3} \). Also, from the domain of \( g \), we know \( x \neq \frac{5}{2} \).

Domain of \( f \circ g : \{ x \mid x \neq \frac{5}{2}, x \neq \frac{14}{3} \} \).
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b. \( (g \circ f)(x) = g(f(x)) = g\left(\frac{2x-1}{x-2}\right) \)
\[
= \frac{2x-1}{x-2} + 4
\]
\[
= \frac{2(2x-1)}{x-2} - 5
\]
\[
= \left(\frac{2x-1}{x-2}\right)(x-2)
\]
\[
= 2(2x-1) - 5(x-2)
\]
\[
= 2x-1 + 4(x-2)
\]
\[
= 2x-1 + 4x - 8
\]
\[
= 6x - 9
\]
Now, \(-x+8\neq0\), so \(x\neq8\). Also, from the domain of \(f\), we know \(x\neq2\).

Domain of \(f \circ g : \{x | x \neq 2, x \neq 8\}\).

c. \( (f \circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x-2}\right) \)
\[
= 2\left(\frac{2x-1}{x-2}\right) - 1
\]
\[
= \frac{4x-2}{x-2} - 1
\]
\[
= \left(\frac{2x-1}{x-2}\right)(x-2)
\]
\[
= 2(2x-1) - 1(x-2)
\]
\[
= 2x - 1 - 2(x-2)
\]
\[
= \frac{4x-2-x+2}{2x-1-2x+4} = \frac{3x}{3} = x
\]
From the domain of \(f\), we know \(x\neq2\).

Domain of \(f \circ f : \{x | x \neq 2\}\).

d. \( (g \circ g)(x) = g(g(x)) = g\left(\frac{x+4}{2x-5}\right) \)
\[
= \frac{x+4}{2x-5} + 4
\]
\[
= \frac{x+4}{2x-5} - 5
\]
\[
= \left(\frac{x+4}{2x-5}\right)(2x-5)
\]
\[
= \frac{2x+4}{2x-5} - 5
\]
\[
= \frac{2x+4+4(2x-5)}{2x-5}
\]
\[
= \frac{2x+4+8x-20}{2x-8+10x+25}
\]
\[
= \frac{9x-16}{8x-33}
\]
Now, \(8x-33\neq0\), so \(x\neq\frac{33}{8}\). Also, from the domain of \(g\), we know \(x\neq\frac{5}{2}\).

Domain of \(f \circ g : \{x | x \neq \frac{5}{2}, x \neq \frac{33}{8}\}\).

45. \( (f \circ g)(x) = f(g(x)) = f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) = x \)
\( (g \circ f)(x) = g(f(x)) = g(2x) = \frac{1}{2}(2x) = x \)

46. \( (f \circ g)(x) = f(g(x)) = f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) = x \)
\( (g \circ f)(x) = g(f(x)) = g(4x) = \frac{1}{4}(4x) = x \)

47. \( (f \circ g)(x) = f(g(x)) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x \)
\( (g \circ f)(x) = g(f(x)) = g\left(x^3\right) = \sqrt[3]{x^3} = x \)

48. \( (f \circ g)(x) = f(g(x)) = f(x-5) = x-5+5 = x \)
\( (g \circ f)(x) = g(f(x)) = g(x+5) = x+5-5 = x \)
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49. \((f \circ g)(x) = f(g(x))\)
   
   \[ = f\left(\frac{1}{2}(x+6)\right) = 2\left(\frac{1}{2}(x+6)\right) - 6 = x + 6 - 6 = x \]

   \((g \circ f)(x) = g(f(x))\)
   
   \[ = g\left(\frac{1}{2}(x+6)\right) = \frac{1}{2}(2x-6) = \frac{1}{2}(2x) = x \]

50. \((f \circ g)(x) = f(g(x))\)
   
   \[ = f\left(\frac{1}{3}(4-x)\right) = 4 - 3\left(\frac{1}{3}(4-x)\right) = 4 - 4 + x = x \]

   \((g \circ f)(x) = g(f(x))\)
   
   \[ = g\left(\frac{1}{3}(4-3x)\right) = \frac{1}{3}(3x) = \frac{1}{3}x \]

51. \((f \circ g)(x) = f(g(x))\)
   
   \[ = f\left(\frac{1}{a}(x-b)\right) = a\left(\frac{1}{a}(x-b)\right) + b = x - b + b = x \]

   \((g \circ f)(x) = g(f(x))\)
   
   \[ = g\left(\frac{1}{a}(x)\right) = \frac{1}{a}(ax + b) = \frac{1}{a}\left((ax + b) - b\right) = \frac{1}{a}(ax) = x \]

52. \((f \circ g)(x) = f(g(x))\)
   
   \[ = f\left(\frac{1}{x}\right) = 1 \cdot \frac{1}{x} = \frac{1}{x} \]

   \((g \circ f)(x) = g(f(x))\)
   
   \[ = g\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x \]

53. \(H(x) = (2x + 3)^3\)

   Answers may vary. One possibility is \(f(x) = x^4\), \(g(x) = 2x + 3\)

54. \(H(x) = \left(1 + x^2\right)^3\)

   Answers may vary. One possibility is \(f(x) = x^3\), \(g(x) = 1 + x^2\)

55. \(H(x) = \sqrt{x^2 + 1}\)

   Answers may vary. One possibility is \(f(x) = \sqrt{x}\), \(g(x) = x^2 + 1\)

56. \(H(x) = \sqrt{1 - x^2}\)

   Answers may vary. One possibility is \(f(x) = \sqrt{x}\), \(g(x) = 1 - x^2\)

57. \(H(x) = |2x + 1|\)

   Answers may vary. One possibility is \(f(x) = |x|\), \(g(x) = 2x + 1\)

58. \(H(x) = |2x^2 + 3|\)

   Answer may vary. One possibility is \(f(x) = |x|\), \(g(x) = 2x^2 + 3\)

59. \(f(x) = 2x^2 - 3x^2 + 4x - 1\) \(g(x) = 2\)

   \((f \circ g)(x) = f(g(x))\)
   
   \[ = f(2) = 2(2)^3 - 3(2)^2 + 4(2) - 1 = 16 - 12 + 8 - 1 = 11 \]

   \((g \circ f)(x) = g(f(x))\)
   
   \[ = g\left(2x^2 - 3x^2 + 4x - 1\right) = 2(2x^3 - 3x^2 + 4x - 1) = 2 \]
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60. \( f(x) = \frac{x+1}{x-1}, \quad x \neq 1 \)

\( (f \circ f)(x) = f(f(x)) \)
\[ = f\left(\frac{x+1}{x-1}\right) \]
\[ = \frac{x+1}{x-1} + 1 \]
\[ = \frac{x+1}{x-1} - 1 \]
\[ = \frac{x+1 + x-1}{x-1} \]
\[ = \frac{2x}{x-1} \]
\[ = \frac{2x}{x-1} \]
\[ = x, \quad x \neq 1 \]

61. \( f(x) = 2x^2 + 5 \) \quad \( g(x) = 3x + a \)

\( (f \circ g)(x) = f(g(x)) = f(3x + a) = 2(3x + a)^2 + 5 \)

When \( x = 0 \), \( (f \circ g)(0) = 23 \).

Solving: \( 2(3 \cdot 0 + a)^2 + 5 = 23 \)
\( 2a^2 + 5 = 23 \)
\( 2a^2 - 18 = 0 \)
\( 2(a+3)(a-3) = 0 \)
\( a = -3 \) or \( a = 3 \)

62. \( f(x) = 3x^2 - 7 \) \quad \( g(x) = 2x + a \)

\( (f \circ g)(x) = f(g(x)) = f(2x + a) = 3(2x + a)^2 - 7 \)

When \( x = 0 \), \( (f \circ g)(0) = 68 \).

Solving: \( 3(2 \cdot 0 + a)^2 - 7 = 68 \)
\( 3a^2 - 7 = 68 \)
\( 3a^2 = 75 = 0 \)
\( 3(a+5)(a-5) = 0 \)
\( a = -5 \) or \( a = 5 \)

63. a. \( (f \circ g)(x) = f(g(x)) \)
\[ = f(cx + d) \]
\[ = a(cx + d) + b \]
\[ = acx + ad + b \]

b. \( (g \circ f)(x) = g(f(x)) \)
\[ = g(ax + b) \]
\[ = c(ax + b) + d \]
\[ = acx + bc + d \]

c. Since the domain of \( f \) is the set of all real numbers and the domain of \( g \) is the set of all real numbers, the domains of both \( f \circ g \) and \( g \circ f \) are all real numbers.

d. \( (f \circ g)(x) = (g \circ f)(x) \)
\[ acx + ad + b = acx + bc + d \]
\[ ad + b = bc + d \]

Thus, \( f \circ g = g \circ f \) when \( ad + b = bc + d \).

64. a. \( (f \circ g)(x) = f(g(x)) \)
\[ = f(mx) \]
\[ = \frac{a(mx)+b}{c(mx)+d} \]
\[ = \frac{amx+b}{cmx+d} \]

b. \( (g \circ f)(x) = g(f(x)) \)
\[ = g\left(\frac{ax+b}{cx+d}\right) \]
\[ = m\left(\frac{ax+b}{cx+d}\right) \]
\[ = \frac{m(ax+b)}{cx+d} \]

Thus the domain of \( f \circ g \) is \( \left\{ x \mid x \neq \frac{d}{cm} \right\} \).

To find the domain of \( g \circ f \), we first recognize that the domain of \( f \) is \( \left\{ x \mid x \neq \frac{d}{c} \right\} \) and the domain of \( g \) is the set of all real numbers. Thus, the domain of \( g \circ f \) is also \( \left\{ x \mid x \neq \frac{d}{c} \right\} \).
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d. \( (f \circ g)(x) = (g \circ f)(x) \)
\[
\begin{align*}
\frac{amx + b}{cmx + d} &= \frac{m(ax + b)}{cx + d} \\
\frac{amx + b}{cmx + d} &= \frac{amx + bm}{cx + d}
\end{align*}
\]
\((amx + bm)(cmx + d) = (amx + b)(cx + d)\)

Now, this equation will only be true if \( m = 1 \). Thus, \( f \circ g = g \circ f \) when \( m = 1 \).

65. \( S(r) = 4\pi r^2 \quad r(t) = \frac{2}{3} t^3, \ t \geq 0 \)

\[
S(r(t)) = S\left(\frac{2}{3} t^3\right)
\]
\[
= 4\pi \left(\frac{2}{3} t^3\right)^2
\]
\[
= 4\pi \left(\frac{4}{9} t^6\right)
\]
\[
= \frac{16}{9} \pi t^6
\]

Thus, \( S(t) = \frac{16}{9} \pi t^6 \).

66. \( V(r) = \frac{4}{3} \pi r^3 \quad r(t) = \frac{2}{3} t^3, \ t \geq 0 \)

\[
V(r(t)) = V\left(\frac{2}{3} t^3\right)
\]
\[
= \frac{4}{3} \pi \left(\frac{2}{3} t^3\right)^3
\]
\[
= \frac{4}{3} \pi \left(\frac{8}{27} t^9\right)
\]
\[
= \frac{32}{81} \pi t^9
\]

Thus, \( V(t) = \frac{32}{81} \pi t^9 \).

67. \( N(t) = 100t - 5t^2, \ 0 \leq t \leq 10 \)

\[
C(N) = 15,000 + 8000N
\]

\[
C(N(t)) = C\left(100t - 5t^2\right)
\]
\[
= 15,000 + 8000 (100t - 5t^2)
\]
\[
= 15,000 + 800,000t - 40,000t^2
\]

Thus, \( C(t) = 15,000 + 800,000t - 40,000t^2 \).

68. \( A(r) = \pi r^2 \quad r(t) = 200\sqrt{t} \)

\[
A(r(t)) = A\left(200\sqrt{t}\right) = \pi \left(200\sqrt{t}\right)^2 = 40,000 \pi t^2
\]

Thus, \( A(t) = 40,000 \pi t^2 \).

69. \( p = -\frac{1}{4} x + 100, \ 0 \leq x \leq 400 \)

\[
\frac{1}{4} x = 100 - p
\]
\[
x = 4(100 - p)
\]

\[
C = \sqrt{x} + 600
\]
\[
= \sqrt{4(100 - p)} + 600
\]
\[
= \frac{2\sqrt{100 - p}}{25} + 600, \ 0 \leq p \leq 100
\]

Thus, \( C(p) = \frac{2\sqrt{100 - p}}{25} + 600, \ 0 \leq p \leq 100 \).

70. \( p = -\frac{1}{5} x + 200, \ 0 \leq x \leq 1000 \)

\[
\frac{1}{5} x = 200 - p
\]
\[
x = 5(200 - p)
\]

\[
C = \sqrt{x} + 400
\]
\[
= \sqrt{5(200 - p)} + 400
\]
\[
= \frac{\sqrt{1000 - 5p}}{10} + 400, \ 0 \leq p \leq 200
\]

Thus, \( C(p) = \frac{\sqrt{1000 - 5p}}{10} + 400, \ 0 \leq p \leq 200 \).

71. \( V = \pi r^2 h \quad h = 2r \)

\( V(r) = \pi r^2 (2r) = 2\pi r^3 \)

72. \( V = \frac{1}{3} \pi r^2 h \quad h = 2r \)

\( V(r) = \frac{1}{3} \pi r^2 (2r) = \frac{2}{3} \pi r^3 \)
73. \( f(x) = \) the number of Euros bought for \( x \) dollars; \\ \( g(x) = \) the number of yen bought for \( x \) Euros \\ a. \( f(x) = 0.7143x \) \\ b. \( g(x) = 137.402x \) \\ c. \( (g \circ f)(x) = g\left(f\left(x\right)\right) = g\left(0.7143x\right) = 137.402(0.7143x) = 98.1462486x \) \\ d. \( (g \circ f)(1000) = 98.1462486(1000) = 98,146.2486 \) yen 

74. a. Given \( C(F) = \frac{5}{9}(F - 32) \) and \\ \( K(C) = C + 273 \), we need to find \\ \( K(C(F)) \). \\ \( K(C(F)) = \left[\frac{5}{9}(F - 32)\right] + 273 \) \\ \( = \frac{5}{9}(F - 32) + 273 \) \\ \( = \frac{5}{9}F - \frac{160}{9} + 273 \) \\ \( = \frac{5}{9}F + 2297 \) or \( \frac{5F + 2297}{9} \) \\ b. \( K(C(80)) = \frac{5(80) + 2297}{9} = 299.7 \) kelvins 

75. a. \( f(p) = p - 200 \) \\ b. \( g(p) = 0.80p \) \\ c. \( (f \circ g)(p) = f\left(g\left(p\right)\right) = (0.80p) - 200 = 0.80p - 200 \) \\ This represents the final price when the rebate is issued on the sale price. \\ \( (g \circ f)(p) = g\left(f\left(p\right)\right) = 0.80\left(p - 200\right) = 0.80p - 160 \) \\ This represents the final price when the sale price is calculated after the rebate is given. \\ Applying the 20% first is a better deal since a larger portion will be removed up front.

76. Given that \( f \) and \( g \) are odd functions, we know that \( f(-x) = -f(x) \) and \( g(-x) = -g(x) \) for all \( x \) in the domain of \( f \) and \( g \), respectively. The composite function \( (f \circ g)(x) = f\left(g\left(x\right)\right) \) has the following property: \\ \( (f \circ g)(-x) = f\left(g\left(-x\right)\right) = f\left(-g\left(x\right)\right) = -f\left(g\left(x\right)\right) \) since \( g \) is odd \\ \( = -f\left(g\left(x\right)\right) \) since \( f \) is odd \\ \( = -(f \circ g)(x) \) \\ Thus, \( f \circ g \) is an odd function.

77. Given that \( f \) is odd and \( g \) is even, we know that \( f(-x) = -f(x) \) and \( g(-x) = g(x) \) for all \( x \) in the domain of \( f \) and \( g \), respectively. The composite function \( (f \circ g)(x) = f\left(g\left(x\right)\right) \) has the following property: \\ \( (f \circ g)(-x) = f\left(g\left(-x\right)\right) = f\left(g\left(x\right)\right) \) since \( g \) is even \\ \( = (f \circ g)(x) \) \\ Thus, \( f \circ g \) is an even function.

The composite function \( (g \circ f)(x) = g\left(f\left(x\right)\right) \) has the following property: \\ \( (g \circ f)(-x) = g\left(f\left(-x\right)\right) = g\left(-f\left(x\right)\right) \) since \( f \) is odd \\ \( = g\left(f\left(x\right)\right) \) since \( g \) is even \\ \( = (g \circ f)(x) \) \\ Thus, \( g \circ f \) is an even function.

Section 5.2

1. The set of ordered pairs is a function because there are no ordered pairs with the same first element and different second elements.

2. The function \( f(x) = x^2 \) is increasing on the interval \((0, \infty)\). It is decreasing on the interval \((-\infty, 0)\).

3. The function is not defined when \( x^2 + 3x - 18 = 0 \). \\ Solve: \( x^2 + 3x - 18 = 0 \) \\ \( (x + 6)(x - 3) = 0 \) \\ \( x = -6 \) or \( x = 3 \) \\ The domain is \( \{x \mid x \neq -6, x \neq 3\} \).
Section 5.2: One-to-One Functions; Inverse Functions

4. \[
\frac{1}{x} + 1 = \frac{1}{x^2 - 1} \quad \Rightarrow \quad \frac{1}{x^2} - 1 = \frac{1}{x} - x^2 = \frac{1}{x} - \frac{x^2}{x^2} = \frac{1}{x} - \frac{x^2}{x^2} = \frac{x^2}{x^2}.
\]

5. \[f(x_1) \neq f(x_2)\]

6. one-to-one

7. 3

8. \[y = x\]

9. \([4, \infty)\]

10. True

11. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

12. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

13. The function is not one-to-one because there are two different inputs, 20 Hours and 50 Hours, that correspond to the same output, $200.

14. The function is not one-to-one because there are two different inputs, John and Chuck, that correspond to the same output, Phoebe.

15. The function is not one-to-one because there are two distinct inputs, 2 and −3, that correspond to the same output.

16. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

17. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

18. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

19. The function \(f\) is one-to-one because every horizontal line intersects the graph at exactly one point.

20. The function \(f\) is one-to-one because every horizontal line intersects the graph at exactly one point.

21. The function \(f\) is not one-to-one because there are horizontal lines (for example, \(y = 1\)) that intersect the graph at more than one point.

22. The function \(f\) is not one-to-one because there are horizontal lines (for example, \(y = 1\)) that intersect the graph at more than one point.

23. The function \(f\) is one-to-one because every horizontal line intersects the graph at exactly one point.

24. The function \(f\) is not one-to-one because the horizontal line \(y = 2\) intersects the graph at more than one point.

25. To find the inverse, interchange the elements in the domain with the elements in the range:

<table>
<thead>
<tr>
<th>Annual Rainfall</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>460.00</td>
<td>Mt Waialeale, Hawaii</td>
</tr>
<tr>
<td>202.01</td>
<td>Monrovia, Liberia</td>
</tr>
<tr>
<td>196.46</td>
<td>Pago Pago, American Samoa</td>
</tr>
<tr>
<td>191.02</td>
<td>Moulmein, Burma</td>
</tr>
<tr>
<td>182.87</td>
<td>Lae, Papua New Guinea</td>
</tr>
</tbody>
</table>

Domain: \{460.00, 202.01, 196.46, 191.02, 182.87\}

Range: \{Mt Waialeale, Monrovia, Pago Pago, Moulmein, Lea\}

26. To find the inverse, interchange the elements in the domain with the elements in the range:

<table>
<thead>
<tr>
<th>Domestic Gross</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>$461</td>
<td>Star Wars</td>
</tr>
<tr>
<td>$431</td>
<td>The Phantom Menace</td>
</tr>
<tr>
<td>$400</td>
<td>E.T. the Extra Terrestrial</td>
</tr>
<tr>
<td>$357</td>
<td>Jurassic Park</td>
</tr>
<tr>
<td>$330</td>
<td>Forrest Gump</td>
</tr>
</tbody>
</table>

Domain: \{$461, $431, $400, $357, $330\} (in millions)
Chapter 5: Exponential and Logarithmic Functions

Range: {Star Wars, The Phantom Menace, E.T. the Extra Terrestrial, Jurassic Park, Forrest Gump}

27. To find the inverse, interchange the elements in the domain with the elements in the range:

<table>
<thead>
<tr>
<th>Monthly Cost of Life Insurance</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7.09</td>
<td>30</td>
</tr>
<tr>
<td>$8.40</td>
<td>40</td>
</tr>
<tr>
<td>$11.29</td>
<td>45</td>
</tr>
</tbody>
</table>

Domain: {$7.09, $8.40, $11.29}
Range: {30, 40, 45}

28. To find the inverse, interchange the elements in the domain with the elements in the range:

<table>
<thead>
<tr>
<th>Unemployment Rate</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>11%</td>
<td>Virginia</td>
</tr>
<tr>
<td>5.5%</td>
<td>Nevada</td>
</tr>
<tr>
<td>5.1%</td>
<td>Tennessee</td>
</tr>
<tr>
<td>6.3%</td>
<td>Texas</td>
</tr>
</tbody>
</table>

Domain: {11%, 5.5%, 5.1%, 6.3%}
Range: {Virginia, Nevada, Tennessee, Texas}

29. Interchange the entries in each ordered pair:
{(5, -3), (9, -2), (2, -1), (11, 0), (-5, 1)}

Domain: {5, 9, 2, 11, -5}
Range: {-3, -2, -1, 0, 1}

30. Interchange the entries in each ordered pair:
{(2, -2), (6, -1), (8, 0), (-3, 1), (9, 2)}

Domain: {2, 6, 8, -3, 9}
Range: {-2, -1, 0, 1, 2}

31. Interchange the entries in each ordered pair:
{(1, -2), (2, -3), (0, -10), (9, 1), (4, 2)}

Domain: {1, 2, 0, 9, 4}
Range: {-2, -3, -10, 1, 2}

32. Interchange the entries in each ordered pair:
{(-8, -2), (-1, -1), (0, 0), (1, 1), (8, 2)}

Domain: {-8, -1, 0, 1, 8}
Range: {-2, -1, 0, 1, 2}

33. \( f(x) = 3x + 4; \quad g(x) = \frac{1}{3}(x - 4) \)

\[
 f \left( g(x) \right) = f \left( \frac{1}{3}(x - 4) \right) \\
= 3 \left( \frac{1}{3}(x - 4) \right) + 4 \\
= (x - 4) + 4 \\
= x \\

 g \left( f(x) \right) = g(3x + 4) \\
= \frac{1}{3}(3x + 4 - 4) \\
= \frac{1}{3}(3x) \\
= x \\

Thus, \( f \) and \( g \) are inverses of each other.

34. \( f(x) = 3 - 2x; \quad g(x) = \frac{1}{2}(x - 3) \)

\[
 f \left( g(x) \right) = f \left( \frac{1}{2}(x - 3) \right) \\
= 3 - 2\left( \frac{1}{2}(x - 3) \right) \\
= 3 + (x - 3) \\
= x \\

 g \left( f(x) \right) = g(3 - 2x) \\
= \frac{1}{2}(3 - 2x - 3) \\
= \frac{1}{2}(-2x) \\
= x \\

Thus, \( f \) and \( g \) are inverses of each other.

35. \( f(x) = 4x - 8; \quad g(x) = \frac{x}{4} + 2 \)

\[
 f \left( g(x) \right) = f \left( \frac{x}{4} + 2 \right) \\
= 4 \left( \frac{x}{4} + 2 \right) - 8 \\
= x + 8 - 8 \\
= x \\

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Section 5.2: One-to-One Functions; Inverse Functions

\[ g \left( f(x) \right) = g \left( 4x - 8 \right) = \frac{4x - 8}{4} + 2 = x - 2 + 2 = x \]

Thus, \( f \) and \( g \) are inverses of each other.

36. \( f(x) = 2x + 6; \quad g(x) = \frac{1}{2}x - 3 \)

\[ f \left( g(x) \right) = f \left( \frac{1}{2}x - 3 \right) = 2 \left( \frac{1}{2}x - 3 \right) + 6 = x - 6 + 6 = x \]

\[ g \left( f(x) \right) = g(2x + 6) = \frac{1}{2}(2x + 6) - 3 = x + 3 - 3 = x \]

Thus, \( f \) and \( g \) are inverses of each other.

37. \( f(x) = x^3 - 8; \quad g(x) = \sqrt[3]{x + 8} \)

\[ f \left( g(x) \right) = f \left( \sqrt[3]{x + 8} \right) = \left( \sqrt[3]{x + 8} \right)^3 - 8 = x + 8 - 8 = x \]

\[ g \left( f(x) \right) = g(x^3 - 8) = \sqrt[3]{(x^3 - 8) + 8} = \sqrt[3]{x^3} = x \]

Thus, \( f \) and \( g \) are inverses of each other.

38. \( f(x) = (x - 2)^2, x \geq 2; \quad g(x) = \sqrt{x + 2} \)

\[ f \left( g(x) \right) = f \left( \sqrt{x + 2} \right) = \left( \sqrt{x + 2} \right)^2 = \left( \sqrt{x} \right)^2 = x \]

\[ g \left( f(x) \right) = g \left( \left( x - 2 \right)^2 \right) = \sqrt{\left( x - 2 \right)^2} + 2 = x - 2 + 2 = x \]

Thus, \( f \) and \( g \) are inverses of each other.

39. \( f(x) = \frac{1}{x}; \quad g(x) = \frac{1}{x} \)

\[ f \left( g(x) \right) = f \left( \frac{1}{x} \right) = \frac{1}{\frac{1}{x}} = \frac{x}{1} = x \]

\[ g \left( f(x) \right) = g \left( \frac{1}{x} \right) = \frac{1}{\frac{1}{x}} = \frac{x}{1} = x \]

Thus, \( f \) and \( g \) are inverses of each other.

40. \( f(x) = x; \quad g(x) = x \)

\[ f \left( g(x) \right) = f \left( x \right) = x \]

\[ g \left( f(x) \right) = g \left( x \right) = x \]

Thus, \( f \) and \( g \) are inverses of each other.

41. \( f(x) = \frac{2x + 3}{x + 4}; \quad g(x) = \frac{4x - 3}{2 - x} \)

\[ f \left( g(x) \right) = f \left( \frac{4x - 3}{2 - x} \right), x \neq 2 \]

\[ = \frac{2 \left( \frac{4x - 3}{2 - x} \right) + 3}{\frac{4x - 3}{2 - x} + 4} = \frac{2 \left( \frac{4x - 3}{2 - x} \right) + 3}{\frac{4x - 3 + 4}{2 - x}} = \frac{4x - 3 + 4}{2 - x} \frac{2 - x}{2 - x} \]

\[ = 2 \left( \frac{4x - 3}{2 - x} \right) + 3(2 - x) = \frac{8x - 6 + 6 - 3x}{4x - 3 + 8 - 4x} = \frac{5x}{5} = x \]
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\[ g(f(x)) = g\left(\frac{2x+3}{x+4}\right), \quad x \neq -4 \]

\[
= \frac{4}{2} \cdot \frac{2x+3}{x+4} - 3 \\
= 2 \cdot \frac{2x+3}{x+4} - 3 \\
= \left(\frac{4}{2} \cdot \frac{2x+3}{x+4} - 3\right)(x+4) \\
= \frac{2-2x+3}{x+4} (x+4) \\
= \frac{4(2x+3) - 3(x+4)}{2(x+4) - (2x+3)} \\
= \frac{8x + 12 - 3x - 12}{2x + 8 - 2x - 3} \\
= \frac{5x}{5} \\
= x
\]

Thus, \( f \) and \( g \) are inverses of each other.

42. \( f(x) = \frac{x-5}{2x+3} \); \( g(x) = \frac{3x+5}{1-2x} \)

\[ f(g(x)) = f\left(\frac{3x+5}{1-2x}\right), \quad x \neq \frac{1}{2} \]

\[
= \frac{3x+5}{1-2x} - 5 \\
= 2 \cdot \frac{3x+5}{1-2x} + 3 \\
= \left(\frac{3x+5}{1-2x} - 5\right)(1-2x) \\
= \frac{2(3x+5)}{1-2x} + 3(1-2x) \\
= \frac{3x+5 - 5(1-2x)}{2(3x+5) + 3(1-2x)} \\
= \frac{3x+5 - 5 + 10x}{6x + 10 + 3 - 6x} \\
= \frac{13x}{13} \\
= x
\]

Thus, \( f \) and \( g \) are inverses of each other.

43. Graphing the inverse:

\[
\begin{align*}
\text{Graph 1:} & \quad y = x \\
\text{Graph 2:} & \quad (2, 1) \\
\text{Graph 3:} & \quad (1, 0) \\
\text{Graph 4:} & \quad (0, -1) \\
\end{align*}
\]

44. Graphing the inverse:

\[
\begin{align*}
\text{Graph 1:} & \quad y = x \\
\text{Graph 2:} & \quad (0, 1) \\
\text{Graph 3:} & \quad (-1, 0) \\
\text{Graph 4:} & \quad (-2, -2) \\
\end{align*}
\]
Section 5.2: One-to-One Functions; Inverse Functions

45. Graphing the inverse:
   \[ f^{-1}(x) = \frac{1}{3}x \]
   \[ f^{-1}(1) = \frac{1}{3} \]
   \[ f^{-1}(-1) = -\frac{1}{3} \]

46. Graphing the inverse:
   \[ f^{-1}(x) = \frac{1}{4}x \]
   \[ f^{-1}(1) = \frac{1}{4} \]
   \[ f^{-1}(-1) = -\frac{1}{4} \]

47. Graphing the inverse:
   \[ f^{-1}(x) = \frac{x}{3} \]
   \[ f^{-1}(1) = \frac{1}{3} \]
   \[ f^{-1}(-1) = -\frac{1}{3} \]

48. Graphing the inverse:
   \[ f^{-1}(x) = \frac{1}{4}x \]
   \[ f^{-1}(1) = \frac{1}{4} \]
   \[ f^{-1}(-1) = -\frac{1}{4} \]

49. \[ f(x) = 3x \]
    \[ y = 3x \]
    \[ x = 3y \]
    \[ y = \frac{x}{3} \]
    \[ f^{-1}(x) = \frac{1}{3}x \]

Verifying:
\[ f\left(f^{-1}(x)\right) = f\left(\frac{1}{3}x\right) = 3\left(\frac{1}{3}x\right) = x \]
\[ f^{-1}(f(x)) = f^{-1}(3x) = \frac{1}{3}(3x) = x \]

50. \[ f(x) = -4x \]
    \[ y = -4x \]
    \[ x = -4y \]
    \[ y = \frac{x}{-4} \]
    \[ f^{-1}(x) = -\frac{1}{4}x \]

Verifying:
\[ f\left(f^{-1}(x)\right) = f\left(-\frac{1}{4}x\right) = -4\left(-\frac{1}{4}x\right) = x \]
\[ f^{-1}(f(x)) = f^{-1}(-4x) = \frac{1}{4}(-4x) = x \]

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51. \( f(x) = 4x + 2 \)
   \[ y = 4x + 2 \]
   \[ x = 4y + 2 \quad \text{Inverse} \]
   \[ 4y = x - 2 \]
   \[ y = \frac{x - 2}{4} \]
   \[ y = \frac{x}{4} - \frac{1}{2} \]
   \[ f^{-1}(x) = \frac{x}{4} - \frac{1}{2} \]

   Verifying:
   \[ f \left( f^{-1}(x) \right) = f \left( \frac{x}{4} - \frac{1}{2} \right) = 4 \left( \frac{x}{4} - \frac{1}{2} \right) + 2 \]
   \[ = x - 2 + 2 = x \]
   \[ f^{-1} \left( f(x) \right) = f^{-1} \left( 4x + 2 \right) = \frac{4x + 2}{4} - \frac{1}{2} \]
   \[ = x + \frac{1}{2} - \frac{1}{2} = x \]

52. \( f(x) = 1 - 3x \)
   \[ y = 1 - 3x \]
   \[ x = 1 - 3y \quad \text{Inverse} \]
   \[ 3y = 1 - x \]
   \[ y = \frac{1 - x}{3} \]
   \[ f^{-1}(x) = \frac{1 - x}{3} \]

   Verifying:
   \[ f \left( f^{-1}(x) \right) = f \left( \frac{1 - x}{3} \right) = 1 - 3 \left( \frac{1 - x}{3} \right) \]
   \[ = 1 - (1 - x) = x \]
   \[ f^{-1} \left( f(x) \right) = f^{-1} \left( 1 - 3x \right) = \frac{1 - (1 - 3x)}{3} = \frac{3x}{3} = x \]

53. \( f(x) = x^3 - 1 \)
   \[ y = x^3 - 1 \]
   \[ x = y^3 - 1 \quad \text{Inverse} \]
   \[ y^3 = x + 1 \]
   \[ y = \sqrt[3]{x + 1} \]
   \[ f^{-1}(x) = \sqrt[3]{x + 1} \]

   Verifying:
   \[ f \left( f^{-1}(x) \right) = f \left( \sqrt[3]{x + 1} \right) = \left( \sqrt[3]{x + 1} \right)^3 - 1 \]
   \[ = x + 1 - 1 = x \]
   \[ f^{-1} \left( f(x) \right) = f^{-1} \left( x^3 - 1 \right) = \sqrt[3]{x^3 - 1} + 1 \]
   \[ = \sqrt[3]{x^3} = x \]

54. \( f(x) = x^3 + 1 \)
   \[ y = x^3 + 1 \]
   \[ x = y^3 + 1 \quad \text{Inverse} \]
   \[ y^3 = x - 1 \]
   \[ y = \sqrt[3]{x - 1} \]
   \[ f^{-1}(x) = \sqrt[3]{x - 1} \]
Verifying:
\[ f\left(f^{-1}(x)\right) = f\left(\sqrt[3]{x-1}\right) = \left(\sqrt[3]{x-1}\right)^3 + 1 = x + 1 = x \]
\[ f^{-1}(f(x)) = f^{-1}\left(x^3 + 1\right) = \sqrt[3]{x^3 + 1} - 1 = \sqrt[3]{x} = x \]
\[ f(x) = x^3 + 1 \]
\[ f^{-1}(x) = \sqrt[3]{x} - 1 \]

55. \( f(x) = x^2 + 4, \ x \geq 0 \)

\[ y = x^2 + 4, \ x \geq 0 \]
\[ x = y^2 + 4, \ y \geq 0 \quad \text{Inverse} \]
\[ y^2 = x - 4, \ x \geq 4 \]
\[ y = \sqrt{x - 4}, \ x \geq 4 \]
\[ f^{-1}(x) = \sqrt{x - 4}, \ x \geq 4 \]

Verifying:
\[ f\left(f^{-1}(x)\right) = f\left(\sqrt[3]{x-4}\right) = \left(\sqrt[3]{x} - 4\right)^2 + 4 = x - 4 + 4 = x \]
\[ f^{-1}(f(x)) = f^{-1}\left(x^2 + 4\right) = \sqrt[3]{x^2 + 4} - 4 = \sqrt[3]{x^2} = x \]
\[ f(x) = x^2 + 4, \ y = x \]

56. \( f(x) = x^2 + 9, \ x \geq 0 \)

\[ y = x^2 + 9, \ x \geq 0 \]
\[ x = y^2 + 9, \ y \geq 0 \quad \text{Inverse} \]
\[ y^2 = x - 9, \ x \geq 9 \]
\[ y = \sqrt{x - 9}, \ x \geq 9 \]
\[ f^{-1}(x) = \sqrt{x - 9}, \ x \geq 9 \]

Verifying:
\[ f\left(f^{-1}(x)\right) = f\left(\sqrt{x-9}\right) = \left(\sqrt{x} - 9\right)^2 + 9 = x - 9 + 9 = x \]
\[ f^{-1}(f(x)) = f^{-1}\left(x^2 + 9\right) = \sqrt{x^2 + 9} - 9 = \sqrt{x^2} = x \]
\[ f(x) = x^2 + 9, \ y = x \]

57. \( f(x) = \frac{4}{x} \)

\[ y = \frac{4}{x} \]
\[ x = \frac{y}{x} \quad \text{Inverse} \]
\[ xy = 4 \]
\[ y = \frac{4}{x} \]
\[ f^{-1}(x) = \frac{4}{x} \]

Verifying:
\[ f\left(f^{-1}(x)\right) = f\left(\frac{4}{x}\right) = \frac{4}{\frac{4}{x}} = 4 \left(\frac{x}{4}\right) = x \]
Chapter 5: Exponential and Logarithmic Functions

58. \( f(x) = \frac{3}{x} \)

\[ y = \frac{3}{x} \]

\[ x = \frac{3}{y} \quad \text{Inverse} \]

\( xy = 3 \)

\[ y = \frac{3}{x} \]

\( f^{-1}(x) = \frac{3}{x} \)

Verifying:

\[ f\left( f^{-1}(x) \right) = f\left( \frac{3}{x} \right) = -\frac{3}{3} = -3 \cdot \left( -\frac{x}{3} \right) = x \]

\[ f^{-1}(f(x)) = f^{-1}\left( \frac{3}{x} \right) = -\frac{3}{3} = -3 \cdot \left( -\frac{x}{3} \right) = x \]

59. \( f(x) = \frac{1}{x-2} \)

\[ y = \frac{1}{x-2} \]

\[ x = \frac{1}{y-2} \quad \text{Inverse} \]

\[ xy - 2x = 1 \]

\[ xy = 2x + 1 \]

\[ y = \frac{2x + 1}{x} \]

\[ f^{-1}(x) = \frac{2x + 1}{x} \]

Verifying:

\[ f\left( f^{-1}(x) \right) = f\left( \frac{2x + 1}{x} \right) = \frac{1}{\frac{2x + 1}{x} - 2} \]

\[ = \frac{1 \cdot x}{\left( \frac{2x + 1}{x} - 2 \right) x} \]

\[ = \frac{x}{\frac{x}{2x + 1 - 2x}} \]

\[ = \frac{x}{\frac{x}{1}} \]

\[ = x = x \]

\[ f^{-1}(f(x)) = f^{-1}\left( \frac{1}{x-2} \right) = \frac{2 \left( \frac{1}{x-2} \right) + 1}{\frac{1}{x-2}} \]

\[ = \frac{\left( 2 \cdot \frac{1}{x-2} \right) + 1 (x-2)}{\frac{1}{x-2}} \]

\[ = \frac{2 + (x-2)}{1} \]

\[ = \frac{x}{1} = x \]
Section 5.2: One-to-One Functions; Inverse Functions

60. \( f(x) = \frac{4}{x+2} \)
   \[ y = \frac{4}{x+2} \]
   \[ x = \frac{4}{y+2} \]
   Inverse
   \[ x(y + 2) = 4 \]
   \[ xy + 2x = 4 \]
   \[ xy = 4 - 2x \]
   \[ y = \frac{4 - 2x}{x} \]
   \[ f^{-1}(x) = \frac{4 - 2x}{x} \] or \( f^{-1}(x) = \frac{4}{x} - 2 \)

Verifying:
\[ f(f^{-1}(x)) = f\left(\frac{4 - 2x}{x}\right) = \frac{4}{4 - 2x + 2} \]
\[ \frac{4x}{4 - 2x + 2} = \frac{4x}{4 - 2x + 2} = \frac{4x}{4} = x \]

\[ f^{-1}(f(x)) = f^{-1}\left(\frac{4}{x + 2}\right) = \frac{4 - 2\left(\frac{4}{x + 2}\right)}{4 - 2\left(\frac{4}{x + 2}\right)}(x + 2) \]
\[ \frac{4x + 8 - 8}{4} = \frac{4x}{4} = x \]

61. \( f(x) = \frac{2}{3 + x} \)
   \[ y = \frac{2}{3 + x} \]
   \[ x = \frac{2}{3 + y} \]
   Inverse
   \[ x(3 + y) = 2 \]
   \[ 3x + xy = 2 \]
   \[ xy = 2 - 3x \]
   \[ y = \frac{2 - 3x}{x} \]
   \[ f^{-1}(x) = \frac{2 - 3x}{x} \]

Verifying:
\[ f\left(f^{-1}(x)\right) = f\left(\frac{2 - 3x}{x}\right) = \frac{2}{3 + \frac{2 - 3x}{x}} \]
\[ \frac{2x}{3 + 2 - 3x} = \frac{2x}{3x + 2 - 3x} = \frac{2x}{2} = x \]

\[ f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{2}{3 + x}\right) = \frac{2 - 3\left(\frac{2}{3 + x}\right)}{3 + x} \]
\[ \frac{2 - 3\left(\frac{2}{3 + x}\right)}{3 + x} = \frac{2}{3 + x} \]
\[ = \frac{2(3 + x) - 3(2)}{2} = \frac{6 + 2x - 6}{2} = \frac{2x}{2} = x \]
62. \( f(x) = \frac{4}{2 - x} \)

\[ y = \frac{4}{2 - x} \]

\[ x = \frac{4}{2 - y} \text{ Inverse} \]

\[ x(2 - y) = 4 \]
\[ 2x - xy = 4 \]
\[ xy = 2x - 4 \]

\[ y = \frac{2x - 4}{x} \]
\[ y = 2 - \frac{4}{x} \]

\[ f^{-1}(x) = 2 - \frac{4}{x} \]

Verifying:
\[
\begin{align*}
\left(f \left(f^{-1}(x)\right)\right) &= f\left(2 - \frac{4}{x}\right) \\
&= \frac{4}{2 - \left(2 - \frac{4}{x}\right)} \\
&= \frac{4}{2 - 2 + \frac{4}{x}} \\
&= \frac{4}{\frac{4}{x}} \\
&= \frac{4x}{4} \\
&= x
\end{align*}
\]

63. \( f(x) = \frac{3x}{x + 2} \)

\[ y = \frac{3x}{x + 2} \]

\[ x = \frac{3y}{y + 2} \text{ Inverse} \]

\[ x(y + 2) = 3y \]
\[ xy + 2x = 3y \]
\[ xy - 3y = -2x \]
\[ y(x - 3) = -2x \]
\[ y = -\frac{2x}{x - 3} \]

\[ f^{-1}(x) = -\frac{2x}{x - 3} \]

64. \( f(x) = \frac{-2x}{x - 1} \)

\[ y = \frac{-2x}{x - 1} \]

\[ x = \frac{2y}{y - 1} \text{ Inverse} \]

\[ x(y - 1) = -2y \]
\[ xy - x = -2y \]
\[ xy + 2y = x \]
\[ y(x + 2) = x \]
\[ y = \frac{x}{x + 2} \]

\[ f^{-1}(x) = \frac{x}{x + 2} \]
Section 5.2: One-to-One Functions; Inverse Functions

Verifying:

\[ f^{-1}(x) = f\left(\frac{x}{3x-2}\right) = \frac{2\left(\frac{x}{3x-2}\right)}{3\left(\frac{x}{3x-2}\right) - 1} \]

\[
= \frac{2x}{3(3x-2)}(3x-2)
\]

\[
= \frac{2x}{3(3x-2)} - \frac{2x}{3(3x-2)} - 1
\]

\[
= \frac{2x}{3x - (3x-2)}
\]

\[
= \frac{2x}{2} = x
\]

Verifying:

\[ f^{-1}(f(x)) = f\left(\frac{x}{x+2}\right) = \frac{2\left(\frac{x}{x+2}\right)}{x + 2 - 1} \]

\[
= \frac{2x}{(x+2)(x+2) - 1}
\]

\[
= \frac{2x}{x^2 + 4x + 4 - 1}
\]

\[
= \frac{2x}{x^2 + 4x + 3}
\]

\[
= \frac{2x}{x+1}
\]

\[
= \frac{-2x}{-2x + 2x - 2}
\]

\[
= -2x
\]

\[
= x
\]

65. \[ f(x) = \frac{2x}{3x-1} \]

\[
y = \frac{2x}{3x-1}
\]

\[
x = \frac{2y}{3y-1} \quad \text{Inverse}
\]

\[
3xy - x = 2y
\]

\[
3xy - 2y = x
\]

\[
y(3x-2) = x
\]

\[
y = \frac{x}{3x-2}
\]

\[
f^{-1}(x) = \frac{x}{3x-2}
\]

66. \[ f(x) = \frac{-3x+1}{x} \]

\[
y = \frac{-3x+1}{x}
\]

\[
x = \frac{3y+1}{y} \quad \text{Inverse}
\]

\[
xy = -(3y+1)
\]

\[
xy = -3y - 1
\]

\[
xy + 3y = -1
\]

\[
y(x+3) = -1
\]

\[
y = -\frac{1}{x+3}
\]

\[
f^{-1}(x) = \frac{-1}{x+3}
\]
Chapter 5: Exponential and Logarithmic Functions

Verifying:

\[ f \left( f^{-1}(x) \right) = f \left( \frac{-1}{x + 3} \right) \]
\[ = 3 \left( \frac{-1}{x + 3} \right) + 1 = \frac{-3}{x + 3} + 1 \]
\[ = \frac{-3}{x + 3} \cdot \frac{x + 3}{1} \]
\[ = -3 + (x + 3) \]
\[ = x \]

\[ f^{-1}(f(x)) = f^{-1}\left( \frac{3x + 1}{x - 1} \right) \]
\[ = \frac{-1}{\left( \frac{3x + 1}{x - 3} \right) + 3} = \frac{1}{\frac{3x + 1}{x - 3} + 3} \]
\[ = \frac{1}{\frac{3x + 1}{x - 3} \cdot \frac{x - 3}{x - 3}} = \frac{x - 3}{3x + 1 - 3x} \]
\[ = \frac{x}{1} = x \]

67. \[ f(x) = \frac{3x + 4}{2x - 3} \]
\[ y = \frac{3x + 4}{2x - 3} \]
\[ x = \frac{3y + 4}{2y - 3} \]
\[ x(2y - 3) = 3y + 4 \]
\[ 2xy - 3x = 3y + 4 \]
\[ 2xy - 3y = 3x + 4 \]
\[ y(2x - 3) = 3x + 4 \]
\[ y = \frac{3x + 4}{2x - 3} \]
\[ f^{-1}(x) = \frac{3x + 4}{2x - 3} \]

Verifying:

\[ f \left( f^{-1}(x) \right) = f \left( \frac{3x + 4}{2x - 3} \right) = \frac{3}{2} \left( \frac{3x + 4}{2x - 3} \right) + 4 \]
\[ = \frac{3}{2} \left( \frac{3x + 4}{2x - 3} \right) + 4(2x - 3) \]
\[ = \frac{3(3x + 4) + 4(2x - 3)}{2(3x + 4) - 3(2x - 3)} \]
\[ = \frac{9x + 12 + 8x - 12}{6x + 8 - 6x + 9} = \frac{17x}{17} = x \]

68. \[ f(x) = \frac{2x - 3}{x + 4} \]
\[ y = \frac{2x - 3}{x + 4} \]
\[ x = \frac{2y - 3}{y + 4} \]
\[ x(y + 4) = 2y - 3 \]
\[ xy + 4x = 2y - 3 \]
\[ xy - 2y = -4x - 3 \]
\[ y(x - 2) = -4(x + 3) \]
\[ y = \frac{-4(x + 3)}{x - 2} = \frac{4x + 3}{2 - x} \]
\[ f^{-1}(x) = \frac{4x + 3}{2 - x} \]
Section 5.2: One-to-One Functions; Inverse Functions

Verifying:

\[ f\left(f^{-1}(x)\right) = f\left(\frac{4x + 3}{2 - x}\right) = \frac{2\left(\frac{4x + 3}{2 - x}\right) - 3}{\frac{4x + 3}{2 - x} + 4} \]
\[ = \frac{2(4x + 3) - 3(2 - x)}{4x + 3 + 4(2 - x)} \]
\[ = \frac{8x + 6 - 6 + 3x}{4x + 3 + 8 - 4x} \]
\[ = \frac{11x}{11} \]
\[ = x \]

Verifying:

\[ f\left(f^{-1}(x)\right) = f\left(\frac{-2x + 3}{x - 2}\right) = \frac{2\left(\frac{-2x + 3}{x - 2}\right) + 3}{\frac{-2x + 3}{x - 2} + 2} \]
\[ = \frac{\left(2\left(\frac{-2x + 3}{x - 2}\right) + 3\right)(x - 2)}{\left(\frac{-2x + 3}{x - 2} + 2\right)(x - 2)} \]
\[ = \frac{2(-2x + 3) + 3(x - 2)}{-2x + 3 + 2(x - 2)} \]
\[ = \frac{-4x + 6 + 3x - 6}{-2x + 3 + 2x - 4} \]
\[ = \frac{-x}{-1} = x \]

Verifying:

\[ f\left(f^{-1}(x)\right) = f\left(\frac{2x + 3}{x + 2}\right) = \frac{-2\left(\frac{2x + 3}{x + 2}\right) + 3}{\frac{2x + 3}{x + 2} - 2} \]
\[ = \frac{-2\left(\frac{2x + 3}{x + 2}\right) + 3}{\frac{2x + 3}{x + 2} - 2} \]
\[ = \frac{-2\left(\frac{2x + 3}{x + 2}\right) + 3}{\frac{2x + 3}{x + 2} - 2} \]
\[ = \frac{-4x - 6 + 3x + 6}{2x + 3 - 2x - 4} \]
\[ = \frac{-x}{-1} = x \]

69. \[ f(x) = \frac{2x + 3}{x + 2} \]
\[ y = \frac{2x + 3}{x + 2} \]
\[ x = \frac{2y + 3}{y + 2} \]
Inverse
\[ xy + 2x = 2y + 3 \]
\[ xy - 2y = -2x + 3 \]
\[ y(x - 2) = -2x + 3 \]
\[ y = \frac{-2x + 3}{x - 2} \]
\[ f^{-1}(x) = \frac{-2x + 3}{x - 2} \]

70. \[ f(x) = \frac{-3x - 4}{x - 2} \]
\[ y = \frac{-3x - 4}{x - 2} \]
\[ x = \frac{-3y - 4}{y - 2} \]
Inverse
\[ x(y - 2) = -3y - 4 \]
\[ xy - 2x = -3y - 4 \]
\[ xy + 3y = 2x - 4 \]
\[ y(x + 3) = 2x - 4 \]
\[ y = \frac{2x - 4}{x + 3} \]
\[ f^{-1}(x) = \frac{2x - 4}{x + 3} \]
Chapter 5: Exponential and Logarithmic Functions

Verifying:
\[ f \left( f^{-1}(x) \right) = \frac{2x - 4}{x + 3} \]
\[ = -3 \left( \frac{2x - 4}{x + 3} \right) - 4 \]
\[ = \frac{2x - 4}{x + 3} - 2 \]
\[ = \frac{-3(2x - 4) - 4(x + 3)}{2x - 4 - 2(x + 3)} \]
\[ = \frac{-6x + 12 - 4x - 12}{2x - 4 - 2x - 6} \]
\[ = \frac{-10x}{-10} \]
\[ = x \]

71. 
\[ f(x) = \frac{x^2 - 4}{2x^2}, \ x > 0 \]
\[ y = \frac{\sqrt{y}^2 - 4}{2\sqrt{y}^2}, \ y > 0 \]
\[ x = \frac{\sqrt{x}^2 - 4}{2\sqrt{x}^2}, \ y > 0 \quad \text{Inverse} \]
\[ 2x^2 = y^2 - 4, \ x < \frac{1}{2} \]
\[ 2x^2 - y^2 = -4, \ x < \frac{1}{2} \]
\[ y^2 (2x - 1) = -4, \ x < \frac{1}{2} \]
\[ y^2 (1 - 2x) = 4, \ x < \frac{1}{2} \]
\[ y^2 = \frac{4}{1 - 2x}, \ x < \frac{1}{2} \]
\[ y = \frac{\sqrt{4}}{\sqrt{1 - 2x}}, \ x < \frac{1}{2} \]
\[ y = \frac{2}{\sqrt{1 - 2x}}, \ x < \frac{1}{2} \]
\[ f^{-1}(x) = \frac{2}{\sqrt{1 - 2x}}, \ x < \frac{1}{2} \]

Verifying:
\[ f \left( f^{-1}(x) \right) = f \left( \frac{2}{\sqrt{1 - 2x}} \right) = \frac{\left( \frac{2}{\sqrt{1 - 2x}} \right)^2 - 4}{2 \left( \frac{2}{\sqrt{1 - 2x}} \right)^2} \]
\[ = \frac{4}{1 - 2x} - 4 \]
\[ = \frac{4}{1 - 2x} - 4 \left( \frac{1}{2} \left( \frac{4}{1 - 2x} \right) \right) (1 - 2x) \]
\[ = \frac{4 - 4(1 - 2x)}{2(4)} = \frac{8 - 8x}{8} = x \]
Section 5.2: One-to-One Functions; Inverse Functions

\[
f^{-1}(f(x)) = f^{-1}\left(\frac{x^2 - 4}{2x^2}\right) = \frac{2}{\sqrt{1 - \frac{4}{x^2}}} = \frac{2}{\sqrt{1 - \frac{4}{x^2}}}
\]

Verifying:

\[
f\left(f^{-1}(x)\right) = f\left(\sqrt{\frac{3}{3x-1}}\right) = \left(\frac{3}{3x-1}\right)^2 + 3
\]

72. \[f(x) = \frac{x^2 + 3}{3x^2}, \quad x > 0\]
\[y = \frac{x^2 + 3}{3x^2}, \quad x > 0\]
\[x = \frac{y^2 + 3}{3y^2}, \quad y > 0\]
Inverse

\[3xy^2 = y^2 + 3, \quad x > \frac{1}{3}\]

\[3xy^2 - y^2 = 3, \quad x > \frac{1}{3}\]

\[y^2 (3x - 1) = 3, \quad x > \frac{1}{3}\]

y = \frac{3}{x^2 - 1}, \quad x > \frac{1}{3}

\[f^{-1}(x) = \frac{3}{3x-1}, \quad x > \frac{1}{3}\]

73. a. Because the ordered pair \((-1,0)\) is on the graph, \(f(-1) = 0\).

b. Because the ordered pair \((1,2)\) is on the graph, \(f(1) = 2\).

c. Because the ordered pair \((0,1)\) is on the graph, \(f^{-1}(1) = 0\).

d. Because the ordered pair \((1,2)\) is on the graph, \(f^{-1}(2) = 1\).

74. a. Because the ordered pair \(\left(2, \frac{1}{2}\right)\) is on the graph, \(f(2) = \frac{1}{2}\).

b. Because the ordered pair \((1,0)\) is on the graph, \(f(1) = 0\).
c. Because the ordered pair \((1,0)\) is on the graph, \(f^{-1}(0) = 1\).

d. Because the ordered pair \((0,-1)\) is on the graph, \(f^{-1}(-1) = 0\).

75. Since \(f(7) = 13\), we have \(f^{-1}(13) = 7\); the input of the function is the output of the inverse when the output of the function is the input of the inverse.

76. Since \(g(-5) = 3\), we have \(g^{-1}(3) = -5\); the input of the function is the output of the inverse when the output of the function is the input of the inverse.

77. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for \(f^{-1}\):
   Domain: \([-2, \infty)\)  Range: \([5, \infty)\)

78. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for \(f^{-1}\):
   Domain: \([5, \infty)\)  Range: \([0, \infty)\)

79. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for \(g^{-1}\):
   Domain: \([0, \infty)\)  Range: \((-\infty, 0]\)

80. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for \(g^{-1}\):
   Domain: \((0, 8)\)  Range: \([0, 15]\)

81. Since \(f(x)\) is increasing on the interval \((0,5)\), it is one-to-one on the interval and has an inverse, \(f^{-1}(x)\). In addition, we can say that \(f^{-1}(x)\) is increasing on the interval \((f(0), f(5))\).

82. Since \(f(x)\) is decreasing on the interval \((0,5)\), it is one-to-one on the interval and has an inverse, \(f^{-1}(x)\). In addition, we can say that \(f^{-1}(x)\) is decreasing on the interval \((f(5), f(0))\).

83. \(f(x) = mx + b, \ m \neq 0\)
   \[y = mx + b\]
   \[x = my + b\]  Inverse
   \[x - b = my\]
   \[
y = \frac{1}{m}(x - b)\]
   \[f^{-1}(x) = \frac{1}{m}(x - b), \ m \neq 0\]

84. \(f(x) = \sqrt{r^2 - x^2}, \ 0 \leq x \leq r\)
   \[y = \sqrt{r^2 - x^2}\]
   \[x = \sqrt{r^2 - y^2}\]  Inverse
   \[x^2 = r^2 - y^2\]
   \[y^2 = r^2 - x^2\]
   \[
y = \sqrt{r^2 - x^2}\]
   \[f^{-1}(x) = \sqrt{r^2 - x^2}, \ 0 \leq x \leq r\]

85. If \((a,b)\) is on the graph of \(f\), then \((b,a)\) is on the graph of \(f^{-1}\). Since the graph of \(f^{-1}\) lies in quadrant I, both coordinates of \((a,b)\) are positive, which means that both coordinates of \((b,a)\) are positive. Thus, the graph of \(f^{-1}\) must lie in quadrant I.

86. If \((a,b)\) is on the graph of \(f\), then \((b,a)\) is on the graph of \(f^{-1}\). Since the graph of \(f\) lies in quadrant II, \(a\) must be negative and \(b\) must be positive. Thus, \((b,a)\) must be a point in quadrant IV, which means the graph of \(f^{-1}\) lies in quadrant IV.

87. Answers may vary. One possibility follows:
   \(f(x) = |x|, \ x \geq 0\)  is one-to-one.
   Thus, \(f(x) = x, \ x \geq 0\)
   \[y = x, \ x \geq 0\]
   \[f^{-1}(x) = x, \ x \geq 0\]
Section 5.2: One-to-One Functions; Inverse Functions

88. Answers may vary. One possibility follows:
   \(f(x) = x^4, \ x \geq 0\) is one-to-one.

   Thus, \(f(x) = x^4, \ x \geq 0\)

   \(y = x^4\) \quad \text{Inverse}

   \(y = \sqrt[4]{x}, \ x \geq 0\)

   \(f^{-1}(x) = \sqrt[4]{x}, \ x \geq 0\)

89. a. \(d = 6.97r - 90.39\)

   \(d + 90.39 = 6.97r\)

   \(\frac{d + 90.39}{6.97} = r\)

   Therefore, we would write

   \(r(d) = \frac{d + 90.39}{6.97}\)

b. \(r(d(r)) = \frac{(6.97r - 90.39) + 90.39}{6.97}\)

   \(= \frac{6.97r + 90.39 - 90.39}{6.97} = \frac{6.97r}{6.97} = r\)

   \(d(r(d)) = 6.97 \left(\frac{d + 90.39}{6.97}\right) - 90.39\)

   \(= d + 90.39 - 90.39\)

   \(= d\)

c. \(r(300) = \frac{300 + 90.39}{6.97} = 56.01\)

   If the distance required to stop was 300 feet,
   the speed of the car was roughly 56 miles
   per hour.

90. a. \(H(C) = 2.15C - 10.53\)

   \(H = 2.15C - 10.53\)

   \(H + 10.53 = 2.15C\)

   \(\frac{H + 10.53}{2.15} = C\)

   \(C(H) = \frac{H + 10.53}{2.15}\)

b. \(H(C(H)) = 2.15 \left(\frac{H + 10.53}{2.15}\right) - 10.53\)

   \(= H + 10.53 - 10.53\)

   \(= H\)

91. a. 6 feet = 72 inches

   \(W(72) = 50 + 2.3(72 - 60)\)

   \(= 50 + 2.3(12) = 50 + 27.6 = 77.6\)

   The ideal weight of a 6-foot male is 77.6 kilograms.

b. \(W = 50 + 2.3(h - 60)\)

   \(W - 50 = 2.3h - 138\)

   \(W + 88 = 2.3h\)

   \(\frac{W + 88}{2.3} = h\)

   Therefore, we would write

   \(h(W) = \frac{W + 88}{2.3}\)

c. \(h(W(h)) = \frac{(50 + 2.3(h - 60)) + 88}{2.3}\)

   \(= \frac{50 + 2.3h - 138 + 88}{2.3} = \frac{2.3h}{2.3} = h\)

   \(W(h(W)) = 50 + 2.3 \left(\frac{W + 88}{2.3} - 60\right)\)

   \(= 50 + W + 88 - 138 = W\)

d. \(h(80) = \frac{80 + 88}{2.3} = \frac{168}{2.3} = 73.04\)

   The height of a male who is at his ideal
   weight of 80 kg is roughly 73 inches.

92. a. \(F = \frac{9}{5}C + 32\)

   \(F - 32 = \frac{9}{5}C\)

   \(\frac{5}{9}(F - 32) = C\)

   Therefore, we would write

   \(C(F) = \frac{5}{9}(F - 32)\)
**Chapter 5: Exponential and Logarithmic Functions**

b. \[ C(F(C)) = \frac{5}{9} \left( \frac{9}{5} C + 32 \right) - 32 \]
\[ = \frac{5}{9} \cdot \frac{9}{5} C = C \]
\[ F(C(F)) = \frac{9}{5} \left( \frac{5}{9} (F - 32) \right) + 32 \]
\[ = F - 32 + 32 = F \]

c. \[ C(70) = \frac{5}{9} (70 - 32) = \frac{5}{9} (38) = 21.1^\circ C \]

93. a. From the restriction given in the problem statement, the domain is \( \{ g \mid 33,950 \leq g \leq 82,250 \} \) or \([33950,82250]\).

b. \[ T(33,950) = 4675 + 0.25(33,950 - 33,950) \]
\[ = 4675 \]
\[ T(82,250) = 4675 + 0.25(82,250 - 33,950) \]
\[ = 16,750 \]

Since \( T \) is linear and increasing, we have that the range is \( \{ T \mid 4675 \leq T \leq 16,750 \} \) or \([4675,16750]\).

c. \[ T = 4675 + 0.25(33,950 - 33,950) \]
\[ T - 4675 = 0.25(33,950 - 33,950) \]
\[ \frac{T - 4675}{0.25} = g - 33,950 \]
\[ \frac{T - 4675}{0.25} + 33,950 = g \]

Therefore, we would write \[ g(T) = \frac{T - 4675}{0.25} + 33,950 \]

Domain: \( \{ T \mid 4675 \leq T \leq 16,750 \} \)
Range: \( \{ g \mid 33,950 \leq g \leq 82,250 \} \)

94. a. From the restriction given in the problem statement, the domain is \( \{ g \mid 16,700 \leq g \leq 67,900 \} \) or \([16700,67900]\).

b. \[ T(16,700) = 1670 + 0.15(16,700 - 16,700) \]
\[ = 1670 \]
\[ T(67,900) = 1670 + 0.15(67,900 - 16,700) \]
\[ = 9350 \]

Since \( T \) is linear and increasing, we have that the range is \( \{ T \mid 1670 \leq T \leq 9350 \} \) or \([1670,9350]\).

c. \[ T = 1670 + 0.15(g - 16,700) \]
\[ T - 1670 = 0.15(g - 16,700) \]
\[ \frac{T - 1670}{0.15} = g - 16,700 \]
\[ \frac{T - 1670}{0.15} + 16,700 = g \]

We would write \( g(T) = \frac{T - 1670}{0.15} + 16,700 \).

Domain: \( \{ T \mid 1670 \leq T \leq 9350 \} \)
Range: \( \{ g \mid 16,700 \leq g \leq 67,900 \} \)

95. a. The graph of \( H \) is symmetric about the y-axis. Since \( t \) represents the number of seconds after the rock begins to fall, we know that \( t \geq 0 \). The graph is strictly decreasing over its domain, so it is one-to-one.

b. \[ H = 100 - 4.9t^2 \]
\[ H + 4.9r^2 = 100 \]
\[ 4.9r^2 = 100 - H \]
\[ r^2 = \frac{100 - H}{4.9} \]
\[ t = \sqrt[2]{\frac{100 - H}{4.9}} \]

Therefore, we would write \( t(H) = \sqrt[2]{\frac{100 - H}{4.9}} \).

(Note: we only need the principal square root since we know \( t \geq 0 \))

\[ H(t(H)) = 100 - 4.9 \left( \frac{100 - H}{4.9} \right)^2 \]
\[ = 100 - 4.9 \left( \frac{100 - H}{4.9} \right) \]
\[ = 100 - 4.9 + H \]
\[ = H \]
\[ t(H(t)) = \sqrt[2]{\frac{100 - (100 - 4.9t^2)}{4.9}} \]
\[ = \sqrt[2]{\frac{4.9t^2}{4.9}} = t \]

(since \( t \geq 0 \))

c. \[ t(80) = \sqrt[2]{\frac{100 - 80}{4.9}} = 2.02 \]

It will take the rock about 2.02 seconds to fall 80 meters.
Section 5.3: Exponential Functions

96. a. \[ T(l) = 2\pi \sqrt{\frac{l}{32.2}} \]
   \[ T = 2\pi \sqrt{\frac{l}{32.2}} \]
   \[ \frac{T}{2\pi} = \sqrt{\frac{l}{32.2}} \]
   \[ \frac{T^2}{4\pi^2} = \frac{l}{32.2} \]
   \[ l = \frac{32.2T^2}{4\pi^2} \]
   \[ l(T) = \frac{8.05T^2}{\pi^2} = 8.05 \left( \frac{T}{\pi} \right)^2, \quad T > 0 \]

   b. \[ l(3) = 8.05 \left( \frac{3}{\pi} \right)^2 = 7.34 \]

   A pendulum whose period is 3 seconds will be about 7.34 feet long.

97. \[ f(x) = \frac{ax+b}{cx+d} \]
   \[ y = \frac{ax+b}{cx+d} \]
   \[ x = \frac{ay+b}{cy+d} \] Inverse
   \[ x(cy+d) = ay+b \]
   \[ cx+y = ay+b \]
   \[ cxy + dx = ay+b \]
   \[ cx - ay = b - dx \]
   \[ y(cx-a) = b - dx \]
   \[ y = \frac{b - dx}{cx-a} \]
   \[ f^{-1}(x) = \frac{-dx+b}{cx-a} \]

   Now, \( f = f^{-1} \) provided that \( \frac{ax+b}{cx+d} = \frac{-dx+b}{cx-a} \).

   This is only true if \( a = -d \).

98. Yes. In order for a one-to-one function and its inverse to be equal, its graph must be symmetric about the line \( y = x \). One such example is the function \( f(x) = \frac{1}{x} \).

99. Answers will vary.

100. Answers will vary. One example is

   \[ f(x) = \begin{cases} \frac{1}{x}, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases} \]

   This function is one-to-one since the graph passes the Horizontal Line Test. However, the function is neither increasing nor decreasing on its domain.

101. No, not every odd function is one-to-one. For example, \( f(x) = x^3 - x \) is an odd function, but it is not one-to-one.

102. \( C^{-1}(800,000) \) represents the number of cars manufactured for $800,000.

103. If a horizontal line passes through two points on a graph of a function, then the \( y \) value associated with that horizontal line will be assigned to two different \( x \) values which violates the definition of one-to-one.

Section 5.3

1. \( 4^3 = 64; \quad 8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 4; \quad 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \)

2. \( x^2 + 3x - 4 = 0 \)
   \( (x+4)(x-1) = 0 \)
   \( x + 4 = 0 \) or \( x - 1 = 0 \)
   The solution set is \( \{-4,1\} \).

3. False. To obtain the graph of \( y = (x-2)^3 \), we would shift the graph of \( y = x^3 \) to the right 2 units.
4. \[
\frac{f(4) - f(0)}{4 - 0} = \frac{3(4) - 5 - [3(0) - 5]}{4} = \frac{(12 - 5) - (0 - 5)}{4} = \frac{7 - (-5)}{4} = \frac{12}{4} = 3
\]

5. True

6. exponential function; growth factor; initial value

7. a

8. True

9. False. The range will be \( \{x \mid x > 0\} \) or \( (0, \infty) \).

10. True

11. \((-1, \frac{1}{a})\), \((0, 1)\), \((1, a)\)

12. 1

13. 4

14. False; for example, the point \((1, 3)\) is on the first graph and \((1, \frac{1}{3})\) is on the other.

15. a. \(3^{2.2} = 11.212\)
   b. \(3^{2.23} = 11.587\)
   c. \(3^{2.230} = 11.664\)
   d. \(3^{5.5} = 11.665\)

16. a. \(5^{1.7} = 15.426\)
   b. \(5^{1.73} = 16.189\)
   c. \(5^{1.732} = 16.241\)
   d. \(5^{6.7} = 16.242\)

17. a. \(2^{3.14} = 8.815\)
   b. \(2^{3.141} = 8.821\)
   c. \(2^{3.1415} = 8.824\)
   d. \(2^e = 8.825\)

18. a. \(2^{3.7} = 6.498\)
   b. \(2^{2.71} = 6.543\)
   c. \(2^{2.718} = 6.580\)
   d. \(2^e = 6.581\)

19. a. \(3.1^{2.7} = 21.217\)
   b. \(3.142^{2.71} = 22.217\)
   c. \(3.141^{2.718} = 22.440\)
   d. \(\pi^e = 22.459\)

20. a. \(2.7^{3.1} = 21.738\)
   b. \(2.71^{3.14} = 22.884\)
   c. \(2.718^{3.141} = 23.119\)
   d. \(e^x = 23.141\)

21. \(e^{1.2} = 3.320\)

22. \(e^{-1.3} = 0.273\)

23. \(e^{-0.85} = 0.427\)

24. \(e^{2.1} = 8.166\)

25. | \(x\) | \(y = f(x)\) | \(\frac{\Delta y}{\Delta x}\) | \(\frac{f(x+1)}{f(x)}\) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>3</td>
<td>(\frac{6}{3} = 2)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>(\frac{6 - 3}{0 - (-1)} = 3)</td>
<td>(\frac{12}{6} = 2)</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>(\frac{12 - 6}{1 - 0} = 6)</td>
<td>(\frac{18}{12} = \frac{3}{2})</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Not a linear function since the average rate of change is not constant.

Not an exponential function since the ratio of consecutive terms is not constant.

26. | \(x\) | \(y = g(x)\) | \(\frac{\Delta y}{\Delta x}\) | \(\frac{g(x+1)}{g(x)}\) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>2</td>
<td></td>
<td>(\frac{5}{2})</td>
</tr>
</tbody>
</table>
Section 5.3: Exponential Functions

<table>
<thead>
<tr>
<th>x</th>
<th>y = H(x)</th>
<th>Δy/Δx</th>
<th>H(x+1)/H(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1/4</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3/4</td>
<td>1/4</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>64/16</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Not a linear function since the average rate of change is not constant.

The ratio of consecutive outputs is a constant, 4. This is an exponential function with growth factor \( a = 4 \). The initial value of the exponential function is \( C = 1 \). Therefore, the exponential function that models the data is \( H(x) = Ca^x = 1 \cdot (4)^x = 4^x \).

<table>
<thead>
<tr>
<th>x</th>
<th>y = f(x)</th>
<th>Δy/Δx</th>
<th>f(x+1)/f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>3/2</td>
<td>3</td>
<td>3/2</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>3</td>
<td>3/2</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>3</td>
<td>3/2</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>3</td>
<td>3/2</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
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<td></td>
</tr>
</tbody>
</table>

Not a linear function since the average rate of change is not constant.

The ratio of consecutive outputs is a constant, 2. This is an exponential function with growth factor \( a = 2 \). The initial value of the exponential function is \( C = 3 \). Therefore, the exponential function that models the data is \( f(x) = Ca^x = 3 \cdot (2)^x = 3 \cdot 2^x \).
Chapter 5: Exponential and Logarithmic Functions

### Not a linear function since the average rate of change is not constant.

Not an exponential function since the ratio of consecutive terms is not constant.

<table>
<thead>
<tr>
<th></th>
<th>y = H(x)</th>
<th>Δy/Δx</th>
<th>H(x+1)</th>
<th>H(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>2</td>
<td>4</td>
<td>4/2 = 2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>4-2/0-(-1) = 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>6-4/1-0 = 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>8-6/2-1 = 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>10-8/3-2 = 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Not an exponential function since the ratio of consecutive terms is not constant.

The average rate of change is a constant, 2.

Therefore, this is a linear function. In a linear function the average rate of change is the slope \( m \).

So, \( m = 2 \). When \( x = 0 \) we have \( y = 4 \) so the y-intercept is \( b = 4 \). The linear function that models this data is \( H(x) = mx + b = 2x + 4 \).

### Not a linear function since the average rate of change is not constant.

The ratio of consecutive outputs is a constant, \( \frac{1}{2} \).

This is an exponential function with growth factor \( a = \frac{1}{2} \). The initial value of the exponential function is \( C = \frac{1}{4} \). Therefore, the exponential function that models the data is \( F(x) = Ca^x = \frac{1}{4}\left(\frac{1}{2}\right)^x \).

### Using the graph of \( y = 2^x \), shift the graph up 1 unit.

**Domain:** All real numbers

**Range:** \( \{ y \mid y > 1 \} \) or \((1, \infty)\)

**Horizontal Asymptote:** \( y = 1 \)

### Using the graph of \( y = 3^x \), shift the graph down 2 units.

**Domain:** All real numbers
Section 5.3: Exponential Functions

Range: \{y \mid y > -2\} or \((-2, \infty)\)
Horizontal Asymptote: \(y = -2\)

43. \(f(x) = 3^{x-1}\)
Using the graph of \(y = 3^x\), shift the graph right 1 unit.
Domain: All real numbers
Range: \{y \mid y > 0\} or \((0, \infty)\)
Horizontal Asymptote: \(y = 0\)

44. \(f(x) = 2^{x+2}\)
Using the graph of \(y = 2^x\), shift the graph left 2 units.
Domain: All real numbers
Range: \{y \mid y > 0\} or \((0, \infty)\)
Horizontal Asymptote: \(y = 0\)

45. \(f(x) = 3 \cdot \left(\frac{1}{2}\right)^x\)
Using the graph of \(y = \left(\frac{1}{2}\right)^x\), vertically stretch the graph by a factor of 3. That is, for each point on the graph, multiply the y-coordinate by 3.
Domain: All real numbers
Range: \{y \mid y > 0\} or \((0, \infty)\)
Horizontal Asymptote: \(y = 0\)

46. \(f(x) = 4 \cdot \left(\frac{1}{3}\right)^x\)
Using the graph of \(y = \left(\frac{1}{3}\right)^x\), vertically stretch the graph by a factor of 4. That is, for each point on the graph, multiply the y-coordinate by 4.
Domain: All real numbers
Range: \{y \mid y > 0\} or \((0, \infty)\)
47. \( f(x) = 3^{-x} - 2 \)
Using the graph of \( y = 3^x \), reflect the graph about the y-axis, and shift down 2 units.
Domain: All real numbers
Range: \( \{ y \mid y > -2 \} \) or \( (-2, \infty) \)
Horizontal Asymptote: \( y = -2 \)

48. \( f(x) = -3^x + 1 \)
Using the graph of \( y = 3^x \), reflect the graph about the x-axis, and shift up 1 unit.
Domain: All real numbers
Range: \( \{ y \mid y < 1 \} \) or \( (-\infty, 1) \)
Horizontal Asymptote: \( y = 1 \)

49. \( f(x) = 2 + 4^{x-1} \)
Using the graph of \( y = 4^x \), shift the graph to the right one unit and up 2 units.
Domain: All real numbers
Range: \( \{ y \mid y > 2 \} \) or \( (2, \infty) \)
Horizontal Asymptote: \( y = 2 \)

50. \( f(x) = 1 - 2^{x+3} \)
Using the graph of \( y = 2^x \), shift the graph to the left 3 units, reflect about the x-axis, and shift up 1 unit.
Domain: All real numbers
Range: \( \{ y \mid y < 1 \} \) or \( (-\infty, 1) \)
Horizontal Asymptote: \( y = 1 \)

51. \( f(x) = 2 + 3^{\sqrt{2}} \)
Using the graph of \( y = 3^x \), stretch the graph horizontally by a factor of 2, and shift up 2 units.
Domain: All real numbers
Range: \( \{ y \mid y > 2 \} \) or \( (2, \infty) \)
Horizontal Asymptote: \( y = 2 \)
52. \( f(x) = 1 - 2^{-x/3} \)
Using the graph of \( y = 2^x \), stretch the graph horizontally by a factor of 3, reflect about the y-axis, reflect about the x-axis, and shift up 1 unit.
Domain: All real numbers
Range: \( \{y \mid y < 1\} \) or \((-\infty, 1)\)
Horizontal Asymptote: \( y = 1 \)

53. \( f(x) = e^{-x} \)
Using the graph of \( y = e^x \), reflect the graph about the y-axis.
Domain: All real numbers
Range: \( \{y \mid y > 0\} \) or \((0, \infty)\)
Horizontal Asymptote: \( y = 0 \)

54. \( f(x) = -e^x \)
Using the graph of \( y = e^x \), reflect the graph about the x-axis.
Domain: All real numbers
Range: \( \{y \mid y < 0\} \) or \((-\infty, 0)\)
Horizontal Asymptote: \( y = 0 \)

55. \( f(x) = e^{x+2} \)
Using the graph of \( y = e^x \), shift the graph 2 units to the left.
Domain: All real numbers
Range: \( \{y \mid y > 0\} \) or \((0, \infty)\)
Horizontal Asymptote: \( y = 0 \)

56. \( f(x) = e^x - 1 \)
Using the graph of \( y = e^x \), shift the graph down 1 unit.
Domain: All real numbers
Range: \( \{y \mid y > -1\} \) or \((-1, \infty)\)
Horizontal Asymptote: \( y = -1 \)
57. \( f(x) = 5 - e^{-x} \)

Using the graph of \( y = e^x \), reflect the graph about the \( y \)-axis, reflect about the \( x \)-axis, and shift up 5 units.

Domain: All real numbers
Range: \( \{ y \mid y < 5 \} \) or \( (-\infty, 5) \)

Horizontal Asymptote: \( y = 5 \)

58. \( f(x) = 9 - 3e^{-x} \)

Using the graph of \( y = e^x \), reflect the graph about the \( y \)-axis, stretch vertically by a factor of 3, reflect about the \( x \)-axis, and shift up 9 units.

Domain: All real numbers
Range: \( \{ y \mid y < 9 \} \) or \( (-\infty, 9) \)

Horizontal Asymptote: \( y = 9 \)

59. \( f(x) = 2 - e^{-x/2} \)

Using the graph of \( y = e^x \), reflect the graph about the \( y \)-axis, stretch horizontally by a factor of 2, reflect about the \( x \)-axis, and shift up 2 units.

Domain: All real numbers
Range: \( \{ y \mid y < 2 \} \) or \( (-\infty, 2) \)

Horizontal Asymptote: \( y = 2 \)

60. \( f(x) = 7 - 3e^{-2x} \)

Using the graph of \( y = e^x \), reflect the graph about the \( y \)-axis, shrink horizontally by a factor of \( \frac{1}{2} \), stretch vertically by a factor of 3, reflect about the \( x \)-axis, and shift up 7 units.

Domain: All real numbers
Range: \( \{ y \mid y < 7 \} \) or \( (-\infty, 7) \)

Horizontal Asymptote: \( y = 7 \)

61. \( 7^x = 7^3 \)

We have a single term with the same base on both sides of the equation. Therefore, we can set the exponents equal to each other: \( x = 3 \).

The solution set is \( \{3\} \).

62. \( 5^x = 5^{-6} \)

We have a single term with the same base on both sides of the equation. Therefore, we can set the
exponents equal to each other:  \( x = -6 \).
The solution set is \( \{-6\} \).

63.  \( 2^{-x} = 16 \)
\( 2^{-x} = 2^4 \)
\( -x = 4 \)
\( x = -4 \)
The solution set is \( \{-4\} \).

64.  \( 3^{-x} = 81 \)
\( 3^{-x} = 3^4 \)
\( -x = 4 \)
\( x = -4 \)
The solution set is \( \{-4\} \).

65.  \( \left( \frac{1}{5} \right)^x = \frac{1}{25} \)
\( \left( \frac{1}{5} \right)^x = \frac{1}{5^2} \)
\( \left( \frac{1}{5} \right)^x = \left( \frac{1}{5} \right)^2 \)
\( x = 2 \)
The solution set is \( \{2\} \).

66.  \( \left( \frac{1}{4} \right)^x = \frac{1}{64} \)
\( \left( \frac{1}{4} \right)^x = \frac{1}{4^3} \)
\( \left( \frac{1}{4} \right)^x = \left( \frac{1}{4} \right)^3 \)
\( x = 3 \)
The solution set is \( \{3\} \).

67.  \( 2^{2x-1} = 4 \)
\( 2^{2x-1} = 2^2 \)
\( 2x - 1 = 2 \)
\( 2x = 3 \)
\( x = \frac{3}{2} \)
The solution set is \( \left\{ \frac{3}{2} \right\} \).

68.  \( 5^{x+3} = \frac{1}{5} \)
\( 5^{x+3} = 5^{-1} \)
\( x + 3 = -1 \)
\( x = -4 \)
The solution set is \( \{-4\} \).

69.  \( 3^x = 9^x \)
\( 3^x = (3^2)^x \)
\( 3^x = 3^{2x} \)
\( x^3 = 2x \)
\( x^3 - 2x = 0 \)
\( x(x^2 - 2) = 0 \)
\( x = 0 \) or \( x^2 - 2 = 0 \)
\( x^2 = 2 \)
\( x = \pm\sqrt{2} \)
The solution set is \( \{-\sqrt{2}, 0, \sqrt{2}\} \).

70.  \( 4^{x^2} = 2^x \)
\( (2^2)^{x^2} = 2^x \)
\( 2^{2x^2} = 2^x \)
\( 2x^2 = x \)
\( 2x^2 - x = 0 \)
\( x(2x - 1) = 0 \)
\( x = 0 \) or \( 2x - 1 = 0 \)
\( 2x = 1 \)
\( x = \frac{1}{2} \)
The solution set is \( \left\{ 0, \frac{1}{2} \right\} \).

71.  \( 8^{-x+14} = 16^x \)
\( (2^3)^{-x+14} = (2^4)^x \)
\( 2^{-3x+42} = 2^{4x} \)
\( -3x + 42 = 4x \)
\( 42 = 7x \)
\( 6 = x \)
The solution set is \( \{6\} \).
Chapter 5: Exponential and Logarithmic Functions

72. \( 9^{-x+15} = 27^x \)
\[
\left(3^2\right)^{-x+15} = \left(3^3\right)^x \\
3^{-2x+30} = 3^{3x} \\
-2x + 30 = 3x \\
30 = 5x \\
6 = x
\]
The solution set is \( \{6\} \).

73. \( 3^{x^2-7} = 27^{2x} \)
\[
3^{x^2-7} = (3^3)^{2x} \\
3^{x^2-7} = 3^{6x} \\
x^2 - 7 = 6x \\
x^2 - 6x - 7 = 0 \\
(x - 7)(x + 1) = 0 \\
x - 7 = 0 \text{ or } x + 1 = 0 \\
x = 7 \quad x = -1
\]
The solution set is \( \{-1, 7\} \).

74. \( 5^{x^2+8} = 125^{2x} \)
\[
5^{x^2+8} = (5^3)^{2x} \\
5^{x^2+8} = 5^{6x} \\
x^2 + 8 = 6x \\
x^2 - 6x + 8 = 0 \\
(x - 4)(x - 2) = 0 \\
x - 4 = 0 \text{ or } x - 2 = 0 \\
x = 4 \quad x = 2
\]
The solution set is \( \{2, 4\} \).

75. \( 4^x \cdot 2^{x^2} = 16^2 \)
\[
\left(2^2\right)^x \cdot 2^{x^2} = \left(2^4\right)^2 \\
2^x \cdot 2^{x^2} = 2^8 \\
2^{2x+x^2} = 2^8 \\
x^2 + 2x = 8 \\
x^2 + 2x - 8 = 0 \\
(x + 4)(x - 2) = 0 \\
x + 4 = 0 \text{ or } x - 2 = 0 \\
x = -4 \quad x = 2
\]
The solution set is \( \{-4, 2\} \).

76. \( 9^{2x} \cdot 27^{x^2} = 3^{-1} \)
\[
\left(3^2\right)^{2x} \cdot (3^3)^{x^2} = 3^{-1} \\
3^{4x} \cdot 3^{3x^2} = 3^{-1} \\
3x^2 + 4x = -1 \\
3x^2 + 4x + 1 = 0 \\
(3x + 1)(x + 1) = 0 \\
x + 1 = 0 \text{ or } x + 1 = 0 \\
x = -1 \quad x = -1 \\
x = \frac{-1}{3}
\]
The solution set is \( \{-1, \frac{-1}{3}\} \).

77. \( e^x = e^{3x+8} \)
\[
x = 3x + 8 \\
-2x = 8 \\
x = -4
\]
The solution set is \( \{-4\} \).

78. \( e^{3x} = e^{2-x} \)
\[
3x = 2 - x \\
4x = 2 \\
x = \frac{1}{2}
\]
The solution set is \( \left\{\frac{1}{2}\right\} \).

79. \( e^{x^2} = e^{3x} \cdot \frac{1}{e^2} \)
\[
e^{x^2} = e^{3x} \cdot e^{-2} \\
e^{x^2} = e^{3x-2} \\
x^2 = 3x - 2 \\
x^2 - 3x + 2 = 0 \\
(x - 2)(x - 1) = 0 \\
x - 2 = 0 \text{ or } x - 1 = 0 \\
x = 2 \quad x = 1
\]
The solution set is \( \{1, 2\} \).
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80. (\(e^x\))^4 \cdot e^{x^2} = e^{12}
   \(e^x \cdot e^x = e^{12}
   e^{4x + x^2} = e^{12}
   x^2 + 4x = 12
   x^2 + 4x - 12 = 0
   (x + 6)(x - 2) = 0
   x = -6 or x = 2
   The solution set is \{-6, 2\}.

81. If \(4^x = 7\), then \((4^x)^2 = 7^2
   4^{-2x} = \frac{1}{7^2}
   4^{-2x} = \frac{1}{49}

82. If \(2^x = 3\), then \((2^x)^2 = 3^2
   2^{-2x} = \frac{1}{3^2}
   \left(2^2\right)^{-x} = \frac{1}{9}
   4^{-x} = \frac{1}{9}

83. If \(3^{-x} = 2\), then \(\left(3^{-x}\right)^2 = 2^{-2}
   3^{2x} = \frac{1}{2^2}
   3^{2x} = \frac{1}{4}

84. If \(5^{-x} = 3\), then \(\left(5^{-x}\right)^{-3} = 3^{-3}
   5^{3x} = \frac{1}{3^3}
   5^{3x} = \frac{1}{27}

85. We need a function of the form \(f(x) = k \cdot a^{p-x}\),
   with \(a > 0, a \neq 1\). The graph contains the points
   \((-1, \frac{1}{3}), (0, 1), (1, 3),\) and \((2, 9)\). In other words,
   \(f(-1) = \frac{1}{3}, f(0) = 1, f(1) = 3,\) and \(f(2) = 9\).
   Therefore, \(f(0) = k \cdot a^{p(0)}\)
   \(1 = k \cdot a^0\)
   \(1 = k \cdot 1\)
   \(1 = k\)
   \(3 = a^p\)
   Let’s use \(a = 3, p = 1\). Then \(f(x) = 3^x\). Now we need to verify that this function yields the other known points on the graph.
   \(f(-1) = 3^{-1} = \frac{1}{3};\)
   \(f(2) = 3^2 = 9\)
   So we have the function \(f(x) = 3^x\).

86. We need a function of the form \(f(x) = k \cdot a^{p-x}\),
   with \(a > 0, a \neq 1\). The graph contains the points
   \((-1, \frac{1}{5}), (0, 1), (1, 5)\). In other words,
   \(f(-1) = \frac{1}{5}, f(0) = 1, f(1) = 5\). Therefore,
   \(f(0) = k \cdot a^{p(0)}\)
   \(1 = k \cdot a^0\)
   \(1 = k \cdot 1\)
   \(1 = k\)
   \(5 = a^p\)
   Let’s use \(a = 5, p = 1\). Then \(f(x) = 5^x\). Now we need to verify that this function yields the other known point on the graph.
   \(f(-1) = 5^{-1} = \frac{1}{5}\)
   So we have the function \(f(x) = 5^x\).
87. We need a function of the form \( f(x) = k \cdot a^{p^x} \), with \( a > 0, \ a \neq 1 \). The graph contains the points \((-1, -\frac{1}{6}), (0, -1), (1, -6), \) and \((2, -36)\). In other words, \( f(-1) = -\frac{1}{6}, \ f(0) = -1, \ f(1) = -6, \) and \( f(2) = -36 \). Therefore, \( f(0) = k \cdot a^{p^{(0)}} \) and \( f(1) = -a^{p^{(1)}} \). 
\[ -1 = k \cdot a^0 \quad -6 = a^p \]
\[ -1 = k \cdot 1 \quad 6 = a^p \]
\[ -1 = k \]
Let’s use \( a = 6, \ p = 1 \). Then \( f(x) = -6^x \).
Now we need to verify that this function yields the other known points on the graph.
\( f(-1) = -6^{-1} = -\frac{1}{6}; \ f(2) = -6^2 = -36 \)
So we have the function \( f(x) = -6^x \).

88. We need a function of the form \( f(x) = k \cdot a^{p^x} \), with \( a > 0, \ a \neq 1 \). The graph contains the points \((-1, -\frac{1}{e}), (0, -1), (1, -e), \) and \((2, -e^2)\). In other words, \( f(-1) = -\frac{1}{e}, \ f(0) = -1, \ f(1) = -e, \) and \( f(2) = -e^2 \). Therefore, \( f(0) = k \cdot a^{p^{(0)}} \)
\[ -1 = k \cdot a^0 \]
\[ -1 = k \cdot 1 \]
\[ -1 = k \]
and \( f(1) = -a^{p^{(1)}} \)
\[ -e = -a^p \]
\[ e = a^p \]
Let’s use \( a = e, \ p = 1 \). Then \( f(x) = -e^x \). Now we need to verify that this function yields the other known points on the graph.
\( f(-1) = -e^{-1} = -\frac{1}{e} \)
\( f(2) = -e^2 \)
So we have the function \( f(x) = -e^x \).

89. We need a function of the form \( f(x) = k \cdot a^{p^x} + b \), with \( a > 0, \ a \neq 1 \) and \( b \) is the vertical shift of 2 units upward. The graph contains the points \((0, 3), \) and \((1, 5)\). In other words, \( f(0) = 1 \) and \( f(1) = 3 \). We can assume the graph has the same shape as the graph of \( f(x) = k \cdot a^{p^x} \). The reference (unshifted) graph would contain the points \((0,1), \) and \((1,3)\).
Therefore, \( f(0) = k \cdot a^{p^{(0)}} \) and \( f(1) = a^{p^{(1)}} \)
\[ 1 = k \cdot a^0 \]
\[ 3 = a^p \]
\[ 1 = k \]
\[ 1 = k \]
Let’s use \( a = 3, \ p = 1 \). Then \( f(x) = 3^x \). To shift the graph up by 2 units we would have \( f(x) = 3^x + 2 \). Now we need to verify that this function yields the other known points on the graph.
\( f(0) = 3^0 + 2 = 3 \)
\( f(1) = 3^1 + 2 = 5 \)
So we have the function \( f(x) = 3^x + 2 \).

90. We need a function of the form \( f(x) = k \cdot a^{p^x} + b \), with \( a > 0, \ a \neq 1 \) and \( b \) is the vertical shift of 3 units downward. The graph contains the points \((0,2), \) and \((-2,1)\). In other words, \( f(0) = -2 \) and \( f(-2) = 1 \). We can assume the graph has the same shape as the graph of \( f(x) = k \cdot a^{p^x} \). The reference (unshifted) graph would contain the points \((0,1), \) and \((-2,4)\).
Therefore, \( f(0) = k \cdot a^{p^{(0)}} \) and \( f(-2) = a^{p^{(-2)}} \)
\[ 1 = k \cdot a^0 \]
\[ \frac{1}{4} = a^{2p} \]
\[ 1 = k \cdot 1 \]
\[ \frac{1}{2} = a^p \]
Let’s use \( a = \frac{1}{2}, \ p = 1 \). Then \( f(x) = \frac{1}{2}x \). To shift the graph down by 3 units we would have \( f(x) = \frac{1}{2}x - 3 \). Now we need to verify that this function yields the other known points on the graph.
\( f(0) = \frac{1}{2} \cdot 0 - 3 = -2 \)
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\[ f(-2) = \frac{1^{-2}}{2} - 3 = 4 - 3 = 1 \]

So we have the function \( f(x) = \frac{1}{2} - 3 \).

91. a. \( f(4) = 2^4 = 16 \)
   The point (4,16) is on the graph of \( f \).

   b. \( f(x) = \frac{1}{16} \)
      \[ 2^x = \frac{1}{16} \]
      \[ 2^x = \frac{1}{2^4} \]
      \[ 2^x = 2^{-4} \]
      \[ x = -4 \]
      The point \((-4, \frac{1}{16})\) is on the graph of \( f \).

92. a. \( f(4) = 3^4 = 81 \)
   The point (4,81) is on the graph of \( f \).

   b. \( f(x) = \frac{1}{9} \)
      \[ 3^x = \frac{1}{9} \]
      \[ 3^x = \frac{1}{3^2} \]
      \[ 3^x = 3^{-2} \]
      \[ x = -2 \]
      The point \((-2, \frac{1}{9})\) is on the graph of \( f \).

93. a. \( g(-1) = 4^{-1} + 2 = \frac{1}{4} + 2 = \frac{9}{4} \)
   The point \((-1, \frac{9}{4})\) is on the graph of \( g \).

   b. \( g(x) = 66 \)
      \[ 4^x + 2 = 66 \]
      \[ 4^x = 64 \]
      \[ 4^x = 4^3 \]
      \[ x = 3 \]
      The point (3,66) is on the graph of \( g \).

94. a. \( g(-1) = 5^{-1} - 3 = \frac{1}{5} - 3 = -\frac{14}{5} \)
   The point \((-1, -\frac{14}{5})\) is on the graph of \( g \).

   b. \( g(x) = 122 \)
      \[ 5^x - 3 = 122 \]
      \[ 5^x = 125 \]
      \[ 5^x = 5^3 \]
      \[ x = 3 \]
      The point (3,122) is on the graph of \( g \).

95. a. \( H(-6) = \left(\frac{1}{2}\right)^{-6} - 4 = (2)^6 - 4 = 60 \)
   The point (-6,60) is on the graph of \( H \).

   b. \( H(x) = 12 \)
      \[ \left(\frac{1}{2}\right)^x - 4 = 12 \]
      \[ \left(\frac{1}{2}\right)^x = 16 \]
      \[ (2)^{-x} = 2^4 \]
      \[ -x = 4 \]
      \[ x = -4 \]
      The point (-4,12) is on the graph of \( H \).

   c. \( \left(\frac{1}{2}\right)^x - 4 = 0 \)
      \[ \left(\frac{1}{2}\right)^x = 4 \]
      \[ (2^{-1})^x = 2^4 \]
      \[ 2^{-x} = 2^2 \]
      \[ -x = 2 \]
      \[ x = -2 \]
      The zero of \( H \) is \( x = -2 \).
96. a. \( F(-5) = \left(\frac{1}{3}\right)^{-5} - 3 = (3)^5 - 3 = 240 \)
   The point \((-5, 240)\) is on the graph of \(F\).

b. \( F(x) = 24 \)
   \( \left(\frac{1}{3}\right)^x - 3 = 24 \)
   \( \left(\frac{1}{3}\right)^x = 27 \)
   \( 3^{-x} = 3^1 \)
   \( -x = 3 \)
   \( x = -3 \)
   The point \((-3, 24)\) is on the graph of \(F\).

c. \( \left(\frac{1}{3}\right)^x - 3 = 0 \)
   \( \left(\frac{1}{3}\right)^x = 3 \)
   \( (3^{-1})^x = 3^1 \)
   \( 3^{-x} = 3^1 \)
   \( -x = 1 \)
   \( x = -1 \)
   The zero of \(F\) is \( x = -1 \).

97. \( f(x) = \begin{cases} 
  e^{-x} & \text{if } x < 0 \\
  e^x & \text{if } x \geq 0 
\end{cases} \)
   Domain: \((-\infty, \infty)\)
   Range: \(\{y \mid y \geq 1\} \) or \([1, \infty)\)
   Intercept: \((0,1)\)

98. \( f(x) = \begin{cases} 
  e^x & \text{if } x < 0 \\
  e^{-x} & \text{if } x \geq 0 
\end{cases} \)
   Domain: \((-\infty, \infty)\)
   Range: \(\{y \mid 0 < y \leq 1\} \) or \((0,1]\)
   Intercept: \((0,1)\)

99. \( f(x) = \begin{cases} 
  -e^{-x} & \text{if } x < 0 \\
  -e^x & \text{if } x \geq 0 
\end{cases} \)
   Domain: \((-\infty, \infty)\)
   Range: \(\{y \mid -1 \leq y < 0\} \) or \([-1,0)\)
   Intercept: \((0,-1)\)

100. \( f(x) = \begin{cases} 
  -e^{-x} & \text{if } x < 0 \\
  -e^x & \text{if } x \geq 0 
\end{cases} \)
    Domain: \((-\infty, \infty)\)
    Range: \(\{y \mid y \leq -1\} \) or \((-\infty,-1]\)
    Intercept: \((0,-1)\)
101. \( p(n) = 100(0.97)^n \)
   a. \( p(10) = 100(0.97)^{10} \approx 74\% \) of light
   b. \( p(25) = 100(0.97)^{25} \approx 47\% \) of light

102. \( p(h) = 760e^{-0.145h} \)
   a. \( p(2) = 760e^{-0.145(2)} = 760e^{-0.290} = 568.68 \text{ mm of Hg} \)
   b. \( p(10) = 760e^{-0.145(10)} = 760e^{-1.45} = 178.27 \text{ mm of Hg} \)

103. \( p(x) = 16,630(0.90)^x \)
   a. \( p(3) = 16,630(0.90)^3 \approx $12,123 \)
   b. \( p(9) = 16,630(0.90)^9 \approx $6,443 \)

104. \( A(n) = Ae^{-0.35n} \)
   a. \( A(3) = 100e^{-0.35(3)} = 100e^{-1.05} = 34.99 \text{ square millimeters} \)
   b. \( A(10) = 100e^{-0.35(10)} = 100e^{-3.5} = 3.02 \text{ square millimeters} \)

105. \( D(h) = 5e^{-0.4h} \)

   \( D(1) = 5e^{-0.4(1)} = 5e^{-0.4} = 3.35 \)

   After 1 hours, 3.35 milligrams will be present.

   \( D(6) = 5e^{-0.4(6)} = 5e^{-2.4} = 0.45 \text{ milligrams} \)

   After 6 hours, 0.45 milligrams will be present.

106. \( N = P\left(1 - e^{-0.15t}\right) \)

   \( N(3) = 1000\left(1 - e^{-0.15(3)}\right) = 1000\left(1 - e^{-0.45}\right) = 362 \)

   After 3 days, 362 students will have heard the rumor.

107. \( F(t) = 1 - e^{-0.1t} \)
   a. \( F(10) = 1 - e^{-0.1(10)} = 1 - e^{-1} = 0.632 \)

The probability that a car will arrive within 10 minutes of 12:00 PM is 0.632.

b. \( F(40) = 1 - e^{-0.1(40)} = 1 - e^{-4} = 0.982 \)

The probability that a car will arrive within 40 minutes of 12:00 PM is 0.982.

c. As \( t \to \infty, F(t) = 1 - e^{-0.1t} \to 1 - 0 = 1 \)

d. Graphing the function:

\[ y = 1 - e^{-0.1t} \]

\[ 0 \leq t \leq 40 \]

108. \( F(t) = 1 - e^{-0.15t} \)
   a. \( F(15) = 1 - e^{-0.15(15)} = 1 - e^{-2.25} = 0.895 \)

The probability that a car will arrive within 15 minutes of 5:00 PM is 0.895.

b. \( F(30) = 1 - e^{-0.15(30)} = 1 - e^{-4.5} = 0.989 \)

The probability that a car will arrive within 30 minutes of 5:00 PM is 0.989.

c. As \( t \to \infty, F(t) = 1 - e^{-0.15t} \to 1 - 0 = 1 \)

d. Graphing the function:

\[ y = 1 - e^{-0.15t} \]

\[ 0 \leq t \leq 40 \]

e. \( F(6) = 0.60 \), so 6 minutes are needed for the probability to reach 60%.

\[ y = 1 - e^{-0.15t} \]

\[ 0 \leq t \leq 40 \]
109. \( P(x) = \frac{20^x e^{-20}}{x!} \)

a. \( P(15) = \frac{20^{15} e^{-20}}{15!} = 0.0516 \) or 5.16% 

The probability that 15 cars will arrive between 5:00 PM and 6:00 PM is 5.16%.

b. \( P(20) = \frac{20^{20} e^{-20}}{20!} = 0.0888 \) or 8.88% 

The probability that 20 cars will arrive between 5:00 PM and 6:00 PM is 8.88%.

110. \( P(x) = \frac{4^x e^{-4}}{x!} \)

a. \( P(5) = \frac{4^5 e^{-4}}{5!} = 0.1563 \) or 15.63% 

The probability that 5 people will arrive within the next minute is 15.63%.

b. \( P(8) = \frac{4^8 e^{-4}}{8!} = 0.0298 \) or 2.98% 

The probability that 8 people will arrive within the next minute is 2.98%.

111. \( R = 10 \left( \frac{4221}{T+459.4} \right)^2 \left( \frac{4221}{D+459.4} \right)^2 \)

a. \( R = 10 \left( \frac{4221}{50+459.4} \right)^2 = 70.95\% \)

b. \( R = 10 \left( \frac{4221}{68+459.4} \right)^2 = 72.62\% \)

c. \( R = 10 \left( \frac{4221}{T+459.4} \right)^2 \left( \frac{4221}{D+459.4} \right)^2 = 10^3 = 100\% \)

112. \( L(t) = 500 \left( 1 - e^{-0.0061 t} \right) \)

a. \( L(30) = 500 \left( 1 - e^{-0.0061(30)} \right) \)

\[ = 500 \left( 1 - e^{-0.183} \right) = 84 \]

The student will learn about 84 words after 30 minutes.

b. \( L(60) = 500 \left( 1 - e^{-0.0061(60)} \right) \)

\[ = 500 \left( 1 - e^{-0.366} \right) = 153 \]

The student will learn about 153 words after 60 minutes.

113. \( I = \frac{E}{R} \left( 1 - e^{-\left( \frac{R}{T} \right) t} \right) \)

a. \( I_1 = \frac{120}{10} \left[ 1 - \left( \frac{10}{3} \right)^{0.3} \right] = 12 \left[ 1 - e^{-0.6} \right] = 5.414 \) amperes after 0.3 second

\( I_1 = \frac{120}{10} \left[ 1 - \left( \frac{10}{3} \right)^{0.5} \right] = 12 \left[ 1 - e^{-1} \right] = 7.585 \) amperes after 0.5 second

\( I_1 = \frac{120}{10} \left[ 1 - \left( \frac{10}{3} \right)^{1} \right] = 12 \left[ 1 - e^{-2} \right] = 10.376 \) amperes after 1 second

b. As \( t \to \infty, e^{-\left( \frac{10}{3} \right) t} \to 0 \). Therefore, as,

\[ t \to \infty, I_1 = \frac{120}{10} \left[ 1 - \left( \frac{10}{3} \right)^{0.3} \right] \to 12 \left[ 1 - 0 \right] = 12 \]

which means the maximum current is 12 amperes.

c. See the graph at the end of the solution.

d. \( I_2 = \frac{120}{5} \left[ 1 - e^{-\left( \frac{5}{10} \right)^{0.3}} \right] = 24 \left[ 1 - e^{-0.15} \right] \)

\[ = 3.343 \text{ amperes after 0.3 second} \]

\( I_2 = \frac{120}{5} \left[ 1 - e^{-\left( \frac{5}{10} \right)^{0.5}} \right] = 24 \left[ 1 - e^{-0.25} \right] \)

\[ = 5.309 \text{ amperes after 0.5 second} \]

\( I_2 = \frac{120}{5} \left[ 1 - e^{-\left( \frac{5}{10} \right)^{1}} \right] = 24 \left[ 1 - e^{-0.5} \right] \)

\[ = 9.443 \text{ amperes after 1 second} \]

e. As \( t \to \infty, e^{-\left( \frac{5}{10} \right) t} \to 0 \). Therefore, as,

\[ t \to \infty, I_1 = \frac{120}{5} \left[ 1 - e^{-\left( \frac{10}{3} \right)^{0.3}} \right] \to 24 \left[ 1 - 0 \right] = 24 \]

which means the maximum current is 24 amperes.

f. See the graph that follows.
114. \( I = \frac{E}{R} \cdot e^{\left(\frac{-t}{RC}\right)} \)

a. \( I_1 = \frac{120}{2000} e^{\left(\frac{-0}{2000 \cdot 1}\right)} = \frac{120}{2000} e^0 = 0.06 \) amperes initially.

\( I_1 = \frac{120}{2000} e^{\left(\frac{-1000}{2000 \cdot 1}\right)} = \frac{120}{2000} e^{-1/2} = 0.0364 \) amperes after 1000 microseconds

\( I_1 = \frac{120}{2000} e^{\left(\frac{-3000}{2000 \cdot 1}\right)} = \frac{120}{2000} e^{-1.5} = 0.0134 \) amperes after 3000 microseconds

b. The maximum current occurs at \( t = 0 \).

Therefore, the maximum current is 0.06 amperes.

c. Graphing the function:

\[ y = 0.06e^{-\frac{t}{2000}} \]

\( (0, 0.06) \)

\[ y = 0.12e^{-\frac{t}{1000}} \]

\( (0, 0.12) \)

d. \( I_2 = \frac{120}{1000} e^{\left(\frac{-1000}{1000 \cdot 2}\right)} = \frac{120}{1000} e^0 = 0.12 \) amperes initially.

\( I_2 = \frac{120}{1000} e^{\left(\frac{-1000}{1000 \cdot 2}\right)} = \frac{120}{1000} e^{-1/2} = 0.0728 \) amperes after 1000 microseconds

\( I_2 = \frac{120}{1000} e^{\left(\frac{-3000}{1000 \cdot 2}\right)} = \frac{120}{1000} e^{-1.5} = 0.0268 \) amperes after 3000 microseconds

e. The maximum current occurs at \( t = 0 \).

Therefore, the maximum current is 0.12 amperes.

f. Graphing the functions:
115. Since the growth rate is 3 then \( a = 3 \). So we have

\[
\begin{align*}
f(x) &= C \cdot 3^x \\
f(6) &= C \cdot 3^6 \\
12 &= C \cdot 3^6 \\
\frac{12}{3^6} &= C
\end{align*}
\]

So \( f(7) = 36 \)

116. 
\[
2 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots + \frac{1}{n!}
\]

\( n = 4; \quad 2 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} = 2.7083 \)

\( n = 6; \quad 2 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} = 2.7181 \)

\( n = 8; \quad 2 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \frac{1}{9!} + \frac{1}{10!} = 2.7182788 \)

\( n = 10; \quad 2 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \frac{1}{9!} + \frac{1}{10!} + \frac{1}{11!} = 2.7182818128 \)

\( e = 2.718281828 \)

117. 
\[
2 + 1 = 3
\]

\[
2 + 1 = 2.5 < e
\]

\[
2 + 1 = 2.8 > e
\]

\[
2 + 1 = 2.7 < e
\]

\[
2 + 1 = 2.721649485 > e
\]

118. \( f(x) = a^x \)

\[
\frac{f(x + h) - f(x)}{h} = \frac{a^{x+h} - a^x}{h}
\]

\[
= \frac{a^x(a^h - 1)}{h}
\]

119. \( f(x) = a^x \)

\( f(A + B) = a^{A+B} = a^A \cdot a^B = f(A) \cdot f(B) \)

120. \( f(x) = a^x \)

\( f(-x) = a^{-x} = \frac{1}{a^x} = \frac{1}{f(x)} \)

121. \( f(x) = a^x \)

\( f(\alpha x) = a^{\alpha x} = (a^x)^\alpha = [f(x)]^\alpha \)

122. \( \sinh x = \frac{1}{2} (e^x - e^{-x}) \)
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a.  \( f(-x) = \sinh(-x) \)
    \[ = \frac{1}{2}(e^{-x} - e^{x}) \]
    \[ = -\frac{1}{2}(e^{x} - e^{-x}) \]
    \[ = -\sinh x \]
    \[ = -f(x) \]

Therefore, \( f(x) = \sinh x \) is an odd function.

b.  Let \( Y_1 = \frac{1}{2}(e^x - e^{-x}) \).

123.  \( \cosh x = \frac{1}{2}(e^x + e^{-x}) \)

a.  \( f(-x) = \cosh(-x) \)
    \[ = \frac{1}{2}(e^{-x} + e^{x}) \]
    \[ = \frac{1}{2}(e^{x} + e^{-x}) \]
    \[ = \cosh x \]
    \[ = f(x) \]

Thus, \( f(x) = \cosh x \) is an even function.

b.  Let \( Y_1 = \frac{1}{2}(e^x + e^{-x}) \).

124.  \( f(x) = 2^{(2^x)} + 1 \)
    \[ f(1) = 2^{(2^1)} + 1 = 2^2 + 1 = 4 + 1 = 5 \]
    \[ f(2) = 2^{(2^2)} + 1 = 2^4 + 1 = 16 + 1 = 17 \]
    \[ f(3) = 2^{(2^3)} + 1 = 2^8 + 1 = 256 + 1 = 257 \]
    \[ f(4) = 2^{(2^4)} + 1 = 2^{16} + 1 = 65,536 + 1 = 65,537 \]
    \[ f(5) = 2^{(2^5)} + 1 = 2^{32} + 1 = 4,294,967,296 + 1 \]
    \[ = 4,294,967,297 \]
    \[ = 641 \times 6,700,417 \]

125.  Since the number of bacteria doubles every minute, half of the container is full one minute before it is full. Thus, it takes 59 minutes to fill the container.

126.  Answers will vary.

127.  Answers will vary.

128.  Given the function \( f(x) = a^x \), with \( a > 1 \),
      If \( x > 0 \), the graph becomes steeper as \( a \)
      increases.
      If \( x < 0 \), the graph becomes less steep as \( a \)
      increases.

129.  Using the laws of exponents, we have:
    \[ a^{-x} = \frac{1}{a^x} = \left(\frac{1}{a}\right)^x \]. So \( y = a^{-x} \) and
    \[ y = \left(\frac{1}{a}\right)^x \] will have the same graph.
Section 5.4

1. a.  \(3x - 7 \leq 8 - 2x\)
\[5x \leq 15\]
\[x \leq 3\]
The solution set is \(\{x | x \leq 3\}\).

b.  \(x^2 - x - 6 > 0\)
We graph the function \(f(x) = x^2 - x - 6\).
The intercepts are
\(y\)-intercept: \(f(0) = -6\)
\(x\)-intercepts: \(x^2 - x - 6 = 0\)
\((x + 2)(x - 3) = 0\)
\(x = -2, x = 3\)
The vertex is at \(x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}\). Since
\[f\left(\frac{1}{2}\right) = -\frac{25}{4},\] the vertex is \(\left(\frac{1}{2}, -\frac{25}{4}\right)\).

The graph is above the \(x\)-axis when \(x < -2\) or \(x > 3\). Since the inequality is strict, the solution set is \(\{x | x < -2\ or\ x > 3\}\) or, using interval notation, \((-\infty, -2) \cup (3, \infty)\).

2.  \(\frac{x - 1}{x + 4} > 0\)
\[f(x) = \frac{x - 1}{x + 4}\]
f is zero or undefined when \(x = 1\) or \(x = -4\).

<table>
<thead>
<tr>
<th>Interval</th>
<th>((-\infty, -4))</th>
<th>((-4, 1))</th>
<th>((1, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Value</td>
<td>-5</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Value of (f)</td>
<td>6</td>
<td>-\frac{1}{4}</td>
<td>\frac{1}{6}</td>
</tr>
<tr>
<td>Conclusion</td>
<td>positive</td>
<td>negative</td>
<td>positive</td>
</tr>
</tbody>
</table>

The solution set is \(\{x | x < -4\ or\ x > 1\}\) or, using interval notation, \((-\infty, -4) \cup (1, \infty)\).

3.  \(2x + 3 = 9\)
\[2x = 6\]
\[x = 3\]

4.  \(\{x | x > 0\}\) or \((0, \infty)\)

5.  \(\left(\frac{1}{a}, -1\right), (1, 0), (a, 1)\)

6.  1

7.  False. If \(y = \log_a x\), then \(x = a^y\).

8.  True

9.  \(9 = 3^2\) is equivalent to \(2 = \log_3 9\).

10. \(16 = 4^2\) is equivalent to \(2 = \log_4 16\).

11. \(a^2 = 1.6\) is equivalent to \(2 = \log_a 1.6\).

12. \(a^3 = 2.1\) is equivalent to \(3 = \log_a 2.1\).

13. \(2^x = 7.2\) is equivalent to \(x = \log_2 7.2\).

14. \(3^t = 4.6\) is equivalent to \(x = \log_3 4.6\).

15. \(e^x = 8\) is equivalent to \(x = \ln 8\).

16. \(e^{2x} = M\) is equivalent to \(2x = \ln M\).

17. \(\log_2 8 = 3\) is equivalent to \(2^3 = 8\).

18. \(\log_3 \left(\frac{1}{9}\right) = -2\) is equivalent to \(3^{-2} = \frac{1}{9}\).

19. \(\log_3 3 = 6\) is equivalent to \(a^6 = 3\).

20. \(\log_4 4 = 2\) is equivalent to \(b^2 = 4\).

21. \(\log_3 2 = x\) is equivalent to \(3^x = 2\).

22. \(\log_2 6 = x\) is equivalent to \(2^x = 6\).

23. \(\ln 4 = x\) is equivalent to \(e^x = 4\).

24. \(\ln x = 4\) is equivalent to \(e^x = x\).
25. \( \log_2 1 = 0 \) since \( 2^0 = 1 \).

26. \( \log_8 8 = 1 \) since \( 8^1 = 8 \).

27. \( \log_5 25 = 2 \) since \( 5^2 = 25 \).

28. \( \log_3 \left( \frac{1}{9} \right) = -2 \) since \( 3^{-2} = \frac{1}{9} \).

29. \( \log_{\sqrt{2}} 16 = -4 \) since \( \left( \frac{1}{2} \right)^{-4} = 2^4 = 16 \).

30. \( \log_{\sqrt{3}} 9 = -2 \) since \( \left( \frac{1}{3} \right)^{-2} = 3^2 = 9 \).

31. \( \log_{10} \sqrt{10} = \frac{1}{2} \) since \( 10^{1/2} = \sqrt{10} \).

32. \( \log_5 \sqrt{25} = \frac{2}{3} \) since \( 5^{2/3} = 25^{1/3} = \sqrt[3]{25} \).

33. \( \log_{\sqrt{2}} 4 = 4 \) since \( \left( \sqrt{2} \right)^4 = 4 \).

34. \( \log_{\sqrt{3}} 9 = 4 \) since \( \left( \sqrt{3} \right)^4 = 9 \).

35. \( \ln e = \frac{1}{2} \) since \( e^{1/2} = \sqrt{e} \).

36. \( \ln e^3 = 3 \) since \( e^3 = e^3 \).

37. \( f(x) = \ln(x-3) \) requires \( x-3 > 0 \).
\( x-3 > 0 \)
\( x > 3 \)

The domain of \( f \) is \( \{x \mid x > 3\} \) or \((3,\infty)\).

38. \( g(x) = \ln(x-1) \) requires \( x-1 > 0 \).
\( x-1 > 0 \)
\( x > 1 \)

The domain of \( g \) is \( \{x \mid x > 1\} \) or \((1,\infty)\).

39. \( F(x) = \log_2 x^2 \) requires \( x^2 > 0 \).
\( x^2 > 0 \) for all \( x \neq 0 \).

The domain of \( F \) is \( \{x \mid x \neq 0\} \).

40. \( H(x) = \log_5 x^3 \) requires \( x^3 > 0 \).
\( x^3 > 0 \) for all \( x > 0 \).

The domain of \( H \) is \( \{x \mid x > 0\} \) or \((0,\infty)\).

41. \( f(x) = 3-2\log_4 \left( \frac{x}{2} - 5 \right) \) requires \( \frac{x}{2} - 5 > 0 \).
\( \frac{x}{2} - 5 > 0 \)
\( x > 10 \)

The domain of \( f \) is \( \{x \mid x > 10\} \) or \((10,\infty)\).

42. \( g(x) = 8+5\ln(2x+3) \) requires \( 2x+3 > 0 \).
\( 2x+3 > 0 \)
\( 2x > -3 \)
\( x > -\frac{3}{2} \)

The domain of \( g \) is \( \{x \mid x > -\frac{3}{2}\} \) or \((-\frac{3}{2},\infty)\).

43. \( f(x) = \ln \left( \frac{1}{x+1} \right) \) requires \( \frac{1}{x+1} > 0 \).
\( p(x) = \frac{1}{x+1} \) is undefined when \( x = -1 \).

<table>
<thead>
<tr>
<th>Interval</th>
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<th>((-1,\infty))</th>
</tr>
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<tbody>
<tr>
<td>Test Value</td>
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<td>0</td>
</tr>
<tr>
<td>Value of ( p )</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>Conclusion</td>
<td>negative</td>
<td>positive</td>
</tr>
</tbody>
</table>

The domain of \( f \) is \( \{x \mid x > -1\} \) or \((-1,\infty)\).

44. \( g(x) = \ln \left( \frac{1}{x-5} \right) \) requires \( \frac{1}{x-5} > 0 \).
\( p(x) = \frac{1}{x-5} \) is undefined when \( x = 5 \).

<table>
<thead>
<tr>
<th>Interval</th>
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<th>((5,\infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Value</td>
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<td>6</td>
</tr>
<tr>
<td>Value of ( p )</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>Conclusion</td>
<td>negative</td>
<td>positive</td>
</tr>
</tbody>
</table>

The domain of \( g \) is \( \{x \mid x > 5\} \) or \((5,\infty)\).
**Chapter 5: Exponential and Logarithmic Functions**

45. \( g(x) = \log_5 \left( \frac{x+1}{x} \right) \) requires \( \frac{x+1}{x} > 0 \).
   \[
p(x) = \frac{x+1}{x}
   \]
is zero or undefined when \( x = -1 \) or \( x = 0 \).

<table>
<thead>
<tr>
<th>Interval</th>
<th>(−∞,−1)</th>
<th>(−1,0)</th>
<th>(0,∞)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Value</td>
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<td>−( \frac{1}{2} )</td>
<td>1</td>
</tr>
<tr>
<td>Value of ( p )</td>
<td>( \frac{1}{2} )</td>
<td>−1</td>
<td>2</td>
</tr>
<tr>
<td>Conclusion</td>
<td>positive</td>
<td>negative</td>
<td>positive</td>
</tr>
</tbody>
</table>

The domain of \( g \) is \( \{ x \mid x < -1 \text{ or } x > 0 \} \); \( (−∞,−1) \cup (0,∞) \).

46. \( h(x) = \log_3 \left( \frac{x}{x−1} \right) \) requires \( \frac{x}{x−1} > 0 \).
   \[
p(x) = \frac{x}{x−1}
   \]
is zero or undefined when \( x = 0 \) or \( x = 1 \).

<table>
<thead>
<tr>
<th>Interval</th>
<th>(−∞,0)</th>
<th>(0,1)</th>
<th>(1,∞)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Value</td>
<td>−1</td>
<td>−( \frac{1}{2} )</td>
<td>2</td>
</tr>
<tr>
<td>Value of ( p )</td>
<td>( \frac{1}{2} )</td>
<td>−1</td>
<td>2</td>
</tr>
<tr>
<td>Conclusion</td>
<td>positive</td>
<td>negative</td>
<td>positive</td>
</tr>
</tbody>
</table>

The domain of \( h \) is \( \{ x \mid x < 0 \text{ or } x > 1 \} \); \( (−∞,0) \cup (1,∞) \).

47. \( f(x) = \sqrt{\ln x} \) requires \( \ln x \geq 0 \) and \( x > 0 \)
   \[
   \ln x \geq 0 \\
x \geq e^0 \\
x \geq 1
   \]
The domain of \( h \) is \( \{ x \mid x \geq 1 \} \) or \( [1,∞) \).

48. \( g(x) = \frac{1}{\ln x} \) requires \( \ln x \neq 0 \) and \( x > 0 \)
   \[
   \ln x \neq 0 \\
x \neq e^0 \\
x \neq 1
   \]
The domain of \( h \) is \( \{ x \mid x > 0 \text{ and } x \neq 1 \} \); \( (0,1) \cup (1,∞) \).

49. \( \ln \left( \frac{5}{3} \right) = 0.511 \)

50. \( \frac{\ln 5}{3} = 0.536 \)

51. \( \frac{\ln 10}{0.04} = 30.099 \)

52. \( \frac{\ln 2}{-0.1} = 4.055 \)

53. \( \frac{\ln 4 + \ln 2}{\log 4 + \log 2} = 2.303 \)

54. \( \frac{\log 15 + \log 20}{\ln 15 + \ln 20} = 0.434 \)

55. \( \frac{2\ln 5 + \log 50}{\log 4 - \ln 2} = -53.991 \)

56. \( \frac{3\log 80 - \ln 5}{\log 5 + \ln 20} = 1.110 \)

57. If the graph of \( f(x) = \log_a x \) contains the point \( (2,2) \), then \( f(2) = \log_a 2 = 2 \). Thus,
   \[ a^2 = 2 \]
   \[ a = \pm 2 \]
Since the base \( a \) must be positive by definition, we have that \( a = 2 \).

58. If the graph of \( f(x) = \log_a x \) contains the point \( \left( \frac{1}{2},-4 \right) \), then \( f \left( \frac{1}{2} \right) = \log_a \left( \frac{1}{2} \right) = -4 \). Thus,
   \[ \log_a \left( \frac{1}{2} \right) = -4 \]
   \[ a^{-4} = \frac{1}{2} \]
   \[ \frac{1}{a^4} = \frac{1}{2} \]
   \[ a^4 = 2 \]
   \[ a = 2^{1/4} = 1.189 \]
59. \( f(x) = 3^x \)  
\[ f^{-1}(x) = \log_3 x \]
\[ f^{-1}(1) = \log_3 1 = 0 \]

60. \( f(x) = 4^x \)  
\[ f^{-1}(x) = \log_4 x \]
\[ f^{-1}(1) = \log_4 1 = 0 \]

61. \( f(x) = \left(\frac{1}{2}\right)^x \)  
\[ f^{-1}(x) = \log_{\frac{1}{2}} x \]
\[ f^{-1}(1) = \log_{\frac{1}{2}} 1 = 0 \]

62. \( f(x) = \left(\frac{1}{3}\right)^x \)  
\[ f^{-1}(x) = \log_{\frac{1}{3}} x \]
\[ f^{-1}(1) = \log_{\frac{1}{3}} 1 = 0 \]

63. \( f(x) = \ln(x+4) \)  
\[ a. \quad \text{Domain: } (-4, \infty) \]
\[ b. \quad \text{Using the graph of } y = \ln x, \text{ shift the graph 4 units to the left.} \]
\[ c. \quad \text{Range: } (-\infty, \infty) \]
\[ \text{Vertical Asymptote: } x = -4 \]
\[ d. \quad f(x) = \ln(x+4) \]
\[ y = \ln(x+4) \]
\[ x = \ln(y+4) \quad \text{Inverse} \]
\[ y + 4 = e^x \]
\[ y = e^x - 4 \]
\[ f^{-1}(x) = e^x - 4 \]
\[ e. \quad \text{The domain of the inverse found in part (d) is all real numbers.} \]
\[ \text{Since the domain of } f \text{ is the range of } f^{-1}, \]
\[ \text{we can use the result from part (a) to say that the range of } f^{-1} \text{ is } (-4, \infty). \]
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72. \( f(x) = \ln(x - 3) \)

a. Domain: \((3, \infty)\)

b. Using the graph of \( y = \ln x \), shift the graph 3 units to the right.

c. Range: \((-\infty, \infty)\)
   Vertical Asymptote: \(x = 3\)

d. \( f(x) = \ln(x - 3) \)
   \( y = \ln(x - 3) \)
   \( x = \ln(y - 3) \) Inverse
   \( y - 3 = e^x \)
   \( y = e^x + 3 \)
   \( f^{-1}(x) = e^{x+3} \)

e. The domain of the inverse found in part (d) is all real numbers.
   Since the domain of \( f \) is the range of \( f^{-1} \), we can use the result from part (a) to say that the range of \( f^{-1} \) is \((3, \infty)\).

73. \( f(x) = 2 + \ln x \)

a. Domain: \((0, \infty)\)

b. Using the graph of \( y = \ln x \), shift up 2 units.

c. Range: \((-\infty, \infty)\)
   Vertical Asymptote: \(x = 0\)

d. \( f(x) = 2 + \ln x \)
   \( y = 2 + \ln x \)
   \( x = 2 + \ln y \) Inverse
   \( x - 2 = \ln y \)
   \( y = e^{x-2} \)
   \( f^{-1}(x) = e^{x-2} \)

e. The domain of the inverse found in part (d) is all real numbers.
   Since the domain of \( f \) is the range of \( f^{-1} \), we can use the result from part (a) to say that the range of \( f^{-1} \) is \((0, \infty)\).
f. Using the graph of \( y = e^x \), shift the graph 2 units to the right.

74. \( f(x) = -\ln(-x) \)
   a. Domain: \((-\infty, 0)\)
   b. Using the graph of \( y = \ln x \), reflect the graph about the \( y \)-axis, and reflect about the \( x \)-axis.
   c. Range: \((-\infty, \infty)\)
   Vertical Asymptote: \( x = 0 \)
   d. \[ f(x) = -\ln(-x) \]
      \[ y = -\ln(-x) \]
      \[ x = -\ln(-y) \]
      Inverse
      \[ -x = \ln(-y) \]
      \[ -y = e^{-x} \]
      \[ y = -e^{-x} \]
      \[ f^{-1}(x) = -e^{-x} \]
   e. The domain of the inverse found in part (d) is all real numbers.
      Since the domain of \( f \) is the range of \( f^{-1} \), we can use the result from part (a) to say that the range of \( f^{-1} \) is \((-\infty, 0)\).

f. Using the graph of \( y = e^x \), reflect the graph about the \( y \)-axis, and reflect about the \( x \)-axis.

75. \( f(x) = \ln(2x) - 3 \)
   a. Domain: \((0, \infty)\)
   b. Using the graph of \( y = \ln x \), compress the graph horizontally by a factor of \( \frac{1}{2} \), and shift down 3 units.
   c. Range: \((-\infty, \infty)\)
   Vertical Asymptote: \( x = 0 \)
   d. \[ f(x) = \ln(2x) - 3 \]
      \[ y = \ln(2x) - 3 \]
      \[ x = \ln(2y) - 3 \]
      Inverse
      \[ x + 3 = \ln(2y) \]
      \[ 2y = e^{x+3} \]
      \[ y = \frac{1}{2} e^{x+3} \]
      \[ f^{-1}(x) = \frac{1}{2} e^{x+3} \]
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e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of \( f \) is the range of \( f^{-1} \), we can use the result from part (a) to say that the range of \( f^{-1} \) is \((0, \infty)\).

f. Using the graph of \( y = e^x \), reflect the graph about the y-axis, and reflect about the x-axis.

76. \( f(x) = -2 \ln(x + 1) \)

a. Domain: \((-1, \infty)\)

b. Using the graph of \( y = \ln x \), shift the graph to the left 1 unit, reflect about the x-axis and stretch vertically by a factor of 2.

c. Range: \((-\infty, \infty)\)
Vertical Asymptote: \( x = -1 \)

d. \( f(x) = -2 \ln(x + 1) \)
\[ y = -2 \ln(x + 1) \]
\[ x = -2 \ln(y + 1) \quad \text{Inverse} \]
\[ -\frac{x}{2} = \ln(y + 1) \]
\[ y + 1 = e^{-x/2} \]
\[ y = e^{-x/2} - 1 \]
\[ f^{-1}(x) = e^{-x/2} - 1 \]

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of \( f \) is the range of \( f^{-1} \), we can use the result from part (a) to say that the range of \( f^{-1} \) is \((-1, \infty)\).

f. Using the graph of \( y = e^x \), reflect the graph about the y-axis, stretch horizontally by a factor of 2, and shift down 1 unit.

77. \( f(x) = \log(x - 4) + 2 \)

a. Domain: \((4, \infty)\)

b. Using the graph of \( y = \log x \), shift the graph 4 units to the right and 2 units up.

c. Range: \((-\infty, \infty)\)
Vertical Asymptote: \( x = 4 \)

d. \( f(x) = \log(x - 4) + 2 \)
\[ y = \log(x - 4) + 2 \]
\[ x = \log(y - 4) + 2 \quad \text{Inverse} \]
\[ x - 2 = \log(y - 4) \]
\[ y - 4 = 10^{x-2} \]
\[ y = 10^{x-2} + 4 \]
\[ f^{-1}(x) = 10^{x-2} + 4 \]

e. The domain of the inverse found in part (d) is all real numbers.
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Since the domain of \( f \) is the range of \( f^{-1} \), we can use the result from part (a) to say that the range of \( f^{-1} \) is \((4, \infty)\).

**f.** Using the graph of \( y = 10^x \), shift the graph 2 units to the right and 4 units up.

78. \( f(x) = \frac{1}{2} \log x - 5 \)
   
   a. Domain: \((0, \infty)\)
   
   b. Using the graph of \( y = \log x \), compress the graph vertically by a factor of \( \frac{1}{2} \) and shift it 5 units down.

79. \( f(x) = \frac{1}{2} \log (2x) \)
   
   a. Domain: \((0, \infty)\)
   
   b. Using the graph of \( y = \log x \), compress the graph horizontally by a factor of \( \frac{1}{2} \), and compress vertically by a factor of \( \frac{1}{2} \).
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80. \( f(x) = \log(-2x) \)
   
a. Domain: \((-\infty, 0)\)
   
b. Using the graph of \( y = \log x \), reflect the graph across the \( y \)-axis and compress horizontally by a factor of \( \frac{1}{2} \).
   
   ![Graph of \( f(x) = \log(-2x) \)]
   
   c. Range: \((-\infty, \infty)\)
   
   Vertical Asymptote: \( x = 0 \)
   
   d. \( f(x) = \frac{1}{2} \log(2x) \)
      
      \[ y = \frac{1}{2} \log(2x) \]
      
      \[ x = \frac{1}{2} \log(2y) \quad \text{Inverse} \]
      
      \[ 2x = \log(2y) \]
      
      \[ 2y = 10^{2x} \]
      
      \[ y = \frac{1}{2} \cdot 10^{2x} \]
      
      \[ f^{-1}(x) = \frac{1}{2} \cdot 10^{2x} \]
   
   e. The domain of the inverse found in part (d) is all real numbers.

   Since the domain of \( f \) is the range of \( f^{-1} \), we can use the result from part (a) to say that the range of \( f^{-1} \) is \((0, \infty)\).
   
   f. Using the graph of \( y = 10^x \), compress the graph horizontally by a factor of \( \frac{1}{2} \), and compress vertically by a factor of \( \frac{1}{2} \).

   ![Graph of \( f(x) = \frac{1}{2} \log(2x) \)]

   ![Graph of \( y = 10^x \)]
81. \( f(x) = 3 + \log_2 (x + 2) \)
   a. Domain: \((-2, \infty)\)
   b. Using the graph of \( y = \log_2 x \), shift 2 units to the left, and shift up 3 units.
   
   c. Range: \((-\infty, \infty)\)
      Vertical Asymptote: \( x = -2 \)
   d. \[ f(x) = 3 + \log_2 (x + 2) \]
      \[ y = 3 + \log_2 (x + 2) \]
      \[ x = 3 + \log_2 (y + 2) \]
      Inverse \[ x - 3 = \log_2 (y + 2) \]
      \[ y + 2 = 3^{x-3} \]
      \[ y = 3^{x-3} - 2 \]
      \[ f^{-1}(x) = 3^{x-3} - 2 \]
   e. The domain of the inverse found in part (d) is all real numbers.
      Since the domain of \( f \) is the range of \( f^{-1} \), we can use the result from part (a) to say that the range of \( f^{-1} \) is \((-2, \infty)\).
   f. Using the graph of \( y = 3^x \), shift 3 units to the right, and shift down 2 units.

82. \( f(x) = 2 - \log_3 (x + 1) \)
   a. Domain: \((-1, \infty)\)
   b. Using the graph of \( y = \log_3 x \), shift 1 unit to the left, reflect the graph about the \( x \)-axis, and shift 2 units up.
   
   c. Range: \((-\infty, \infty)\)
      Vertical Asymptote: \( x = -1 \)
   d. \[ f(x) = 2 - \log_3 (x + 1) \]
      \[ y = 2 - \log_3 (x + 1) \]
      \[ x = 2 - \log_3 (y + 1) \]
      Inverse \[ x - 2 = -\log_3 (y + 1) \]
      \[ 2 - x = \log_3 (y + 1) \]
      \[ y + 1 = 3^{2-x} \]
      \[ y = 3^{2-x} - 1 \]
      \[ f^{-1}(x) = 3^{2-x} - 1 \]
   e. The domain of the inverse found in part (d) is all real numbers.
      Since the domain of \( f \) is the range of \( f^{-1} \), we can use the result from part (a) to say that the range of \( f^{-1} \) is \((-1, \infty)\).
   f. Using the graph of \( y = 3^x \), reflect the graph about the \( y \)-axis, shift 2 units to the right, and shift down 1 unit.
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83. \( f(x) = e^{x+2} - 3 \)
   a. Domain: \((-\infty, \infty)\)

   b. Using the graph of \( y = e^x \), shift the graph two units to the left, and shift 3 units down.

   ![Graph of \( y = e^x \) shifted two units left and three units down]

   c. Range: \((-3, \infty)\)
   Horizontal Asymptote: \( y = -3 \)

   d. \( f(x) = e^{x+2} - 3 \)
      
      \[
      \begin{align*}
      y &= e^{x+2} - 3 \\
      x &= e^{y+2} - 3 \text{ Inverse} \\
      x + 3 &= e^{y+2} \\
      y + 2 &= \ln(x + 3) \\
      y &= \ln(x + 3) - 2 \\
      f^{-1}(x) &= \ln(x + 3) - 2
      \end{align*}
      \]

   e. For the domain of \( f^{-1} \) we need
      \[x + 3 > 0 \Rightarrow x > -3\]
      So the domain of the inverse found in part (d) is \((-3, \infty)\).

      Since the domain of \( f \) is the range of \( f^{-1} \),
      we can use the result from part (a) to say that the range of \( f^{-1} \) is \((-\infty, \infty)\).

f. Using the graph of \( y = \ln x \), shift 3 units to the left, and shift down 2 units.

84. \( f(x) = 3e^x + 2 \)
   a. Domain: \((-\infty, \infty)\)

   b. Using the graph of \( y = e^x \), stretch the graph vertically by a factor of 3, and shift 2 units up.

   ![Graph of \( y = e^x \) stretched vertically by 3 and shifted up 2 units]

   c. Range: \((2, \infty)\)
   Horizontal Asymptote: \( y = 2 \)

   d. \( f(x) = 3e^x + 2 \)
      
      \[
      \begin{align*}
      y &= 3e^x + 2 \\
      x &= 3e^y + 2 \text{ Inverse} \\
      x - 2 &= 3e^y \\
      \frac{x-2}{3} &= e^y \\
      y &= \ln\left(\frac{x-2}{3}\right) \\
      f^{-1}(x) &= \ln\left(\frac{x-2}{3}\right)
      \end{align*}
      \]
e. For the domain of \( f^{-1} \) we need
\[
\frac{x - 2}{3} > 0
\]
\[
x - 2 > 0
\]
\[
x > 2
\]
The domain of the inverse found in part (d) is \((2, \infty)\).

Since the domain of \( f \) is the range of \( f^{-1} \), we can use the result from part (a) to say that the range of \( f^{-1} \) is \((-\infty, \infty)\).

f. Using the graph of \( y = \ln x \), shift 3 units to the left, and shift down 2 units.

85. \( f(x) = 2^{x/3} + 4 \)
   a. Domain: \((-\infty, \infty)\)
   b. Using the graph of \( y = 2^x \), stretch the graph horizontally by a factor of 3, and shift 4 units up.
   c. Range: \((4, \infty)\)
   Horizontal Asymptote: \( y = 4 \)

86. \( f(x) = -3^{x+1} \)
   a. Domain: \((-\infty, \infty)\)
   b. Using the graph of \( y = 3^x \), shift the graph to the left 1 unit, and reflect about the \( x \)-axis.
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c. Range: $(−\infty, 0)$
   Horizontal Asymptote: $y = 0$

d. $f(x) = −3^{x+1}$
   $y = −3^{x+1}$
   $x = −3^{x+1}$  Inverse
   $-x = 3^{x+1}$
   $y + 1 = \log_3(−x)$
   $y = \log_3(−x) − 1$
   $f^{-1}(x) = \log_3(−x) − 1$

e. For the domain of $f^{-1}$ we need
   $−x > 0$
   $x < 0$
   The domain of the inverse found in part (d) is $(−\infty, 0)$.

Since the domain of $f$ is the range of $f^{-1}$, we can use the result from part (a) to say that the range of $f^{-1}$ is $(−\infty, \infty)$.

f. Using the graph of $y = \log_3 x$, reflect the graph across the $y$-axis, and shift down 1 unit.

87. $\log_3 x = 2$
    $x = 3^2$
    $x = 9$
    The solution set is $\{9\}$.

88. $\log_5 x = 3$
    $x = 5^3$
    $x = 125$
    The solution set is $\{125\}$.

89. $\log_2 (2x + 1) = 3$
    $2x + 1 = 2^3$
    $2x + 1 = 8$
    $2x = 7$
    $x = \frac{7}{2}$

The solution set is $\left\{\frac{7}{2}\right\}$.

90. $\log_3 (3x − 2) = 2$
    $3x − 2 = 3^2$
    $3x − 2 = 9$
    $3x = 11$
    $x = \frac{11}{3}$

The solution set is $\left\{\frac{11}{3}\right\}$.

91. $\log_x 4 = 2$
    $x^2 = 4$
    $x = 2$  $(x \neq −2$, base is positive)$

The solution set is $\{2\}$.

92. $\log_x \left(\frac{1}{8}\right) = 3$
    $x^3 = \frac{1}{8}$
    $x = \frac{1}{2}$

The solution set is $\left\{\frac{1}{2}\right\}$.

93. $\ln e^x = 5$
    $e^x = e^5$
    $x = 5$

The solution set is $\{5\}$.

94. $\ln e^{-2x} = 8$
    $e^{-2x} = e^8$
    $−2x = 8$
    $x = −4$

The solution set is $\{-4\}$.
Section 5.4: Logarithmic Functions

95. \( \log_4 64 = x \)
   \( 4^x = 64 \)
   \( 4^x = 4^3 \)
   \( x = 3 \)
   The solution set is \( \{3\} \).

96. \( \log_5 625 = x \)
   \( 5^x = 625 \)
   \( 5^x = 5^4 \)
   \( x = 4 \)
   The solution set is \( \{4\} \).

97. \( \log_3 243 = 2x + 1 \)
   \( 3^{2x+1} = 243 \)
   \( 3^{2x+1} = 3^5 \)
   \( 2x + 1 = 5 \)
   \( 2x = 4 \)
   \( x = 2 \)
   The solution set is \( \{2\} \).

98. \( \log_6 36 = 5x + 3 \)
   \( 6^{5x+3} = 36 \)
   \( 6^{5x+3} = 6^2 \)
   \( 5x + 3 = 2 \)
   \( 5x = -1 \)
   \( x = -\frac{1}{5} \)
   The solution set is \( \{-\frac{1}{5}\} \).

99. \( e^{3x} = 10 \)
   \( 3x = \ln 10 \)
   \( x = \frac{\ln 10}{3} \)
   The solution set is \( \left\{ \frac{\ln 10}{3} \right\} \).

100. \( e^{-2x} = \frac{1}{3} \)
    \(-2x = \ln \left( \frac{1}{3} \right) \)
    \(-2x = -\ln 3 \)
    \(2x = \ln 3 \)
    \(x = \frac{\ln 3}{2} \)
    The solution set is \( \left\{ \frac{\ln 3}{2} \right\} \).

101. \( e^{2x+5} = 8 \)
    \(2x + 5 = \ln 8 \)
    \(2x = -5 + \ln 8 \)
    \(x = \frac{-5 + \ln 8}{2} \)
    The solution set is \( \left\{ \frac{-5 + \ln 8}{2} \right\} \).

102. \( e^{-2x+1} = 13 \)
    \(-2x + 1 = \ln 13 \)
    \(-2x = -1 + \ln 13 \)
    \(x = \frac{-1 + \ln 13}{-2} = \frac{1 - \ln 13}{2} \)
    The solution set is \( \left\{ \frac{1 - \ln 13}{2} \right\} \).

103. \( \log_3 \left( x^2 + 1 \right) = 2 \)
    \(x^2 + 1 = 3^2 \)
    \(x^2 + 1 = 9 \)
    \(x^2 = 8 \)
    \(x = \pm \sqrt{8} = \pm 2\sqrt{2} \)
    The solution set is \( \left\{ -2\sqrt{2}, 2\sqrt{2} \right\} \).

104. \( \log_5 \left( x^2 + x + 4 \right) = 2 \)
    \(x^2 + x + 4 = 5^2 \)
    \(x^2 + x + 4 = 25 \)
    \(x^2 + x - 21 = 0 \)
    \(x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-21)}}{2(1)} = \frac{-1 \pm \sqrt{85}}{2} \)
    The solution set is \( \left\{ -\frac{1 - \sqrt{85}}{2}, -\frac{1 + \sqrt{85}}{2} \right\} \).
105. \[ \log_2 8^x = -3 \]
\[ 8^x = 2^{-3} \]
\[ (2^3)^x = 2^{-3} \]
\[ 2^{3x} = 2^{-3} \]
\[ 3x = -3 \]
\[ x = -1 \]
The solution set is \( \{-1\} \).

106. \[ \log_3 3^x = -1 \]
\[ 3^x = 3^{-1} \]
\[ x = -1 \]
The solution set is \( \{-1\} \).

107. \[ 5e^{0.2x} = 7 \]
\[ e^{0.2x} = \frac{7}{5} \]
\[ 0.2x = \ln \frac{7}{5} \]
\[ 5 \cdot 0.2x = 5 \ln \frac{7}{5} \]
\[ x = 5 \ln \frac{7}{5} \]
The solution set is \( \left\{ 5 \ln \frac{7}{5} \right\} \).

108. \[ 8 \cdot 10^{2x-7} = 3 \]
\[ 10^{2x-7} = \frac{3}{8} \]
\[ 2x - 7 = \log \frac{3}{8} \]
\[ 2x = 7 + \log \frac{3}{8} \]
\[ x = \frac{1}{2} \left( 7 + \log \frac{3}{8} \right) \]
The solution set is \( \left\{ \frac{1}{2} \left( 7 + \log \frac{3}{8} \right) \right\} \).

109. \[ 2 \cdot 10^{2-x} = 5 \]
\[ 10^{2-x} = \frac{5}{2} \]
\[ 2 - x = \log \frac{5}{2} \]
\[ -x = -2 + \log \frac{5}{2} \]
\[ x = 2 - \log \frac{5}{2} \]
The solution set is \( \left\{ 2 - \log \frac{5}{2} \right\} \).

110. \[ 4e^{x+1} = 5 \]
\[ e^{x+1} = \frac{5}{4} \]
\[ x + 1 = \ln \frac{5}{4} \]
\[ x = -1 + \ln \frac{5}{4} \]
The solution set is \( \left\{ -1 + \ln \frac{5}{4} \right\} \).

111. a. \( G(x) = \log_3 (2x + 1) - 2 \)
We require that \( 2x + 1 \) be positive.
\[ 2x + 1 > 0 \]
\[ 2x > -1 \]
\[ x > -\frac{1}{2} \]
Domain: \( \left\{ x \mid x > -\frac{1}{2} \right\} \) or \( \left( -\frac{1}{2}, \infty \right) \)

b. \( G(40) = \log_3 (2 \cdot 40 + 1) - 2 \)
\[ = \log_3 81 - 2 \]
\[ = 4 - 2 \]
\[ = 2 \]
The point \( (40, 2) \) is on the graph of \( G \).

c. \( G(x) = 3 \)
\[ \log_3 (2x + 1) - 2 = 3 \]
\[ \log_3 (2x + 1) = 5 \]
\[ 2x + 1 = 3^5 \]
\[ 2x + 1 = 243 \]
\[ 2x = 242 \]
\[ x = 121 \]
The point \( (121, 3) \) is on the graph of \( G \).
Section 5.4: Logarithmic Functions

113. \( f(x) = \begin{cases} 
\ln(-x) & \text{if } x < 0 \\
\ln x & \text{if } x > 0 
\end{cases} \)

Domain: \( \{ x \mid x \neq 0 \} \)
Range: \( (-\infty, \infty) \)
Intercepts: \( (-1,0), (1,0) \)

114. \( f(x) = \begin{cases} 
\ln(-x) & \text{if } x \leq -1 \\
-\ln(-x) & \text{if } -1 < x < 0 
\end{cases} \)

Domain: \( \{ x \mid x < 0 \} \cup (-\infty, 0) \)
Range: \( \{ y \mid y \geq 0 \} \cup [0, \infty) \)
Intercept: \( (-1,0) \)

115. \( f(x) = \begin{cases} 
-\ln x & \text{if } 0 < x < 1 \\
\ln x & \text{if } x \geq 1 
\end{cases} \)

Domain: \( \{ x \mid x > 0 \} \cup (0, \infty) \)
Range: \( \{ y \mid y \geq 0 \} \cup [0, \infty) \)
Intercept: \( (1,0) \)

112. a. \( F(x) = \log_2 (x+1) - 3 \)
We require that \( x+1 \) be positive.
\( x+1 > 0 \)
\( x > -1 \)
Domain: \( \{ x \mid x > -1 \} \) or \( (-1, \infty) \)

b. \( F(7) = \log_2 (7+1) - 3 \)
\( = \log_2 (8) - 3 \)
\( = 3 - 3 \)
\( = 0 \)
The point \( (7, 0) \) is on the graph of \( F \).

c. \( F(x) = -1 \)
\( \log_2 (x+1) - 3 = -1 \)
\( \log_2 (x+1) = 2 \)
\( x+1 = 2^2 \)
\( x+1 = 4 \)
\( x = 3 \)
The point \( (3, -1) \) is on the graph of \( F \).

d. \( F(x) = 0 \)
\( \log_2 (x+1) - 3 = 0 \)
\( \log_2 (x+1) = 3 \)
\( x+1 = 2^3 \)
\( x+1 = 8 \)
\( x = 7 \)
The zero of \( G \) is \( x = 7 \).
Chapter 5: Exponential and Logarithmic Functions

116. \( f(x) = \begin{cases} \ln x & \text{if } 0 < x < 1 \\ -\ln x & \text{if } x \geq 1 \end{cases} \)

\[
\begin{align*}
\text{Domain: } & \quad \{ x \mid x > 0 \}; \quad (0, \infty) \\
\text{Range: } & \quad \{ y \mid y \leq 0 \}; \quad (-\infty, 0] \\
\text{Intercept: } & \quad (1, 0)
\end{align*}
\]

117. \( \text{pH} = -\log_{10}[H^+] \)

a. \( \text{pH} = -\log_{10}[0.1] = -(1) = 1 \)

b. \( \text{pH} = -\log_{10}[0.01] = -(2) = 2 \)

c. \( \text{pH} = -\log_{10}[0.001] = -(3) = 3 \)

d. As the \( H^+ \) decreases, the pH increases.

e. \( 3.5 = -\log_{10}[H^+] \)

\[
\begin{align*}
[H^+] & = 10^{-3.5} \\
& = 3.16 \times 10^{-4} \\
& = 0.00316
\end{align*}
\]

f. \( 7.4 = -\log_{10}[H^+] \)

\[
\begin{align*}
[H^+] & = 10^{-7.4} \\
& = 3.981 \times 10^{-8} \\
& = 0.00000003981
\end{align*}
\]

118. \( H = -(p_1 \log p_1 + p_2 \log p_2 + \cdots + p_n \log p_n) \)

\[
\begin{align*}
H & = -0.015 \log (0.015) - 0.042 \log (0.042) \\
& \quad -0.129 \log (0.129) - 0.125 \log (0.125) \\
& \quad -0.003 \log (0.003) - 0.686 \log (0.686) \\
& = 0.4327
\end{align*}
\]

b. \( H_{\max} = \log (6) = 0.7782 \)

c. \( E = \frac{H}{H_{\max}} = \frac{0.4327}{0.7782} = 0.5560 \)

d. (answers may vary)

<table>
<thead>
<tr>
<th>Race</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Indian/Native Alaskan</td>
<td>0.009</td>
</tr>
<tr>
<td>Asian</td>
<td>0.036</td>
</tr>
<tr>
<td>Black or African American</td>
<td>0.123</td>
</tr>
<tr>
<td>Hispanic</td>
<td>0.125</td>
</tr>
<tr>
<td>White</td>
<td>0.706</td>
</tr>
<tr>
<td>Native Hawaiian/Pacific Islander</td>
<td>0.001</td>
</tr>
</tbody>
</table>

\( H = -0.009 \log (0.009) - 0.036 \log (0.036) \)

\[
\begin{align*}
& -0.123 \log (0.123) - 0.125 \log (0.125) \\
& -0.706 \log (0.706) - 0.001 \log (0.001) \\
= & 0.4050
\end{align*}
\]

The United States appears to be growing more diverse.

119. \( p = 760e^{-0.145h} \)

a. \( \frac{320}{760} = e^{-0.145h} \)

\[
\ln \left( \frac{320}{760} \right) = -0.145h
\]

\[
h = \frac{\ln \left( \frac{320}{760} \right)}{-0.145} = 5.97
\]

Approximately 5.97 kilometers.

d. \( \frac{667}{760} = e^{-0.145h} \)

\[
\ln \left( \frac{667}{760} \right) = -0.145h
\]

\[
h = \frac{\ln \left( \frac{667}{760} \right)}{-0.145} = 0.90
\]

Approximately 0.90 kilometers.
120. \( A = A_0 e^{-0.35n} \)
   
   a. \( 50 = 100e^{-0.35n} \)
      
      \[ 0.5 = e^{-0.35n} \]
      
      \[ \ln (0.5) = -0.35n \]
      
      \[ t = \frac{\ln (0.5)}{-0.35} = 1.98 \]
      
      Approximately 2 days.

   b. \( 10 = 100e^{-0.35n} \)
      
      \[ 0.1 = e^{-0.35n} \]
      
      \[ \ln (0.1) = -0.35n \]
      
      \[ t = \frac{\ln (0.1)}{-0.35} = 6.58 \]
      
      About 6.58 days, or 6 days and 14 hours.

121. \( F(t) = 1 - e^{-0.1t} \)
   
   a. \( 0.5 = 1 - e^{-0.1t} \)
      
      \[ -0.5 = -e^{-0.1t} \]
      
      \[ 0.5 = e^{-0.1t} \]
      
      \[ \ln (0.5) = -0.1t \]
      
      \[ t = \frac{\ln (0.5)}{-0.1} = 6.93 \]
      
      Approximately 6.93 minutes.

   b. \( 0.8 = 1 - e^{-0.1t} \)
      
      \[ -0.2 = -e^{-0.1t} \]
      
      \[ 0.2 = e^{-0.1t} \]
      
      \[ \ln (0.2) = -0.1t \]
      
      \[ t = \frac{\ln (0.2)}{-0.1} = 16.09 \]
      
      Approximately 16.09 minutes.

   c. It is impossible for the probability to reach 100% because \( e^{-0.1t} \) will never equal zero; thus, \( F(t) = 1 - e^{-0.1t} \) will never equal 1.

122. \( F(t) = 1 - e^{-0.15t} \)
   
   a. \( 0.50 = 1 - e^{-0.15t} \)
      
      \[ -0.5 = -e^{-0.15t} \]
      
      \[ 0.5 = e^{-0.15t} \]
      
      \[ \ln (0.5) = -0.15t \]
      
      \[ t = \frac{\ln (0.5)}{-0.15} = 4.62 \]
      
      Approximately 4.62 minutes, or 4 minutes and 37 seconds.

   b. \( 0.80 = 1 - e^{-0.15t} \)
      
      \[ -0.2 = -e^{-0.15t} \]
      
      \[ 0.2 = e^{-0.15t} \]
      
      \[ \ln (0.2) = -0.15t \]
      
      \[ t = \frac{\ln (0.2)}{-0.15} = 10.73 \]
      
      Approximately 10.73 minutes, or 10 minutes and 44 seconds.

123. \( D = 5e^{-0.4h} \)
   
   \( 2 = 5e^{-0.4h} \)
   
   \( 0.4 = e^{-0.4h} \)
   
   \[ \ln (0.4) = -0.4h \]
   
   \[ h = \frac{\ln (0.4)}{-0.4} = 2.29 \]
   
   Approximately 2.29 hours, or 2 hours and 17 minutes.

124. \( N = P \left( 1 - e^{-0.15d} \right) \)
   
   \( 450 = 1000 \left( 1 - e^{-0.15d} \right) \)
   
   \( 0.45 = 1 - e^{-0.15d} \)
   
   \[ -0.55 = -e^{-0.15d} \]
   
   \( 0.55 = e^{-0.15d} \)
   
   \[ \ln (0.55) = -0.15d \]
   
   \[ d = \frac{\ln (0.55)}{-0.15} = 3.99 \]
   
   Approximately 4 days.
125. \[
I = \frac{E}{R} \left[1 - e^{-\left(\frac{R}{L}\right) t}\right]
\]
Substituting \(E = 12\), \(R = 10\), \(L = 5\), and \(I = 0.5\), we obtain:
\[
0.5 = \frac{12}{10} \left[1 - e^{-\left(\frac{10}{5}\right) t}\right]
\]
\[
\frac{5}{12} = 1 - e^{-2t}
\]
\[
e^{-2t} = \frac{7}{12}
\]
\[
-2t = \ln\left(\frac{7}{12}\right)
\]
\[
t = \frac{\ln\left(\frac{7}{12}\right)}{-2} = 0.2695
\]
It takes approximately 0.2695 second to obtain a current of 0.5 ampere.
Substituting \(E = 12\), \(R = 10\), \(L = 5\), and \(I = 1.0\), we obtain:
\[
1.0 = \frac{12}{10} \left[1 - e^{-\left(\frac{10}{5}\right) t}\right]
\]
\[
\frac{10}{12} = 1 - e^{-2t}
\]
\[
e^{-2t} = \frac{1}{6}
\]
\[
-2t = \ln\left(\frac{1}{6}\right)
\]
\[
t = \frac{\ln\left(\frac{1}{6}\right)}{-2} = 0.8959
\]
It takes approximately 0.8959 second to obtain a current of 0.5 ampere.

Graphing:

126. \(L(t) = A\left(1 - e^{-kt}\right)\)

a. \(20 = 200\left(1 - e^{-k(5)}\right)\)
\[
0.1 = 1 - e^{-5k}
\]
\[
e^{-5k} = 0.9
\]
\[
-5k = \ln 0.9
\]
\[
k = \frac{-\ln 0.9}{5} = 0.0211
\]
b. \(L(10) = 200\left(1 - e^{-\left(\frac{\ln 0.9}{10}\right)}\right)\)
\[
= 200\left(1 - e^{\ln 0.9}\right)
\]
\[
= 38 \text{ words}
\]
c. \(L(15) = 200\left(1 - e^{-\left(\frac{\ln 0.9}{15}\right)}\right)\)
\[
= 200\left(1 - e^{3\ln 0.9}\right)
\]
\[
= 54 \text{ words}
\]
d. \(180 = 200\left(1 - e^{-\left(\frac{\ln 0.9}{180}\right)}\right)\)
\[
0.9 = 1 - e^{\frac{\ln 0.9}{180}}
\]
\[
e^{\frac{\ln 0.9}{180}} = 0.1
\]
\[
\frac{\ln 0.9}{\frac{\ln 0.9}{180}} = \ln 0.1
\]
\[
t = \frac{\ln 0.1}{\ln 0.9} = 109.27 \text{ minutes}
\]

127. \(L\left(10^{-7}\right) = 10 \log \left(\frac{10^{-7}}{10^{-12}}\right)\)
\[
= 10 \log \left(10^5\right)
\]
\[
= 10 \cdot 5
\]
\[
= 50 \text{ decibels}
\]

128. \(L\left(10^{-1}\right) = 10 \log \left(\frac{10^{-1}}{10^{-12}}\right)\)
\[
= 10 \log \left(10^{11}\right)
\]
\[
= 10 \cdot 11
\]
\[
= 110 \text{ decibels}
\]

129. \(L\left(10^{-3}\right) = 10 \log \left(\frac{10^{-3}}{10^{-12}}\right)\)
\[
= 10 \log \left(10^9\right)
\]
\[
= 10 \cdot 9
\]
\[
= 90 \text{ decibels}
\]
130. Intensity of car:
\[ 70 = 10 \log \left( \frac{x}{10^{-12}} \right) \]
\[ 7 = \log \left( \frac{x}{10^{-12}} \right) \]
\[ 10^7 = \frac{x}{10^{-12}} \]
\[ x = 10^{-5} \]

Intensity of truck is \( 10 \cdot 10^{-5} = 10^{-4} \).

\[ L(10^{-4}) = 10 \log \left( \frac{10^{-4}}{10^{-12}} \right) \]
\[ = 10 \log \left( 10^{8} \right) \]
\[ = 10 \cdot 8 \]
\[ = 80 \text{ decibels} \]

131. \( M(125,892) = \log \left( \frac{125,892}{10^{-3}} \right) = 8.1 \)

132. \( M(50,119) = \log \left( \frac{50,119}{10^{-3}} \right) = 7.7 \)

133. \( R = e^{kt} \)

\( \text{a.} \quad 1.4 = e^{k(0.03)} \)
\[ 1.4 = e^{0.03k} \]
\[ \ln(1.4) = 0.03k \]
\[ k = \frac{\ln(1.4)}{0.03} = 11.216 \]

\( \text{b.} \quad R = e^{11.216(0.17)} = e^{1.90672} = 6.73 \)

\( \text{c.} \quad 100 = e^{11.216x} \)
\[ 100 = e^{11.216x} \]
\[ \ln(100) = 11.216x \]
\[ x = \frac{\ln(100)}{11.216} = 0.41 \text{ percent} \]

\( \text{d.} \quad 5 = e^{11.216x} \)
\[ 5 = e^{11.216x} \]
\[ \ln 5 = 11.216x \]
\[ x = \frac{\ln 5}{11.216} = 0.14 \text{ percent} \]

At a percent concentration of 0.14 or higher, the driver should be charged with a DUI.

\( \text{e.} \quad \text{Answers will vary.} \)

134. No. Explanations will vary.

135. If the base of a logarithmic function equals 1, we would have the following:
\[ f(x) = \log_1(x) \]
\[ f^{-1}(x) = 1^x = 1 \quad \text{for every real number} \ x. \]

In other words, \( f^{-1} \) would be a constant function and, therefore, \( f^{-1} \) would not be one-to-one.

136. \( \text{New} = \text{Old} \left( e^{Rt} \right) \)

<table>
<thead>
<tr>
<th>Age</th>
<th>Depreciation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>38,000 = 36,600e^{R(1)}</td>
</tr>
<tr>
<td></td>
<td>[ R = \ln \left( \frac{38,000}{36,600} \right) = 0.03754 = 3.8% ]</td>
</tr>
</tbody>
</table>

| 2   | 38,000 = 32,400e^{R(2)} |
|     | \[ R = \ln \left( \frac{38,000}{32,400} \right) = 0.07971 = 8\% \] |

| 3   | 38,000 = 28,750e^{R(3)} |
|     | \[ R = \ln \left( \frac{38,000}{28,750} \right) = 0.0930 = 9.3\% \] |

| 4   | 38,000 = 25,400e^{R(4)} |
|     | \[ R = \ln \left( \frac{38,000}{25,400} \right) = 0.1007 = 10.1\% \] |
Chapter 5: Exponential and Logarithmic Functions

<table>
<thead>
<tr>
<th>Age</th>
<th>Depreciation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$38,000 = 21,200e^{5R}$</td>
</tr>
</tbody>
</table>

\[
\ln \left( \frac{38,000}{21,200} \right) = 5R
\]

\[
R = \frac{\ln \left( \frac{38,000}{21,200} \right)}{5} = 0.1167 = 11.7\%
\]

Answers will vary.

Section 5.5

1. 0
2. 1
3. \(M\)
4. \(r\)
5. \(\log_a M ; \log_a N\)
6. \(\log_a M ; \log_a N\)
7. \(r \log_a M\)
8. 6
9. 7
10. False: \(\ln(x+3) - \ln(2x) = \ln \left( \frac{x+3}{2x} \right)\)
11. False: \(\log_2 (3x^4) = \log_2 3 + 4 \log_2 x\)
12. False: \(\frac{\ln 8}{\ln 4} = \frac{3}{2}\)
13. \(\log_3 3^{71} = 71\)
14. \(\log_2 2^{-13} = -13\)
15. \(\ln e^{-4} = -4\)
16. \(\ln e^{\sqrt{2}} = \sqrt{2}\)
17. \(2^{\log_2 7} = 7\)
18. \(e^{\ln 8} = 8\)
19. \(\log_8 2 + \log_8 4 = \log_8 (4 \cdot 2) = \log_8 8 = 1\)
20. \(\log_8 9 + \log_8 4 = \log_8 (9 \cdot 4) = \log_8 36 = \log_8 6^2 = 2\)
21. \(\log_8 18 - \log_8 3 = \log_8 \frac{18}{3} = \log_8 6 = 1\)
22. \(\log_8 16 - \log_8 2 = \log_8 \frac{16}{2} = \log_8 8 = 1\)
23. \(\log_2 6 \cdot \log_8 8 = \log_8 8^{\log_2 6}\)
   \[= \log_8 2^{\log_2 6} = \log_8 2^{\log_2 6^1} = \log_8 6^3 = 3\]
24. \(\log_8 8 \cdot \log_8 9 = \log_8 9^{\log_8 8}\)
   \[= \log_8 (3^2)^{\log_8 8} = \log_8 3^{2\log_8 8} = \log_8 3^{\log_8 8^2} = \log_8 8^2 = 2\]
25. \(3^{\log_3 5 - \log_3 4} = 3^{\frac{5}{4}} = \frac{5}{4}\)
26. \(5^{\log_5 6 + \log_5 7} = 5^{\log_5 (6 \cdot 7)} = 5^{\log_5 42} = 42\)
27. \(e^{\log_2 16}\)

Let \(a = \log_2 16\), then \((e^2)^a = 16\).
Section 5.5: Properties of Logarithms

\[ e^{2a} = 16 \]
\[ e^{2a} = 4^2 \]
\[ (e^{2a})^{1/2} = (4^2)^{1/2} \]
\[ e^a = 4 \]
\[ a = \ln 4 \]
Thus, \( e^{\log_9 16} = e^{\ln 4} = 4 \).

28. \( e^{\log_9 9} \)

Let \( a = \log_9 9 \), then \( (e^a)^9 = 9 \).
\[ e^{2a} = 9 \]
\[ e^{2a} = 3^2 \]
\[ (e^{2a})^{1/2} = (3^2)^{1/2} \]
\[ e^a = 3 \]
\[ a = \ln 3 \]
Thus, \( e^{\log_9 9} = e^{\ln 3} = 3 \).

29. \( \ln 6 = \ln(2 \cdot 3) = \ln 2 + \ln 3 = a + b \)

30. \( \ln \frac{2}{3} = \ln 2 - \ln 3 = a - b \)

31. \( \ln 1.5 = \ln \frac{3}{2} = \ln 3 - \ln 2 = b - a \)

32. \( \ln 0.5 = \ln \frac{1}{2} = \ln 1 - \ln 2 = 0 - a = -a \)

33. \( \ln 8 = \ln 2^3 = 3 \cdot \ln 2 = 3a \)

34. \( \ln 27 = \ln 3^3 = 3 \cdot \ln 3 = 3b \)

35. \( \ln \sqrt[5]{6} = \ln 6^{1/5} \)
\[ = \frac{1}{5} \ln 6 \]
\[ = \frac{1}{5} \ln (2 \cdot 3) \]
\[ = \frac{1}{5} (\ln 2 + \ln 3) \]
\[ = \frac{1}{5} (a + b) \)

36. \( \ln \sqrt[5]{\frac{2}{3}} = \ln \left(\frac{2}{3}\right)^{1/5} \)
\[ = \frac{1}{5} \ln \frac{2}{3} \]
\[ = \frac{1}{5} \ln 2 - \frac{1}{5} \ln 3 \]
\[ = \frac{1}{5} (\ln 2 - \ln 3) \]
\[ = \frac{1}{5} (a - b) \)

37. \( \log_9 (25x) = \log_9 25 + \log_9 x = 2 + \log_9 x \)

38. \( \log_9 \frac{x}{9} = \log_9 \frac{x}{3} = \log_9 x - \log_9 3^2 = \log_9 x - 2 \)

39. \( \log_2 z^3 = 3 \log_2 z \)

40. \( \log_7 x^3 = 3 \log_7 x \)

41. \( \ln (e^x) = \ln e + \ln x = 1 + \ln x \)

42. \( \ln e^x = \ln e - \ln x = 1 - \ln x \)

43. \( \ln \left(\frac{x}{e^x}\right) = \ln x - \ln e^x = \ln x - x \)

44. \( \ln (xe^x) = \ln x + \ln e^x = \ln x + x \)

45. \( \log_a (u^2v^3) = \log_a u^2 + \log_a v^3 \)
\[ = 2 \log_a u + 3 \log_a v \)

46. \( \log_2 \left(\frac{a}{b^2}\right) = \log_2 a - \log_2 b^2 = \log_2 a - 2 \log_2 b \)

47. \( \ln \left(\frac{x^2}{\sqrt{1-x}}\right) = \ln x^2 + \ln \sqrt{1-x} \)
\[ = \ln x^2 + \ln (1-x)^{1/2} \]
\[ = 2 \ln x + \frac{1}{2} \ln (1-x) \)

48. \( \ln \left(\sqrt{1+x^2}\right) = \ln x + \ln \sqrt{1+x^2} \)
\[ = \ln x + \ln \left(1 + x^2\right)^{1/2} \]
\[ = \ln x + \frac{1}{2} \ln (1 + x^2) \)
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49. \[ \log_2 \left( \frac{x^3}{x-3} \right) = \log_2 x^3 - \log_2 (x-3) \]
   \[ = 3 \log_2 x - \log_2 (x-3) \]

50. \[ \log_5 \left( \frac{\sqrt{x^2 + 1}}{x^2 - 1} \right) \]
    \[ = \log_5 \left( x^2 + 1 \right)^{1/3} - \log_5 (x^2 - 1) \]
    \[ = \frac{1}{3} \log_5 \left( x^2 + 1 \right) - \log_5 (x^2 - 1) \]
    \[ = \frac{1}{3} \log_5 \left( x^2 + 1 \right) - \log_5 ((x+1)(x-1)) \]
    \[ = \frac{1}{3} \log_5 \left( x^2 + 1 \right) - \log_5 (x+1) - \log_5 (x-1) \]

51. \[ \log \left( \frac{x(x+2)}{(x+3)^2} \right) = \log \left( x(x+2) \right) - \log(x+3)^2 \]
    \[ = \log x + \log(x+2) - 2 \log(x+3) \]

52. \[ \log \left( \frac{x^3 \sqrt{x+1}}{(x-2)^2} \right) = \log \left( x^3 \sqrt{x+1} \right) - \log(x-2)^2 \]
    \[ = \log x^3 + \log(x+1)^{1/2} - 2 \log(x-2) \]
    \[ = 3 \log x + \frac{1}{2} \log(x+1) - 2 \log(x-2) \]

53. \[ \ln \left( \frac{x^2 - x - 2}{(x+4)^3} \right)^{1/3} \]
    \[ = \frac{1}{3} \ln \left( \frac{(x-2)(x+1)}{(x+4)^2} \right) \]
    \[ = \frac{1}{3} \left[ \ln(x-2)(x+1) - \ln(x+4)^2 \right] \]
    \[ = \frac{1}{3} \left[ \ln(x-2) + \ln(x+1) - 2 \ln(x+4) \right] \]
    \[ = \frac{1}{3} \ln(x-2) + \frac{1}{3} \ln(x+1) - \frac{2}{3} \ln(x+4) \]

54. \[ \ln \left( \frac{(x-4)^{2/3}}{x^2 - 1} \right) \]
    \[ = \frac{2}{3} \ln \left( \frac{(x-4)^2}{x^2 - 1} \right) \]
    \[ = \frac{2}{3} \left[ \ln(x-4)^2 - \ln(x^2 - 1) \right] \]
    \[ = \frac{2}{3} \left[ 2 \ln(x-4) - \ln((x+1)(x-1)) \right] \]
    \[ = \frac{2}{3} \left[ 2 \ln(x-4) - \ln(x+1) - \ln(x-1) \right] \]
    \[ = \frac{4}{3} \ln(x-4) - \frac{2}{3} \ln(x+1) - \frac{2}{3} \ln(x-1) \]

55. \[ \ln \left( \frac{5x \sqrt{1+3x}}{(x-4)^3} \right) \]
    \[ = \ln \left( 5x \sqrt{1+3x} \right) - \ln(x-4)^3 \]
    \[ = \ln 5 + \ln x + \ln \sqrt{1+3x} - 3 \ln(x-4) \]
    \[ = \ln 5 + \ln x + \ln(1+3x)^{1/2} - 3 \ln(x-4) \]
    \[ = \ln 5 + \ln x + \frac{1}{2} \ln(1+3x) - 3 \ln(x-4) \]

56. \[ \ln \left( \frac{5x^2 \sqrt{1-x}}{4(x+1)^3} \right) \]
    \[ = \ln \left( 5x^2 \sqrt{1-x} \right) - \ln \left( 4(x+1)^3 \right) \]
    \[ = \ln 5 + \ln x^2 + \ln(1-x)^{1/2} - \left[ \ln 4 + \ln(x+1)^2 \right] \]
    \[ = \ln 5 + 2 \ln x + \frac{1}{3} \ln(1-x) - 4 - 2 \ln(x+1) \]

57. \[ 3 \log_u v + 4 \log_u v = \log_u u^3 + \log_u v^4 \]
    \[ = \log_u \left( u^3 v^4 \right) \]

58. \[ 2 \log_u v - \log_u v = \log_u u^2 - \log_u v \]
    \[ = \log_u \left( \frac{u^2}{v} \right) \]

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Section 5.5: Properties of Logarithms

59. \[ \log_3 \sqrt{x} - \log_3 x^3 = \log_3 \left( \frac{\sqrt{x}}{x^3} \right) = \log_3 \left( \frac{x^{1/2}}{x^3} \right) = \log_3 x^{-5/2} = \log_3 \left( \frac{1}{x^{5/2}} \right) \]

60. \[ \log_2 \left( \frac{1}{x} \right) + \log_2 \left( \frac{1}{x^2} \right) = \log_2 \left( \frac{1}{x \cdot x^2} \right) = \log_2 \left( \frac{1}{x^3} \right) \]

61. \[ \log_4 \left( x^2 - 1 \right) - 5 \log_4 \left( x + 1 \right) = \log_4 \left( x^2 - 1 \right) - \log_4 \left( x + 1 \right)^5 = \log_4 \left( \frac{x^2 - 1}{(x + 1)^5} \right) = \log_4 \left( \frac{(x + 1)(x - 1)}{(x + 1)^5} \right) = \log_4 \left( \frac{x - 1}{(x + 1)^4} \right) \]

62. \[ \log \left( x^2 + 3x + 2 \right) - 2 \log_5 \left( x + 1 \right) = \log \left( x^2 + 3x + 2 \right) - \log_5 \left( x + 1 \right)^2 = \log \left( \frac{x^2 + 3x + 2}{(x + 1)^2} \right) = \log \left( \frac{(x + 2)(x + 1)}{(x + 1)^2} \right) = \log \left( \frac{x + 2}{x + 1} \right) \]

63. \[ \ln \left( \frac{x}{x - 1} \right) + \ln \left( \frac{x + 1}{x} \right) - \ln \left( x^2 - 1 \right) = \ln \left[ \frac{x \cdot (x + 1)}{x(x - 1)} \right] - \ln \left( x^2 - 1 \right) = \ln \left[ \frac{x + 1}{x - 1} \right] + \ln \left( x^2 - 1 \right) = \ln \left[ \frac{x + 1}{(x - 1)(x + 1)} \right] = \ln \left( \frac{1}{x - 1} \right) = \ln(x - 1)^{-2} = -2 \ln(x - 1) \]

64. \[ \log \left( \frac{x^2 + 2x - 3}{x^2 - 4} \right) - \log \left( \frac{x^2 + 7x + 6}{x + 2} \right) = \log \left( \frac{x^2 + 2x - 3}{\left( x^2 - 4 \right) \left( x^2 + 7x + 6 \right)} \right) = \log \left[ \frac{(x + 3)(x - 1)}{(x - 2)(x + 1)} \right] \]

65. \[ 8 \log_2 \sqrt{3x - 2} - \log_2 \left( \frac{4}{x} \right) + \log_2 4 = \log_2 \left( \sqrt{3x - 2} \right)^8 - \log_2 \left( 4 - \log_2 x \right) + \log_2 4 = \log_2 (3x - 2)^4 - \log_2 4 \cdot \log_2 x - \log_2 x + \log_2 4 = \log_2 \left[ x(3x - 2)^4 \right] \]

66. \[ 21 \log_3 \sqrt{x} + \log_3 \left( 9x^3 \right) - \log_9 = \log_3 \left( (\sqrt{x})^{21} \right) + \log_3 \left( 9 \right) + \log_3 \left( x^3 \right) - \log_9 = \log_3 \left( x^{7} \cdot x^2 \right) \]

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67. \(2 \log_a (5x^3) - \frac{1}{2} \log_a (2x + 3)\)
   
   \[= \log_a (5x^3)^2 - \frac{1}{2} \log_a (2x + 3)^{1/2}\]
   
   \[= \log_a (25x^6) - \log_a \sqrt{2x + 3}\]
   
   \[= \log_a \left[ \frac{25x^6}{\sqrt{2x + 3}} \right]\]

68. \(\frac{1}{3} \log(x^3 + 1) + \frac{1}{2} \log(x^2 + 1)\)
   
   \[= \log(x^3 + 1)^{1/3} + \log(x^2 + 1)^{1/2}\]
   
   \[= \log \left[ \sqrt[3]{x^3 + 1} \cdot \sqrt{x^2 + 1} \right]\]

69. \(2 \log_2 (x + 1) - \log_2 (x + 3) - \log_2 (x - 1)\)
   
   \[= \log_2 (x + 1)^2 - \log_2 (x + 3) - \log_2 (x - 1)\]
   
   \[= \log_2 \left[ \frac{(x + 1)^2}{(x + 3)(x - 1)} \right]\]

70. \(3 \log_3 (3x + 1) - 2 \log_3 (2x - 1) - \log_3 x\)
   
   \[= \log_3 (3x + 1)^3 - \log_3 (2x - 1)^2 - \log_3 x\]
   
   \[= \log_3 \left[ \frac{(3x + 1)^3}{(2x - 1)^2} \right]\]

71. \(\log_3 21 = \frac{\log 21}{\log 3} = 2.771\)

72. \(\log_5 18 = \frac{\log 18}{\log 5} = 1.796\)

73. \(\log_{\sqrt[3]{2}} 71 = \frac{\log_{1/3} 71}{\log (1/3)} = \frac{\log 71}{-\log 3} = -3.880\)

74. \(\log_{\sqrt{2}} 15 = \frac{\log_{1/2} 15}{\log (1/2)} = \frac{\log 15}{-\log 2} = -3.907\)

75. \(\log_{\sqrt{2}} 7 = \frac{\log 7}{\log \sqrt{2}} = 5.615\)

76. \(\log_{\sqrt{5}} 8 = \frac{\log 8}{\log \sqrt{5}} = 2.584\)

77. \(\log_e e = \frac{\ln e}{\ln \pi} = 0.874\)

78. \(\log_2 \sqrt{2} = \frac{\ln \sqrt{2}}{\ln \pi} = 0.303\)

79. \(y = \log_4 x = \frac{\ln x}{\ln 4} \text{ or } y = \frac{\log x}{\log 4}\)

80. \(y = \log_5 x = \frac{\ln x}{\ln 5} \text{ or } y = \frac{\log x}{\log 5}\)

81. \(y = \log_2 (x + 2) = \frac{\ln(x + 2)}{\ln 2} \text{ or } y = \frac{\log(x + 2)}{\log 2}\)

82. \(y = \log_4 (x - 3) = \frac{\ln(x - 3)}{\ln 4} \text{ or } y = \frac{\log(x - 3)}{\log 4}\)
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83. \[ y = \log_{x+1}(x+1) = \frac{\ln(x+1)}{\ln(x-1)} \quad \text{or} \quad y = \frac{\log(x+1)}{\log(x-1)} \]

84. \[ y = \log_{x+2}(x-2) = \frac{\ln(x-2)}{\ln(x+2)} \quad \text{or} \quad y = \frac{\log(x-2)}{\log(x+2)} \]

85. \[ f(x) = \ln x; \quad g(x) = e^x; \quad h(x) = x^2 \]
   
a. \[ (f \circ g)(x) = f(g(x)) = \ln(e^x) = x \]
      Domain: \( x \in \mathbb{R} \) or \( (-\infty, \infty) \)
   
b. \[ (g \circ f)(x) = g(f(x)) = e^{\ln x} = x \]
      Domain: \( x > 0 \) or \( (0, \infty) \)
      (Note: the restriction on the domain is due to the domain of \( \ln x \))
   
c. \[ (f \circ g)(5) = 5 \quad \text{[from part (a)]} \]
   
d. \[ (f \circ h)(x) = f(h(x)) = \ln(x^2) \]
      Domain: \( x \neq 0 \) or \( (-\infty, 0) \cup (0, \infty) \)
   
e. \[ (f \circ h)(e) = \ln(e^2) = 2 \ln e = 2 \cdot 1 = 2 \]

86. \[ f(x) = \log_2 x; \quad g(x) = 2^x; \quad h(x) = 4x \]
   
a. \[ (f \circ g)(x) = f(g(x)) = \log_2(2^x) = x \]
      Domain: \( x \in \mathbb{R} \) or \( (-\infty, \infty) \)
   
b. \[ (g \circ f)(x) = g(f(x)) = 2^{\log_2 x} = x \]
      Domain: \( x > 0 \) or \( (0, \infty) \)
      (Note: the restriction on the domain is due to the domain of \( \log_2 x \))
   
c. \[ (f \circ g)(3) = 3 \quad \text{[from part (a)]} \]
   
d. \[ (f \circ h)(x) = f(h(x)) = \log_2(4x) \]
      \[ = \log_2 4 + \log_2 x = 2 + \log_2 x \]
      Domain: \( \{ x | x > 0 \} \) or \( (0, \infty) \)
   
e. \[ (f \circ h)(8) = \log_2(4 \cdot 8) = \log_2 32 = 5 \]
      \[ = 2 + \log_2 8 = 2 + 3 = 5 \]

87. \[ \ln y = \ln x + \ln C \]
      \[ y = Cx \]

88. \[ \ln y = \ln(x + C) \]
      \[ y = x + C \]

89. \[ \ln y = \ln x + \ln(x + 1) + \ln C \]
      \[ \ln y = \ln((x + 1)C) \]
      \[ y = Cx(x + 1) \]

90. \[ \ln y = 2\ln x - \ln(x + 1) + \ln C \]
      \[ \ln y = \ln\left(\frac{x^2C}{x + 1}\right) \]
      \[ y = \frac{C\sqrt{x}}{x + 1} \]

91. \[ \ln y = 3x + \ln C \]
      \[ \ln y = \ln e^{3x} + \ln C \]
      \[ \ln y = \ln(Ce^{3x}) \]
      \[ y = Ce^{3x} \]

92. \[ \ln y = -2x + \ln C \]
      \[ \ln y = \ln e^{-2x} + \ln C \]
      \[ \ln y = \ln(Ce^{-2x}) \]
      \[ y = Ce^{-2x} \]

93. \[ \ln (y - 3) = -4x + \ln C \]
      \[ \ln (y - 3) = \ln e^{-4x} + \ln C \]
      \[ \ln (y - 3) = \ln(Ce^{-4x}) \]
      \[ y - 3 = Ce^{-4x} \]
      \[ y = Ce^{-4x} + 3 \]
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94. \( \ln (y+4) = 5x + \ln C \)
\( \ln (y+4) = \ln e^{5x} + \ln C \)
\( \ln (y+4) = \ln (Ce^{5x}) \)
\( y + 4 = Ce^{5x} \)
\( y = Ce^{5x} - 4 \)

95. \( 3 \ln y = \frac{1}{2} \ln (2x+1) + \frac{1}{3} \ln (x+4) + \ln C \)
\( \ln y^3 = \ln (2x+1)^{\frac{1}{2}} + \ln (x+4)^{\frac{1}{3}} + \ln C \)
\( \ln y^3 = \ln \left[ \frac{C(2x+1)^{1/2}}{(x+4)^{1/3}} \right] \)
\( y^3 = \frac{C(2x+1)^{1/2}}{(x+4)^{1/3}} \)
\( y = \left[ \frac{C(2x+1)^{1/2}}{(x+4)^{1/3}} \right]^{1/3} \)
\( y = \frac{\sqrt[3]{C}(2x+1)^{1/6}}{(x+4)^{1/9}} \)

96. \( 2 \ln y = -\frac{1}{2} \ln x + \frac{1}{3} \ln (x^2 + 1) + \ln C \)
\( \ln y^2 = -\ln x^{1/2} + \ln (x^2 + 1)^{1/3} + \ln C \)
\( \ln y^2 = \ln \left[ \frac{C(x^2+1)^{1/3}}{x^{1/2}} \right] \)
\( y^2 = \frac{C(x^2+1)^{1/3}}{x^{1/2}} \)
\( y = \left[ \frac{C(x^2+1)^{1/3}}{x^{1/2}} \right]^{1/2} \)
\( y = \frac{\sqrt{C}(x^2+1)^{1/6}}{x^{1/4}} \)

97. \( \log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8 = \) \( \frac{\log 3}{\log 2} \cdot \frac{\log 4}{\log 3} \cdot \frac{\log 5}{\log 4} \cdot \frac{\log 6}{\log 5} \cdot \frac{\log 7}{\log 6} \cdot \frac{\log 8}{\log 7} \)
\( = \frac{\log 8}{\log 2} \cdot \frac{\log 3}{\log 2} \cdot \frac{\log 2}{\log 2} \cdot \frac{\log 2}{\log 2} \cdot \frac{\log 2}{\log 2} \cdot \frac{\log 2}{\log 2} \)
\( = 3 \)

98. \( \log_2 4 \cdot \log_4 6 \cdot \log_6 8 = \) \( \frac{\log 4}{\log 2} \cdot \frac{\log 6}{\log 4} \cdot \frac{\log 8}{\log 6} \)
\( = \frac{\log 8}{\log 2} \cdot \frac{\log 2}{\log 2} \cdot \frac{\log 2}{\log 2} \)
\( = \frac{3\log 2}{\log 2} \)
\( = 3 \)

99. \( \log_2 3 \cdot \log_3 4 \cdot \ldots \cdot \log_n (n+1) \cdot \log_{n+1} 2 \)
\( = \frac{\log 3}{\log 2} \cdot \frac{\log 4}{\log 3} \cdot \ldots \cdot \frac{\log (n+1)}{\log n} \cdot \frac{\log 2}{\log (n+1)} \)
\( = \frac{\log 2}{\log 2} \cdot \frac{\log 3}{\log 2} \cdot \ldots \cdot \frac{\log n}{\log n} \)
\( = 1 \)

100. \( \log_2 2 \cdot \log_2 4 \cdot \ldots \cdot \log_2 2^n \)
\( = \frac{\log 2}{\log 2} \cdot \frac{\log 4}{\log 2} \cdot \ldots \cdot \frac{\log 2^n}{\log 2} \)
\( = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n \)
\( = n! \)

101. \( \log_a (x + \sqrt{x^2 - 1}) + \log_a (x - \sqrt{x^2 - 1}) : \)
\( = \log_a \left[ (x + \sqrt{x^2 - 1})(x - \sqrt{x^2 - 1}) \right] \)
\( = \log_a [x^2 - (x^2 - 1)] \)
\( = \log_a [x^2 - x^2 + 1] \)
\( = \log_a 1 \)
\( = 0 \)

102. \( \log_a (\sqrt{x + \sqrt{x - 1}}) + \log_a (\sqrt{x - \sqrt{x - 1}}) \)
\( = \log_a \left[ (\sqrt{x + \sqrt{x - 1}})(\sqrt{x - \sqrt{x - 1}}) \right] \)
\( = \log_a [x - (x - 1)] \)
\( = \log_a [x - x + 1] \)
\( = \log_a 1 \)
\( = 0 \)
103. \[ 2x + \ln (1 + e^{-2x}) = \ln e^{2x} + \ln (1 + e^{-2x}) = \ln (e^{2x}) + \ln (1 + e^{-2x}) = \ln (e^{2x} + e^0) = \ln (e^{2x} + 1) \]

104. \[
\frac{f(x + h) - f(x)}{h} = \frac{\log_a (x + h) - \log_a x}{h}
= \frac{\log_a \left( \frac{x + h}{x} \right)}{h}
= \frac{1}{h} \cdot \log_a \left( 1 + \frac{h}{x} \right)
= \log_a \left( 1 + \frac{h}{x} \right), \quad h \neq 0
\]

105. \( f(x) = \log_a x \) means that \( x = a^{f(a)} \). 
Now, raising both sides to the \(-1\) power, we obtain 
\( x^{-1} = \left( a^{f(a)} \right)^{-1} = a^{-f(a)} = \left( \frac{1}{a} \right)^{f(a)} \).

\( x^{-1} = \left( \frac{1}{a} \right)^{f(a)} \) means that \( \log_{1/a} x^{-1} = f(x) \).
Thus, \( \log_{1/a} x^{-1} = f(x) \)
\(- \log_{1/a} x = f(x) \)
\(- f(x) = \log_{1/a} x \)

106. \( f(AB) = \log_a (AB) \)
= \( \log_a A + \log_a B \)
= \( f(A) + f(B) \)

107. \( f(x) = \log_a x \)
\( f \left( \frac{1}{x} \right) = \log_a \left( \frac{1}{x} \right) \)
= \( \log_a 1 - \log_a x \)
= \( - \log_a x \)
= \(- f(x) \)

108. \( f(x) = \log_a x \)
\( f \left( x^\alpha \right) = \log_a \left( x^\alpha \right) = \alpha \log_a x = \alpha f(x) \)

109. If \( A = \log_a M \) and \( B = \log_a N \), then \( a^A = M \) and \( a^B = N \).
\[ \log_a \left( \frac{M}{N} \right) = \log_a \left( \frac{a^A}{a^B} \right) = \log_a a^{A-B} = A - B = \log_a M - \log_a N \]

110. \[ \log_a \left( \frac{1}{N} \right) = \log_a N^{-1} = -1 \cdot \log_a N = - \log_a N, \quad a \neq 1 \]

111. \( Y_1 = \log x^2 \)
\( Y_2 = 2 \log x \)

The domain of \( Y_1 = \log x^2 \) is \( \{ x | x \neq 0 \} \). The domain of \( Y_2 = 2 \log x \) is \( \{ x | x > 0 \} \). These two domains are different because the logarithm property \( \log a \cdot x^\alpha = n \cdot \log_a x \) holds only when \( \log_a x \) exists.

112. Answers may vary. One possibility follows:
Let \( a = 2 \), \( x = 8 \), and \( r = 3 \). Then
\( (\log_a x)^r = (\log_a 8)^3 = 8 = 27 \). But
\( r \log_a x = 3 \log_a 8 = 3 \cdot 3 = 9 \). Thus,
\( (\log_a 8)^3 \neq 3 \log_a 8 \) and, in general,
\( (\log_a x)^r \neq r \log_a x \).

113. Answers may vary. One possibility follows:
Let \( x = 4 \) and \( y = 4 \). Then
\( \log_2 (x + y) = \log_2 (4 + 4) = \log_2 8 = 3 \). But
\( \log_2 x + \log_2 y = \log_2 4 + \log_2 4 = 2 + 2 = 4 \).
Thus, \( \log_2 (x + y) \neq \log_2 x + \log_2 y \) and, in general,
\( \log_2 (x + y) \neq \log_2 x + \log_2 y \).

114. No. \( \log_a (-5) \) does not exist. The argument of a logarithm must be nonnegative.
Section 5.6

1. \( x^2 - 7x - 30 = 0 \)
   
   \((x + 3)(x - 10) = 0 \)
   
   \( x + 3 = 0 \) or \( x - 10 = 0 \)
   
   \( x = -3 \) or \( x = 10 \)
   
   The solution set is \{-3, 10\}.

2. Let \( u = x + 3 \). Then
   
   \((x + 3)^2 - 4(x + 3) + 3 = 0 \)
   
   \( u^2 - 4u + 3 = 0 \)
   
   \((u - 1)(u - 3) = 0 \)
   
   \( u - 1 = 0 \) or \( u - 3 = 0 \)
   
   \( u = 1 \) or \( u = 3 \)
   
   Back substituting \( u = x + 3 \), we obtain
   
   \( x + 3 = 1 \) or \( x + 3 = 3 \)
   
   \( x = -2 \) or \( x = 0 \)
   
   The solution set is \{-2, 0\}.

3. \( x^3 = x^2 - 5 \)
   
   Using INTERSECT to solve:
   
   \( y_1 = x^3; \ y_2 = x^2 - 5 \)
   
   [Graph of intersecting curves]
   
   Thus, \( x = -1.43 \), so the solution set is \{-1.43\}.

4. \( x^3 - 2x + 2 = 0 \)
   
   Using ZERO to solve:
   
   \( y_1 = x^3 - 2x + 2 \)
   
   [Graph of intersecting curves]
   
   Thus, \( x = -1.77 \), so the solution set is \{-1.77\}.

5. \( \log_{4} x = 2 \)
   
   \( x = 4^2 \)
   
   \( x = 16 \)
   
   The solution set is \{16\}.

6. \( \log (x + 6) = 1 \)
   
   \( x + 6 = 10^1 \)
   
   \( x + 6 = 10 \)
   
   \( x = 4 \)
   
   The solution set is \{4\}.

7. \( \log_{2} (5x) = 4 \)
   
   \( 5x = 2^4 \)
   
   \( 5x = 16 \)
   
   \( x = \frac{16}{5} \)
   
   The solution set is \{16, 5\}.

8. \( \log_{3} (3x - 1) = 2 \)
   
   \( 3x - 1 = 3^2 \)
   
   \( 3x - 1 = 9 \)
   
   \( 3x = 10 \)
   
   \( x = \frac{10}{3} \)
   
   The solution set is \{10, 3\}.

9. \( \log_{4} (x + 2) = \log_{4} 8 \)
   
   \( x + 2 = 8 \)
   
   \( x = 6 \)
   
   The solution set is \{6\}.

10. \( \log_{5} (2x + 3) = \log_{5} 3 \)
    
    \( 2x + 3 = 3 \)
    
    \( 2x = 0 \)
    
    \( x = 0 \)
    
    The solution set is \{0\}.

11. \( \frac{1}{2} \log_{3} x = 2 \log_{3} 2 \)
    
    \( \log_{3} x^{1/2} = \log_{3} 2^2 \)
    
    \( x^{1/2} = 4 \)
    
    \( x = 16 \)
    
    The solution set is \{16\}.
12. \(-2 \log_4 x = \log_4 9\)
\[
\begin{align*}
\log_4 x^2 &= \log_4 9 \\
x^2 &= 9 \\
\frac{1}{x^2} &= 9 \\
x^2 &= \frac{1}{9} \\
x &= \pm \frac{1}{3}
\end{align*}
\]
Since \(\log_4 \left( \frac{1}{3} \right)\) is undefined, the solution set is \(\left\{ \frac{1}{3}, 3 \right\}\).

13. \(3 \log_2 x = -\log_2 27\)
\[
\begin{align*}
\log_2 x^3 &= \log_2 27^{-1} \\
x^3 &= 27^{-1} \\
x^3 &= \frac{1}{27} \\
x &= \frac{1}{3}
\end{align*}
\]
The solution set is \(\left\{ \frac{1}{3} \right\}\).

14. \(2 \log_5 x = 3 \log_5 4\)
\[
\begin{align*}
\log_5 x^2 &= \log_5 4^3 \\
x^2 &= 64 \\
x &= \pm 8
\end{align*}
\]
Since \(\log_5 (-8)\) is undefined, the solution set is \(\{8\}\).

15. \(3 \log_2 (x-1) + \log_2 4 = 5\)
\[
\begin{align*}
\log_2 (x-1)^3 + \log_2 4 &= 5 \\
\log_2 4(x-1)^3 &= 5 \\
4(x-1)^3 &= 2^5 \\
(x-1)^3 &= \frac{32}{4} \\
(x-1)^3 &= 8 \\
x-1 &= 2 \\
x &= 3
\end{align*}
\]
The solution set is \(\{3\}\).

16. \(2 \log_3 (x+4) - \log_3 9 = 2\)
\[
\begin{align*}
\log_3 (x+4)^2 - \log_3 3^2 &= 2 \\
\log_3 (x+4)^2 - 2 &= 2 \\
\log_3 (x+4)^2 &= 4 \\
(x+4)^2 &= 3^4 \\
(x+4)^2 &= 81 \\
x+4 &= \pm 9 \\
x &= -4 \pm 9 \\
x &= 5 \text{ or } x = -13
\end{align*}
\]
Since \(\log_3 (-13+4) = \log_3 (-9)\) is undefined, the solution set is \(\{5\}\).

17. \(\log x + \log (x+15) = 2\)
\[
\begin{align*}
\log (x(x+15)) &= 2 \\
x(x+15) &= 10^2 \\
x^2 + 15x - 100 &= 0 \\
(x+20)(x-5) &= 0 \\
x &= -20 \text{ or } x = 5
\end{align*}
\]
Since \(\log (-20)\) is undefined, the solution set is \(\{5\}\).

18. \(\log x + \log (x-21) = 2\)
\[
\begin{align*}
\log (x(x-21)) &= 2 \\
x(x-21) &= 10^2 \\
x^2 - 21x - 100 &= 0 \\
(x+4)(x-25) &= 0 \\
x &= -4 \text{ or } x = 25
\end{align*}
\]
Since \(\log (-4)\) is undefined, the solution set is \(\{25\}\).

19. \(\log(2x+1) + \log(x-2) = 1\)
\[
\begin{align*}
\log \left( \frac{2x+1}{x-2} \right) &= 1 \\
\frac{2x+1}{x-2} &= 10^1 \\
2x+1 &= 10(x-2) \\
2x+1 &= 10x-20 \\
-8x &= -21 \\
x &= \frac{-21}{-8} = \frac{21}{8}
\end{align*}
\]
The solution set is \(\left\{ \frac{21}{8} \right\}\).
Chapter 5: Exponential and Logarithmic Functions

20. \( \log(2x) - \log(x-3) = 1 \)
\[
\log\left(\frac{2x}{x-3}\right) = 1
\]
\[
\frac{2x}{x-3} = 10^1
\]
\[
x = 10(x-3)
\]
\[
x = 10x - 30
\]
\[
x = -8x = -30
\]
\[
x = -\frac{30}{8} = 15
\]
\[
x = 4
\]
The solution set is \( \left\{ \frac{15}{4} \right\} \).

21. \( \log_2(x+7) + \log_2(x+8) = 1 \)
\[
\log_2[(x+7)(x+8)] = 1
\]
\[
(x+7)(x+8) = 2^1
\]
\[
x^2 + 8x + 7x + 56 = 2
\]
\[
x^2 + 15x + 54 = 0
\]
\[
x + 9 + (x+6) = 0
\]
\[
x = -9 \text{ or } x = -6
\]
Since \( \log_2(-9+7) = \log_2(-2) \) is undefined, the solution set is \( \{-6\} \).

22. \( \log_6(x+4) + \log_6(x+3) = 1 \)
\[
\log_6[(x+4)(x+3)] = 1
\]
\[
(x+4)(x+3) = 6^1
\]
\[
x^2 + 3x + 4x + 12 = 6
\]
\[
x^2 + 7x + 6 = 0
\]
\[
x + 6 + (x+1) = 0
\]
\[
x = -6 \text{ or } x = -1
\]
Since \( \log_6(-6+4) = \log_6(-2) \) is undefined, the solution set is \( \{-1\} \).

23. \( \log_8(x+6) = 1 - \log_8(x+4) \)
\[
\log_8(x+6) + \log_8(x+4) = 1
\]
\[
\log_8[(x+6)(x+4)] = 1
\]
\[
(x+6)(x+4) = 8^1
\]
\[
x^2 + 4x + 6x + 24 = 8
\]
\[
x^2 + 10x + 16 = 0
\]
\[
(x+8)(x+2) = 0
\]
\[
x = -8 \text{ or } x = -2
\]
Since \( \log_8(-8+6) = \log_8(-2) \) is undefined, the solution set is \( \{-2\} \).

24. \( \log_5(x+3) = 1 - \log_5(x-1) \)
\[
\log_5(x+3) + \log_5(x-1) = 1
\]
\[
\log_5[(x+3)(x-1)] = 1
\]
\[
(x+3)(x-1) = 5^1
\]
\[
x^2 + 3x - x - 3 = 5
\]
\[
x^2 + 2x - 8 = 0
\]
\[
(x+4)(x-2) = 0
\]
\[
x = -4 \text{ or } x = 2
\]
Since \( \log_5(-4+3) = \log_5(-1) \) is undefined, the solution set is \( \{2\} \).

25. \( \ln x + \ln(x+2) = 4 \)
\[
\ln(x(x+2)) = 4
\]
\[
x(x+2) = e^4
\]
\[
x^2 + 2x - e^4 = 0
\]
\[
x = \frac{-2 \pm \sqrt{4 - 4(1)(-e^4)}}{2(1)}
\]
\[
x = \frac{-2 \pm \sqrt{4 + 4e^4}}{2}
\]
\[
x = -1 \pm \sqrt{1 + e^4}
\]
\[
x = -1 - \sqrt{1 + e^4} \text{ or } x = -1 + \sqrt{1 + e^4}
\]
\[
x = -8.456 \quad 6.456
\]
Since \( \ln(-8.456) \) is undefined, the solution set is \( \{-1 + \sqrt{1 + e^4}\} = \{6.456\} \).

26. \( \ln(x+1) - \ln x = 2 \)
\[
\ln\left(\frac{x+1}{x}\right) = 2
\]
\[
\frac{x+1}{x} = e^2
\]
\[
x + 1 = e^2x
\]
\[
e^2x - x = 1
\]
\[
x(e^2 - 1) = 1
\]
\[
x = \frac{1}{e^2 - 1} = 0.157
\]
The solution set is \( \left\{ \frac{1}{e^2 - 1} \right\} = \{0.157\} \).
27. \( \log_3 (x+1) + \log_3 (x+4) = 2 \)
\[
\begin{align*}
\log_3 [(x+1)(x+4)] &= 2 \\
(x+1)(x+4) &= 3^2 \\
x^2 + 4x + x + 4 &= 9 \\
x^2 + 5x - 5 &= 0 \\
x &= \frac{-5 \pm \sqrt{5^2 - 4(1)(-5)}}{2(1)} \\
&= \frac{-5 \pm \sqrt{45}}{2} \\
&= \frac{-5 \pm 3\sqrt{5}}{2} \\
x &= \frac{-5 - 3\sqrt{5}}{2} \quad \text{or} \quad x = \frac{-5 + 3\sqrt{5}}{2} \\
&= -5.854 \quad \text{or} \quad 0.854
\end{align*}
\]
Since \( \log_3 (-8.854 + 1) = \log_3 (-7.854) \) is undefined, the solution set is \( \left\{ \frac{-5 + 3\sqrt{5}}{2} \right\} = \{0.854\} \).

28. \( \log_2 (x+1) + \log_2 (x+7) = 3 \)
\[
\begin{align*}
\log_2 [(x+1)(x+7)] &= 3 \\
(x+1)(x+7) &= 2^3 \\
x^2 + 8x + x + 7 &= 8 \\
x^2 + 9x - 1 &= 0 \\
x &= \frac{-8 \pm \sqrt{8^2 - 4(1)(-1)}}{2(1)} \\
&= \frac{-8 \pm \sqrt{64}}{2} \\
&= \frac{-8 \pm 2\sqrt{17}}{2} \\
&= -4 \pm \sqrt{17} \\
x &= -4 - \sqrt{17} \quad \text{or} \quad x = -4 + \sqrt{17} \\
&= -8.123 \quad \text{or} \quad 0.123
\end{align*}
\]
Since \( \log_2 (-8.123 + 1) = \log_2 (-7.123) \) is undefined, the solution set is \( \left\{ -4 + \sqrt{17} \right\} = \{0.123\} \).

29. \( \log_{1/3} (x^2 + x) - \log_{1/3} (x^2 - x) = -1 \)
\[
\begin{align*}
\log_{1/3} \left( \frac{x^2 + x}{x^2 - x} \right) &= -1 \\
x^2 + x &= \left( \frac{1}{3} \right)^{-1} \\
x^2 + x &= 3 \\
x^2 + x &= 3(x^2 - x) \\
x^2 + x &= 3x^2 - 3x \\
x^2 + 4x &= 0 \\
x(x - 2) &= 0 \\
x &= 0 \quad \text{or} \quad x = 2
\end{align*}
\]
Since each of the original logarithms are not defined for \( x = 0 \), but are defined for \( x = 2 \), the solution set is \( \{2\} \).

30. \( \log_4 (x^2 - 9) - \log_4 (x + 3) = 3 \)
\[
\begin{align*}
\log_4 \left( \frac{x^2 - 9}{x + 3} \right) &= 3 \\
\frac{(x-3)(x+3)}{x+3} &= 4^3 \\
x - 3 &= 64 \\
x &= 67
\end{align*}
\]
Since each of the original logarithms is defined for \( x = 67 \), the solution set is \( \{67\} \).

31. \( \log_a (x-1) - \log_a (x+6) = \log_a (x-2) - \log_a (x+3) \)
\[
\begin{align*}
\log_a \left( \frac{x-1}{x+6} \right) &= \log_a \left( \frac{x-2}{x+3} \right) \\
\frac{x-1}{x+6} &= \frac{x-2}{x+3} \\
(x-1)(x+3) &= (x-2)(x+6) \\
x^2 + 2x - 3 &= x^2 + 4x - 12 \\
2x - 3 &= 4x - 12 \\
9 &= 2x \\
x &= \frac{9}{2}
\end{align*}
\]
Since each of the original logarithms is defined for \( x = \frac{9}{2} \), the solution set is \( \left\{ \frac{9}{2} \right\} \).
32. \( \log_a x + \log_a (x - 2) = \log_a (x + 4) \)
\[
\log_a (x(x - 2)) = \log_a (x + 4)
\]
\[
x(x - 2) = x + 4
\]
\[
x^2 - 2x = x + 4
\]
\[
x^2 - 3x - 4 = 0
\]
\[
(x - 4)(x + 1) = 0
\]
\[
x = 4 \text{ or } x = -1
\]
Since \( \log_a (-1) \) is undefined, the solution set is \{4\}.

33. \( 2^{x-5} = 8 \)
\[
2^{x-5} = 2^3
\]
\[
x - 5 = 3
\]
\[
x = 8
\]
The solution set is \{8\}.

34. \( 5^{x} = 25 \)
\[
5^{-x} = 5^2
\]
\[
-x = 2
\]
\[
x = -2
\]
The solution set is \{-2\}.

35. \( 2^x = 10 \)
\[
x = \log_2 10 = \frac{\ln 10}{\ln 2} = 3.322
\]
The solution set is \{\frac{\ln 10}{\ln 2}\} = \{3.322\}.

36. \( 3^x = 14 \)
\[
x = \log_3 14 = \frac{\ln 14}{\ln 3} = 2.402
\]
The solution set is \{\log_3 14\} = \{2.402\}.

37. \( 8^{-x} = 1.2 \)
\[
-x = \log_8 1.2
\]
\[
x = -\log_8 1.2 = -\frac{\log (1.2)}{\log 8} = -0.088
\]
The solution set is \{-\log_8 1.2\} = \{-0.088\}.

38. \( 2^{-x} = 1.5 \)
\[
-x = \log_2 1.5
\]
\[
x = -\log_2 1.5 = -\frac{\log 1.5}{\log 2} = -0.585
\]

The solution set is \{-\log_2 1.5\} = \{-0.585\}.

39. \( 5(2^x) = 8 \)
\[
2^x = \frac{8}{5}
\]
\[
3x = \log_2 \left( \frac{8}{5} \right)
\]
\[
x = \frac{1}{3} \log_2 \left( \frac{8}{5} \right) = \frac{\ln (8/5)}{3 \ln 2} = 0.226
\]

The solution set is \{\frac{1}{3} \log_2 \left( \frac{8}{5} \right)\} = \{0.226\}.

40. \( 0.3(4^{0.2x}) = 0.2 \)
\[
4^{0.2x} = \frac{2}{3}
\]
\[
0.2x = \log_4 \left( \frac{2}{3} \right)
\]
\[
x = \frac{\log_4 (2/3)}{0.2} = \frac{\ln (2/3)}{0.2 \ln 4} = -1.462
\]

The solution set is \{\log_4 (2/3) \} = \{-1.462\}.

41. \( 3^{1-2x} = 4^x \)
\[
\ln (3^{1-2x}) = \ln (4^x)
\]
\[
(1-2x) \ln 3 = x \ln 4
\]
\[
\ln 3 - 2x \ln 3 = x \ln 4
\]
\[
\ln 3 = x(2 \ln 3 + \ln 4)
\]
\[
x = \frac{\ln 3}{2 \ln 3 + \ln 4} = 0.307
\]

The solution set is \{\log_4 (2/3) \} = \{0.307\}.
42. \[ 2^{x+1} = 5^{1-2x} \]
\[
\ln(2^{x+1}) = \ln(5^{1-2x})
\]
\[
(x+1)\ln 2 = (1-2x)\ln 5
\]
\[
x\ln 2 + 2x\ln 5 = 5 - 2x\ln 5
\]
x\ln 2 + 2x\ln 5 = 5 - 2x\ln 2
\[
x(\ln 2 + 25) = \frac{\ln 5}{2}
\]
x(\ln 50) = \frac{\ln 5}{2}
\[
x = \frac{\ln 5}{\ln 50} = 0.234
\]
The solution set is \( \{ \ln \frac{5}{50} \} = \{0.234\} \).

43. \[
\left( \frac{3}{5} \right)^x = 7^{1-x}
\]
\[
\ln \left( \frac{3}{5} \right)^x = \ln \left( 7^{1-x} \right)
\]
x\ln (3/5) = (1-x)\ln 7
\[
x\ln (3/5) = 7 - x\ln 7
\]
x\ln (3/5) + x\ln 7 = 7
\[
x(\ln (3/5) + \ln 7) = 7
\]
x = \frac{\ln 7}{\ln (3/5) + \ln 7} = 1.356

The solution set is \( \left\{ \frac{\ln 7}{\ln (3/5) + \ln 7} \right\} = \{1.356\} \).

44. \[
\left( \frac{4}{3} \right)^{1-x} = 5^x
\]
\[
\ln \left( \frac{4}{3} \right)^{1-x} = \ln (5^x)
\]
\[
(1-x)\ln (4/3) = x\ln 5
\]
\[
\ln (4/3) - x\ln 4/3 = x\ln 5
\]
\[
\ln (4/3) = x\ln 5 + x\ln (4/3)
\]
\[
\ln (4/3) = x(\ln 5 + \ln (4/3))
\]
\[
\ln (4/3) = x\left( \ln \frac{20}{3} \right)
\]
x = \frac{\ln \frac{4}{3}}{\ln \frac{20}{3}} = 0.152

The solution set is \( \left\{ \frac{\ln \frac{4}{3}}{\ln \frac{20}{3}} \right\} = \{0.152\} \).

45. \[ 1.2^x = (0.5)^{-x} \]
\[ \ln 1.2^x = \ln (0.5)^{-x} \]
\[ x\ln 1.2 = -x\ln 0.5 \]
\[ x\ln 1.2 - x\ln 0.5 = 0 \]
\[ x(\ln 1.2 - \ln 0.5) = 0 \]
x = 0

The solution set is \( \{0\} \).

46. \[ 0.3^{3+x} = 1.7^{2x-1} \]
\[ \ln (0.3)^{3+x} = \ln (1.7)^{2x-1} \]
\[ (1+x)\ln 0.3 = (2x-1)\ln 1.7 \]
\[ \ln 0.3 + x\ln 0.3 = 2x\ln 1.7 - \ln 1.7 \]
x\ln 0.3 - 2x\ln 1.7 = -\ln 1.7 - \ln 0.3
\[ x(\ln 0.3 - 2\ln 1.7) = -\ln 1.7 - \ln 0.3 \]
x = \frac{-\ln 1.7 - \ln 0.3}{\ln 0.3 - 2\ln 1.7} = \frac{\ln 0.51}{\ln 2.89/3} = -0.297

The solution set is \( \left\{ \frac{\ln 0.51}{\ln (2.89/3)} \right\} = \{-0.297\} \).

47. \[ \pi^{1-x} = e^x \]
\[ \ln \pi^{1-x} = \ln e^x \]
\[ (1-x)\ln \pi = x \]
\[ \ln \pi - x\ln \pi = x \]
\[ \ln \pi = x + x\ln \pi \]
\[ \ln \pi = x(1 + \ln \pi) \]
x = \frac{\ln \pi}{1 + \ln \pi} = 0.534

The solution set is \( \left\{ \frac{\ln \pi}{1 + \ln \pi} \right\} = \{0.534\} \).

48. \[ e^{x+3} = \pi^x \]
\[ \ln e^{x+3} = \ln \pi^x \]
\[ x + 3 = x\ln \pi \]
\[ 3 = x\ln \pi - x \]
\[ 3 = x(\ln \pi) - x \]
x = \frac{3}{\ln \pi} = 20.728

The solution set is \( \left\{ \frac{3}{\ln \pi} \right\} = \{20.728\} \).
49. \[ 2^{2x} + 2^x - 12 = 0 \]
\[ (2^x)^2 + 2^x - 12 = 0 \]
\[ (2^x - 3)(2^x + 4) = 0 \]
\[ 2^x - 3 = 0 \quad \text{or} \quad 2^x + 4 = 0 \]
\[ 2^x = 3 \quad \text{or} \quad 2^x = -4 \]
\[ \ln(2^x) = \ln 3 \quad \text{No solution} \]
\[ x \ln 2 = \ln 3 \]
\[ x = \frac{\ln 3}{\ln 2} = 1.585 \]
The solution set is \( \{1.585\} \).

50. \[ 3^{2x} + 3^x - 2 = 0 \]
\[ (3^x)^2 + 3^x - 2 = 0 \]
\[ (3^x - 1)(3^x + 2) = 0 \]
\[ 3^x - 1 = 0 \quad \text{or} \quad 3^x + 2 = 0 \]
\[ 3^x = 1 \quad \text{or} \quad 3^x = -2 \]
\[ x = 0 \quad \text{No solution} \]
The solution set is \( \{0\} \).

51. \[ 3^{2x} + 3^{x+1} - 4 = 0 \]
\[ (3^x)^2 + 3^{x+1} - 4 = 0 \]
\[ (3^x - 1)(3^x + 4) = 0 \]
\[ 3^x - 1 = 0 \quad \text{or} \quad 3^x + 4 = 0 \]
\[ 3^x = 1 \quad \text{or} \quad 3^x = -4 \]
\[ x = 0 \quad \text{No solution} \]
The solution set is \( \{0\} \).

52. \[ 2^{2x} + 2^{x+2} - 12 = 0 \]
\[ (2^x)^2 + 2^x \cdot 2^x - 12 = 0 \]
\[ (2^x - 2)(2^x + 6) = 0 \]
\[ 2^x - 2 = 0 \quad \text{or} \quad 2^x + 6 = 0 \]
\[ 2^x = 2 \quad \text{or} \quad 2^x = -6 \]
\[ x = 1 \quad \text{No solution} \]
The solution set is \( \{1\} \).

53. \[ 16^x + 4^{x+1} - 3 = 0 \]
\[ (4^2)^x + 4 \cdot 4^x - 3 = 0 \]
\[ (4^x)^2 + 4 \cdot 4^x - 3 = 0 \]
Let \( u = 4^x \).
\[ u^2 + 4u - 3 = 0 \]
\[ a = 1, b = 4, c = -3 \]
\[ u = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)} = \frac{-4 \pm \sqrt{28}}{2} \]
\[ = \frac{-4 + 2\sqrt{7}}{2} = -2 + \sqrt{7} \]
Therefore, we get \( 4^x = -2 + \sqrt{7} \) or \( 4^x = -2 - \sqrt{7} \)
(we ignore the first solution since \( 4^x \) is never negative).
The solution set is \( \{ \log_4(-2 + \sqrt{7}) \} = \{-0.315\} \).

54. \[ 9^x - 3^{x+1} + 1 = 0 \]
\[ (3^2)^x - 3 \cdot 3^x + 1 = 0 \]
\[ (3^x)^2 - 3 \cdot 3^x + 1 = 0 \]
Let \( u = 3^x \).
\[ u^2 - 3u + 1 = 0 \]
\[ a = 1, b = -3, c = 1 \]
\[ u = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} = \frac{3 \pm \sqrt{5}}{2} \]
Therefore, we get \( 3^x = \frac{3 + \sqrt{5}}{2} \) or \( 3^x = \frac{3 - \sqrt{5}}{2} \)
\[ x = \log_3 \left( \frac{3 \pm \sqrt{5}}{2} \right) \]
The solution set is \( \left\{ \log_3 \left( \frac{3 + \sqrt{5}}{2} \right), \log_3 \left( \frac{3 - \sqrt{5}}{2} \right) \right\} = \{-0.876, 0.876\} \).
55. \(25^x - 8 \cdot 5^x = -16\)
\[
\left(5^2\right)^x - 8 \cdot 5^x = -16
\]
\[
\left(5^x\right)^2 - 8 \cdot 5^x = -16
\]
Let \(u = 5^x\).
\[
u^2 - 8u = -16
\]
\[
u^2 - 8u + 16 = 0
\]
\[
\left(u - 4\right)^2 = 0
\]
\[
u = 4
\]
Therefore, we get
\(5^x = 4\)
\[
x = \log_5 4
\]
The solution set is \(\{\log_5 4\} = \{0.861\}\).

56. \(36^x - 6 \cdot 6^x = -9\)
\[
\left(6^2\right)^x - 6 \cdot 6^x + 9 = 0
\]
\[
\left(6^x\right)^2 - 6 \cdot 6^x + 9 = 0
\]
\[
\left(6^x - 3\right)^2 = 0
\]
\[
6^x = 3
\]
\[
x = \log_6 3
\]
The solution set is \(\{\log_6 3\} = \{0.613\}\).

57. \(3 \cdot 4^x + 4 \cdot 2^x + 8 = 0\)
\[
3 \cdot \left(2^2\right)^x + 4 \cdot 2^x + 8 = 0
\]
\[
3 \cdot \left(2^x\right)^2 + 4 \cdot 2^x + 8 = 0
\]
Let \(u = 2^x\).
\[
3u^2 + 4u + 8 = 0
\]
\[
a = 3, \ b = 4, \ c = 8
\]
\[
u = \frac{-4 \pm \sqrt{4^2 - 4(3)(8)}}{2(3)}
\]
\[
= \frac{-4 \pm \sqrt{-80}}{6} = \text{not real}
\]
The equation has no real solution.

58. \(2 \cdot 49^x + 11 \cdot 7^x + 5 = 0\)
\[
2 \cdot \left(7^2\right)^x + 11 \cdot 7^x + 5 = 0
\]
\[
2 \cdot \left(7^x\right)^2 + 11 \cdot 7^x + 5 = 0
\]
Let \(u = 7^x\).
\[
2u^2 + 11u + 5 = 0
\]
\[
\left(2u + 1\right)\left(u + 5\right) = 0
\]
\[
2u + 1 = 0 \quad \text{or} \quad u + 5 = 0
\]
\[
2u = -1 \quad \text{or} \quad u = -5
\]
\[
u = -\frac{1}{2}
\]
Therefore, we get
\(7^x = -\frac{1}{2}\) or \(7^x = -5\)

Since \(7^x > 0\) for all \(x\), the equation has no real solution.

59. \(4^x - 10 \cdot 4^{-x} = 3\)
Multiply both sides of the equation by \(4^x\).
\[
\left(4^x\right)^2 - 10 \cdot 4^{-x} \cdot 4^x = 3 \cdot 4^x
\]
\[
\left(4^x\right)^2 - 10 = 3 \cdot 4^x
\]
\[
\left(4^x\right)^2 - 3 \cdot 4^x - 10 = 0
\]
\[
\left(4^x - 5\right)\left(4^x + 2\right) = 0
\]
\[
4^x - 5 = 0 \quad \text{or} \quad 4^x + 2 = 0
\]
\[
4^x = 5 \quad \text{or} \quad 4^x = -2 \quad \text{(not real)}
\]
\[
x = \log_4 5
\]
The solution set is \(\{\log_4 5\} = \{1.161\}\).

60. \(3^x - 14 \cdot 3^{-x} = 5\)
Multiply both sides of the equation by \(3^x\).
\[
\left(3^x\right)^2 - 14 \cdot 3^{-x} \cdot 3^x = 5 \cdot 3^x
\]
\[
\left(3^x\right)^2 - 14 = 5 \cdot 3^x
\]
\[
\left(3^x\right)^2 - 5 \cdot 3^x - 14 = 0
\]
\[
\left(3^x - 7\right)\left(3^x + 2\right) = 0
\]
\[
3^x - 7 = 0 \quad \text{or} \quad 3^x + 2 = 0
\]
\[
3^x = 7 \quad \text{or} \quad 3^x = -2 \quad \text{(not real)}
\]
\[
x = \log_3 7
\]
The solution set is \(\{\log_3 7\} = \{1.771\}\).
61. \( \log_5(x+1) - \log_4(x-2) = 1 \)
Using INTERSECT to solve:
\[
y_1 = \frac{\ln(x+1)}{\ln(5)} - \frac{\ln(x-2)}{\ln(4)} \\
y_2 = 1
\]
Thus, \( x = 2.79 \), so the solution set is \( \{2.79\} \).

62. \( \log_2(x-1) - \log_4(x+2) = 2 \)
Using INTERSECT to solve:
\[
y_1 = \frac{\ln(x-1)}{\ln(2)} - \frac{\ln(x+2)}{\ln(4)} \\
y_2 = 2
\]
Thus, \( x = 12.15 \), so the solution set is \( \{12.15\} \).

63. \( e^x = -x \)
Using INTERSECT to solve: \( y_1 = e^x; \ y_2 = -x \)
Thus, \( x = -0.57 \), so the solution set is \( \{-0.57\} \).

64. \( e^{2x} = x + 2 \)
Using INTERSECT to solve: \( y_1 = e^{2x}; \ y_2 = x + 2 \)
Thus, \( x = -1.98 \) or \( x = 0.45 \), so the solution set is \( \{-1.98, 0.45\} \).

65. \( e^x = x^2 \)
Using INTERSECT to solve:
\[
y_1 = e^x; \ y_2 = x^2
\]
Thus, \( x = -0.70 \), so the solution set is \( \{-0.70\} \).

66. \( e^x = x^3 \)
Using INTERSECT to solve:
\[
y_1 = e^x; \ y_2 = x^3
\]
Thus, \( x = 1.86 \) or \( x = 4.54 \), so the solution set is \( \{1.86, 4.54\} \).

67. \( \ln x = -x \)
Using INTERSECT to solve:
\[
y_1 = \ln x; \ y_2 = -x
\]
Thus, \( x = 0.57 \), so the solution set is \( \{0.57\} \).

68. \( \ln(2x) = -x + 2 \)
Using INTERSECT to solve:
\[
y_1 = \ln(2x); \ y_2 = -x + 2
\]
Thus, \( x = 1.16 \), so the solution set is \( \{1.16\} \).
69.  \[ \ln x = x^3 - 1 \]
Using INTERSECT to solve:
\[ y_1 = \ln x; \quad y_2 = x^3 - 1 \]
Thus, \( x = 0.39 \) or \( x = 1 \), so the solution set is \( \{0.39, 1\} \).

70.  \[ \ln x = -x^2 \]
Using INTERSECT to solve:
\[ y_1 = \ln x; \quad y_2 = -x^2 \]
Thus, \( x = 0.65 \), so the solution set is \( \{0.65\} \).

71.  \[ e^x + \ln x = 4 \]
Using INTERSECT to solve:
\[ y_1 = e^x + \ln x; \quad y_2 = 4 \]
Thus, \( x = 1.32 \), so the solution set is \( \{1.32\} \).

72.  \[ e^x - \ln x = 4 \]
Using INTERSECT to solve:
\[ y_1 = e^x - \ln x; \quad y_2 = 4 \]
Thus, \( x = 0.05 \) or \( x = 1.48 \), so the solution set is \( \{0.05, 1.48\} \).

73.  \[ e^{-x} = \ln x \]
Using INTERSECT to solve:
\[ y_1 = e^{-x}; \quad y_2 = \ln x \]
Thus, \( x = 1.31 \), so the solution set is \( \{1.31\} \).

74.  \[ e^{-x} = -\ln x \]
Using INTERSECT to solve:
\[ y_1 = e^{-x}; \quad y_2 = -\ln x \]
Thus, \( x = 0.57 \), so the solution set is \( \{0.57\} \).

75.  \[ \log_2 (x+1) - \log_4 x = 1 \]
\[ \log_2 (x+1) - \log_2 x = 1 \]
\[ \log_2 x = 2 \]
\[ \log_2 (x+1) - \log_2 x = 2 \]
\[ \log_2 \left( \frac{(x+1)^2}{x} \right) = 2 \]
\[ \frac{(x+1)^2}{x} = 2^2 \]
\[ x^2 + 2x + 1 = 4x \]
\[ x^2 - 2x + 1 = 0 \]
\[ (x-1)^2 = 0 \]
\[ x-1 = 0 \]
\[ x = 1 \]
Since each of the original logarithms is defined for \( x = 1 \), the solution set is \( \{1\} \).
76. \( \log_2(3x + 2) - \log_4 x = 3 \)
\( \log_2(3x + 2) - \frac{\log_2 x}{\log_2 4} = 3 \)
\( \log_2(3x + 2) - \frac{\log_2 x}{2} = 3 \)
\( 2 \log_2(3x + 2) - \log_2 x = 6 \)
\( \log_2(3x + 2)^2 - \log_2 x = 6 \)
\( \log_2 \left( \frac{(3x + 2)^2}{x} \right) = 6 \)
\( \frac{(3x + 2)^2}{x} = 26 \)
\( 9x^2 + 12x + 4 = 64x \)
\( 9x^2 - 52x + 4 = 0 \)
\( x = \frac{52 \pm \sqrt{(-52)^2 - 4(9)(4)}}{2(9)} \)
\( = \frac{52 \pm \sqrt{2560}}{18} \)
\( = \frac{52 \pm 16\sqrt{10}}{18} \)
\( = \frac{26 \pm 8\sqrt{10}}{9} \)
\( = 5.700 \) or 0.078

Since each of the original logarithms is defined for \( x = 0.078 \) and \( x = 5.700 \), the solution set is \( \left\{ \frac{26 - 8\sqrt{10}}{9}, \frac{26 + 8\sqrt{10}}{9} \right\} = \{0.078, 5.700\} \).

77. \( \log_{16} x + \log_{4} x + \log_{2} x = 7 \)
\( \log_{2} x + \log_{2} x + \log_{2} x = 7 \)
\( \log_{2} 16 + \log_{2} 4 \)
\( \log_{2} x + \log_{2} x + \log_{2} x = 7 \)
\( \log_{2} x + 2\log_{2} x + 4\log_{2} x = 28 \)
\( 7\log_{2} x = 28 \)
\( \log_{2} x = 4 \)
\( x = 2^4 = 16 \)

Since each of the original logarithms is defined for \( x = 16 \), the solution set is \( \{16\} \).

78. \( \log_{9} x + 3 \log_{3} x = 14 \)
\( \log_{3} \frac{x}{2} + 3 \log_{3} x = 14 \)
\( \log_{3} x = 4 \)
\( x = 3^4 = 81 \)

Since each of the original logarithms is defined for \( x = 81 \), the solution set is \( \{81\} \).

79. \( (\sqrt{2})^{2-x} = 2^{x^2} \)
\( 2^{(\frac{1}{2})2-x} = 2^{x^2} \)
\( \frac{1}{2^3} (2-x) = 2^{x^2} \)
\( \frac{1}{3} (2-x) = x^2 \)
\( 2-x = 3x^2 \)
\( 3x^2 + x - 2 = 0 \)
\( (3x - 2)(x + 1) = 0 \)
\( x = \frac{2}{3} \) or \( x = -1 \)

The solution set is \( \{-1, \frac{2}{3}\} \).

80. \( \log_{2} x \log_{2} x = 4 \)
\( \log_{2} x \cdot \log_{2} x = 4 \)
\( (\log_{2} x)^2 = 4 \)
\( \log_{2} x = -2 \) or \( \log_{2} x = 2 \)
\( x = 2^{-2} \) or \( x = 2^2 \)
\( x = \frac{1}{4} \) or \( x = 4 \)

Since each of the original logarithms is defined for both \( x = \frac{1}{4} \) and \( x = 4 \), the solution set is \( \left\{ \frac{1}{4}, 4 \right\} \).
Section 5.6: Logarithmic and Exponential Equations

81. \[
\frac{e^x + e^{-x}}{2} = 1
\]
\[
e^x + e^{-x} = 2
\]
\[
e^x(e^x + e^{-x}) = 2e^x
\]
\[
e^{2x} + 1 = 2e^x
\]
\[
(e^x)^2 - 2e^x + 1 = 0
\]
\[
(e^x - 1)^2 = 0
\]
\[
e^x - 1 = 0
\]
\[
e^x = 1
\]
\[
x = 0
\]
The solution set is \(\{0\}\).

82. \[
\frac{e^x + e^{-x}}{2} = 3
\]
\[
e^x + e^{-x} = 6
\]
\[
e^x(e^x + e^{-x}) = 6e^x
\]
\[
e^{2x} + 1 = 6e^x
\]
\[
(e^x)^2 - 6e^x + 1 = 0
\]
\[
e^x = \frac{6 \pm \sqrt{(6)^2 - 4(1)(1)}}{2(1)}
\]
\[
e^x = \frac{6 \pm \sqrt{32}}{2} = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}
\]
x = ln(3 - 2\sqrt{2}) or x = ln(3 + 2\sqrt{2})
x = -1.763 or x = 1.763
The solution set is \(\{-1.763, 1.763\}\).

83. \[
\frac{e^x - e^{-x}}{2} = 2
\]
\[
e^x - e^{-x} = 4
\]
\[
e^x(e^x - e^{-x}) = 4e^x
\]
\[
e^{2x} - 1 = 4e^x
\]
\[
(e^x)^2 - 4e^x - 1 = 0
\]
\[
e^x = \frac{(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}
\]
\[
e^x = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}
\]
x = ln(2 - \sqrt{5}) or x = ln(2 + \sqrt{5})
x = ln(-0.236) or x = 1.444
Since ln(-0.236) is undefined, the solution set is \(\{ln(2 + \sqrt{5})\}\) = \{1.444\}.

84. \[
\frac{e^x - e^{-x}}{2} = -2
\]
\[
e^x - e^{-x} = -4
\]
\[
e^x(e^x - e^{-x}) = -4e^x
\]
\[
e^{2x} - 1 = -4e^x
\]
\[
(e^x)^2 + 4e^x - 1 = 0
\]
\[
e^x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-1)}}{2(1)}
\]
\[
e^x = \frac{-4 \pm \sqrt{20}}{2} = \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}
\]
x = ln(-2 - \sqrt{5}) or x = ln(-2 + \sqrt{5})
x = ln(-4.236) or x = -1.444
Since ln(-4.236) is undefined, the solution set is \(\{ln(-2 + \sqrt{5})\}\) = \{-1.444\}.

85. \[
\log_5 x + \log_3 x = 1
\]
\[
\frac{\ln x}{\ln 5} + \frac{\ln x}{\ln 3} = 1
\]
\[
(ln x) \left(\frac{1}{\ln 5} + \frac{1}{\ln 3}\right) = 1
\]
\[
\frac{\ln x}{\ln 5} + \frac{\ln x}{\ln 3} = 1
\]
\[
\ln x = \frac{1}{\frac{1}{\ln 5} + \frac{1}{\ln 3}}
\]
\[
\ln x = \frac{1}{\ln 5 + \ln 3}
\]
x = \frac{\ln x}{(\ln 5)(\ln 3)}
\[
\ln x = \frac{\ln x}{\ln 3 + \ln 5}
\]
\[
\ln x = \frac{(\ln 5)(\ln 3)}{\ln 15}
\]
\[
x = \frac{\ln 15}{(\ln 5)(\ln 3)} = 1.921
\]
The solution set is \(\{\frac{(\ln 5)(\ln 3)}{\ln 15}\}\) = \{1.921\}.
Chapter 5: Exponential and Logarithmic Functions

86. \[ \log_2 x + \log_6 x = 3 \]
\[ \ln x \cdot \ln x + \ln 2 \cdot \ln 6 = 3 \]
\[ (\ln x) \left( \frac{1}{\ln 2} + \frac{1}{\ln 6} \right) = 3 \]
\[ \ln x = \frac{3}{\frac{1}{\ln 2} + \frac{1}{\ln 6}} \]
\[ \ln x = \frac{3(\ln 6)(\ln 2)}{\ln 6 + \ln 2} \]
\[ \ln x = \frac{3(\ln 2)(\ln 6)}{\ln 12} \]
\[ x = e^{\frac{3(\ln 2)(\ln 6)}{\ln 12}} = 4.479 \]
The solution set is \( \{4.479\} \).

87. a. \[ f(x) = 3 \]
\[ \log_2 (x + 3) = 3 \]
\[ x + 3 = 2^3 \]
\[ x + 3 = 8 \]
\[ x = 5 \]
The solution set is \( \{5\} \). The point \((5,3)\) is on the graph of \( f \).

b. \[ g(x) = 4 \]
\[ \log_2 (3x + 1) = 4 \]
\[ 3x + 1 = 2^4 \]
\[ 3x + 1 = 16 \]
\[ 3x = 15 \]
\[ x = 5 \]
The solution set is \( \{5\} \). The point \((5,4)\) is on the graph of \( g \).

c. \[ f(x) = g(x) \]
\[ \log_2 (x + 3) = \log_2 (3x + 1) \]
\[ x + 3 = 3x + 1 \]
\[ 2 = 2x \]
\[ 1 = x \]
The solution set is \( \{1\} \), so the graphs intersect when \( x = 1 \). That is, at the point \((1,2)\).

d. \[(f + g)(x) = 7 \]
\[ \log_2 (x + 3) + \log_2 (3x + 1) = 7 \]
\[ \log_2 [(x + 3)(3x + 1)] = 7 \]
\[ (x + 3)(3x + 1) = 2^7 \]
\[ 3x^2 + 10x + 3 = 128 \]
\[ 3x^2 + 10x - 125 = 0 \]
\[ (3x + 25)(x - 5) = 0 \]
\[ 3x + 25 = 0 \quad \text{or} \quad x - 5 = 0 \]
\[ x = -\frac{25}{3} \quad \text{or} \quad x = 5 \]
The solution set is \( \{5\} \).

e. \[(f - g)(x) = 2 \]
\[ \log_2 (x + 3) - \log_2 (3x + 1) = 2 \]
\[ \log_2 \frac{x + 3}{3x + 1} = 2 \]
\[ \frac{x + 3}{3x + 1} = 2^2 \]
\[ \frac{x + 3}{3x + 1} = 4(3x + 1) \]
\[ x + 3 = 12x + 4 \]
\[ -11x = -1 \]
\[ x = \frac{1}{11} \]
The solution set is \( \left\{ \frac{1}{11} \right\} \).

88. a. \[ f(x) = 2 \]
\[ \log_3 (x + 5) = 2 \]
\[ x + 5 = 3^2 \]
\[ x + 5 = 9 \]
\[ x = 4 \]
The solution set is \( \{4\} \). The point \((4,2)\) is on the graph of \( f \).

b. \[ g(x) = 3 \]
\[ \log_3 (x - 1) = 3 \]
\[ x - 1 = 3^3 \]
\[ x - 1 = 27 \]
\[ x = 28 \]
The solution set is \( \{28\} \). The point \((28,3)\) is on the graph of \( g \).
Section 5.6: Logarithmic and Exponential Equations

c. \[ f(x) = g(x) \]
\[ \log_5(x+5) = \log_5(x-1) \]
\[ x+5 = x-1 \]
\[ 5 = -1 \quad \text{False} \]
This is a contradiction, so the equation has no solution. The graphs do not intersect.

d. \[ (f + g)(x) = 3 \]
\[ \log_3(x+5) + \log_3(x-1) = 3 \]
\[ \log_3[(x+5)(x-1)] = 3 \]
\[ (x+5)(x-1) = 3^3 \]
\[ x^2 + 4x - 5 = 27 \]
\[ x^2 + 4x - 32 = 0 \]
\[ (x+8)(x-4) = 0 \]
\[ x = -8 \quad \text{or} \quad x = 4 \]
The solution set is \( \{4\} \).

e. \[ (f - g)(x) = 2 \]
\[ \log_3(x+5) - \log_3(x-1) = 2 \]
\[ \log_3 \frac{x+5}{x-1} = 2 \]
\[ \frac{x+5}{x-1} = 3^2 \]
\[ x+5 = 9(x-1) \]
\[ x+5 = 9x-9 \]
\[ 14 = 8x \]
\[ x = \frac{7}{4} \]
The solution set is \( \left\{ \frac{7}{4} \right\} \).

89. a.

90. a.

b. \[ f(x) = g(x) \]
\[ 3^{x+1} = 2^{x+2} \]
\[ \ln(3^{x+1}) = \ln(2^{x+2}) \]
\[ (x+1)\ln 3 = (x+2)\ln 2 \]
\[ x\ln 3 + \ln 3 = x\ln 2 + 2\ln 2 \]
\[ x\ln 3 - x\ln 2 = 2\ln 2 - \ln 3 \]
\[ x(\ln 3 - \ln 2) = 2\ln 2 - \ln 3 \]
\[ x = \frac{2\ln 2 - \ln 3}{\ln 3 - \ln 2} = 0.710 \]
\[ f\left(\frac{2\ln 2 - \ln 3}{\ln 3 - \ln 2}\right) = 6.541 \]
The intersection point is roughly \((0.710, 6.541)\).

c. Based on the graph, \( f(x) > g(x) \) for \( x > 0.710 \). The solution set is \( \{x \mid x > 0.710\} \) or \((0.710, \infty)\).
c. Based on the graph, \( f(x) > g(x) \) for \( x > 2.513 \). The solution set is \( \{ x \mid x > 2.513 \} \) or \( (2.513, \infty) \).

91. a., b.

\[ f(x) = 3^x \]
\[ g(x) = 2^{-x^2} \]
\[ (\log_3 10, 10) \]
\[ (1/2, 2\sqrt{2}) \]

92. a., b.

\[ f(x) = 2^x \]
\[ g(x) = 10 \]
\[ (\log_2 12, 12) \]

93. a., b.

\[ f(x) = g(x) \]
\[ 2^{x+1} = 2^{-x^2} \]
\[ x+1 = -x + 2 \]
\[ 2x = 1 \]
\[ x = 1/2 \]
\[ f\left(\frac{1}{2}\right) = 2^{1/2 + 1} = 2^{3/2} = 2\sqrt{2} \]

The intersection point is \( \left(\frac{1}{2}, 2\sqrt{2}\right) \).

94. a., b.

\[ f(x) = 3^{-x+1} \]
\[ g(x) = 3^x - 2 \]
\[ \left(\frac{3}{2}, \frac{\sqrt{3}}{3}\right) \]

95. a., b.

\[ f(x) = g(x) \]
\[ 3^{-x+1} = 3^{x^2} \]
\[ -x + 1 = x - 2 \]
\[ -2x = -3 \]
\[ x = -\frac{3}{2} \]
\[ f\left(-\frac{3}{2}\right) = 3^{-3/2 + 1} = 3^{-1/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \]

The intersection point is \( \left(-\frac{3}{2}, \frac{\sqrt{3}}{3}\right) \).
Section 5.6: Logarithmic and Exponential Equations

95. a. \( f(x) = 2^x - 4 \)
   
   Using the graph of \( y = 2^x \), shift the graph down 4 units.

   \[ f(x) = 2^x - 4 \]

   \[ \begin{array}{c|c|c|c|c|c|c|}
   \hline
   x & -5 & -4 & -3 & -2 & -1 & 0 & 1 \\
   \hline
   y & 1 & 0.5 & 0.25 & 0.125 & 0.0625 & 0.03125 & 0.015625 \\
   \hline
   \end{array} \]

   b. \( f(x) = 0 \)
   
   \[ 2^x - 4 = 0 \]
   
   \[ 2^x = 4 \]
   
   \[ 2^x = 2^2 \]
   
   \[ x = 2 \]
   
   The zero of \( f \) is \( x = 2 \).

   c. Based on the graph, \( f(x) < 0 \) when \( x < 2 \).
   
   The solution set is \( \{ x \mid x < 2 \} \) or \( (-\infty, 2) \).

96. a. \( g(x) = 3^x - 9 \)
   
   Using the graph of \( y = 3^x \), shift the graph down nine units.

   \[ g(x) = 3^x - 9 \]

   \[ \begin{array}{c|c|c|c|c|c|c|}
   \hline
   x & -5 & -4 & -3 & -2 & -1 & 0 & 1 \\
   \hline
   y & 0.2401 & 0.0977 & 0.0321 & 0.0101 & 0.0029 & 0.0008 & 0.0002 \\
   \hline
   \end{array} \]

   b. \( g(x) = 0 \)
   
   \[ 3^x - 9 = 0 \]
   
   \[ 3^x = 9 \]
   
   \[ 3^x = 3^2 \]
   
   \[ x = 2 \]
   
   The zero of \( g \) is \( x = 2 \).

   c. Based on the graph, \( g(x) > 0 \) when \( x > 2 \).
   
   The solution set is \( \{ x \mid x > 2 \} \) or \( (2, \infty) \).

97. a. \( 304(1.009)^{t-2008} = 354 \)
   
   \( (1.009)^{t-2008} = \frac{354}{304} \)
   
   \( \ln(1.009)^{t-2008} = \ln\left(\frac{354}{304}\right) \)
   
   \( (t - 2008) \ln(1.009) = \ln\left(\frac{354}{304}\right) \)
   
   \( t - 2008 = \frac{\ln(354/304)}{\ln(1.009)} \)
   
   \( t = \frac{\ln(354/304)}{\ln(1.009)} + 2008 \)
   
   \( t = 2025 \)
   
   According to the model, the population of the U.S. will reach 354 million people around the beginning of the year 2025.

b. \( 304(1.009)^{t-2008} = 416 \)
   
   \( (1.009)^{t-2008} = \frac{416}{304} \)
   
   \( \ln(1.009)^{t-2008} = \ln\left(\frac{416}{304}\right) \)
   
   \( (t - 2008) \ln(1.009) = \ln\left(\frac{416}{304}\right) \)
   
   \( t - 2008 = \frac{\ln(416/304)}{\ln(1.009)} \)
   
   \( t = \frac{\ln(416/304)}{\ln(1.009)} + 2008 \)
   
   \( t = 2043 \)
   
   According to the model, the population of the U.S. will reach 416 million people in the beginning of the year 2043.

98. a. \( 6.78(1.0114)^{t-2009} = 8.7 \)
   
   \( (1.0114)^{t-2009} = \frac{8.7}{6.78} \)
   
   \( \ln(1.0114)^{t-2009} = \ln\left(\frac{8.7}{6.78}\right) \)
   
   \( (t - 2009) \ln(1.0114) = \ln\left(\frac{8.7}{6.78}\right) \)
   
   \( t - 2009 = \frac{\ln(8.7/6.78)}{\ln(1.0114)} \)
   
   \( t = \frac{\ln(8.7/6.78)}{\ln(1.0114)} + 2009 \)
   
   \( t = 2031 \)
Chapter 5: Exponential and Logarithmic Functions

According to the model, the population of the world will reach 8.7 billion people at the beginning of the year 2031.

b. \[6.78 \times 1.0114^{t-2009} = 14\]
\[\frac{14}{6.78} \approx 2.073\]

According to the model, the population of the world will reach 14 billion people at the beginning of the year 2073.

99. a. \[16,500(0.82)^t = 9,000\]
\[(0.82)^t = \frac{9,000}{16,500}\]
\[\log (0.82)^t = \log \left(\frac{9,000}{16,500}\right)\]
\[t \log (0.82) = \log \left(\frac{9,000}{16,500}\right)\]
\[t = \frac{\log (9,000/16,500)}{\log (0.82)} \approx 3.05\]
According to the model, the car will be worth $9,000 after about 3 years.

b. \[16,500(0.82)^t = 4,000\]
\[(0.82)^t = \frac{4,000}{16,500}\]
\[\log (0.82)^t = \log \left(\frac{4,000}{16,500}\right)\]
\[t \log (0.82) = \log \left(\frac{4,000}{16,500}\right)\]
\[t = \frac{\log (4,000/16,500)}{\log (0.82)} \approx 7.14\]
According to the model, the car will be worth $4,000 after about 7.1 years.

c. \[16,500(0.82)^t = 2,000\]
\[(0.82)^t = \frac{2,000}{16,500}\]
\[\log (0.82)^t = \log \left(\frac{2,000}{16,500}\right)\]
\[t \log (0.82) = \log \left(\frac{2,000}{16,500}\right)\]
\[t = \frac{\log (2,000/16,500)}{\log (0.82)} \approx 10.63\]
According to the model, the car will be worth $2,000 after about 10.6 years.

100. a. \[16,775(0.905)^t = 15,000\]
\[(0.905)^t = \frac{15,000}{16,775}\]
\[\log (0.905)^t = \log \left(\frac{15,000}{16,775}\right)\]
\[t \log (0.905) = \log \left(\frac{15,000}{16,775}\right)\]
\[t = \frac{\log (15,000/16,775)}{\log (0.905)} \approx 1.1\]
According to the model, the car will be worth $15,000 after about 1.1 years.

b. \[16,775(0.905)^t = 8,000\]
\[(0.905)^t = \frac{8,000}{16,775}\]
\[\log (0.905)^t = \log \left(\frac{8,000}{16,775}\right)\]
\[t \log (0.905) = \log \left(\frac{8,000}{16,775}\right)\]
\[t = \frac{\log (8,000/16,775)}{\log (0.905)} \approx 7.4\]
According to the model, the car will be worth $8,000 after about 7.4 years.
Section 5.7: Financial Models

c. \[ 16,775(0.905)^t = 4,000 \]
\[ (0.905)^t = \frac{4,000}{16,775} \]
\[ \log (0.905)^t = \log \left( \frac{4,000}{16,775} \right) \]
\[ t \log (0.905) = \log \left( \frac{4,000}{16,775} \right) \]
\[ t = \frac{\log (4,000/16,775)}{\log (0.905)} = 14.4 \]

According to the model, the car will be worth $4,000 after about 14.4 years.

101. Solution A: change to exponential expression; square root method; meaning of ±; solve.

Solution B: \( \log_a M^r = r \log_a M \); divide by 2; change to exponential expression; solve.

The power rule \( \log_a M^r = r \log_a M \) only applies when \( M > 0 \). In this equation, \( M = x^{-1} \).

Now, \( x = -2 \) causes \( M = -2^{-1} = -0.5 \). Thus, if we use the power rule, we lose the valid solution \( x = -2 \).

Section 5.7

1. \( P = \$500, \ r = 0.06, \ t = 6 \) months = 0.5 year
\( I = Prt = (500)(0.06)(0.5) = \$15.00 \)

2. \( P = \$5000, \ t = 9 \) months = 0.75 year, \( I = \$500 \)
\[ 500 = 5000r(0.75) \]
\[ r = \frac{500}{5000(0.75)} = \frac{2}{15} = 13 \frac{1}{3} \% \]

The per annum interest rate was \( 13 \frac{1}{3} \% \).

3. principal
4. I; Prt; simple interest
5. 4
6. effective rate of interest

7. \( P = \$100, \ r = 0.04, \ n = 4, \ t = 2 \)
\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} = 100 \left( 1 + \frac{0.04}{4} \right)^{4(2)} = \$108.29 \]

8. \( P = \$50, \ r = 0.06, \ n = 12, \ t = 3 \)
\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} = 50 \left( 1 + \frac{0.06}{12} \right)^{12(3)} = \$59.83 \]

9. \( P = \$500, \ r = 0.08, \ n = 4, \ t = 2.5 \)
\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} = 500 \left( 1 + \frac{0.08}{4} \right)^{4(2.5)} = \$609.50 \]

10. \( P = \$300, \ r = 0.12, \ n = 12, \ t = 1.5 \)
\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} = 300 \left( 1 + \frac{0.12}{12} \right)^{12(1.5)} = \$358.84 \]

11. \( P = \$600, \ r = 0.05, \ n = 365, \ t = 3 \)
\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} = 600 \left( 1 + \frac{0.05}{365} \right)^{365(3)} = \$697.09 \]

12. \( P = \$700, \ r = 0.06, \ n = 365, \ t = 2 \)
\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} = 700 \left( 1 + \frac{0.06}{365} \right)^{365(2)} = \$789.24 \]

13. \( P = \$1000, \ r = 0.11, \ t = 2 \)
\[ A = Pe^{rt} = 1000e^{0.11(2)} = \$1246.08 \]

14. \( P = \$400, \ r = 0.07, \ t = 3 \)
\[ A = Pe^{rt} = 400e^{0.07(3)} = \$493.47 \]

15. \( P = \$100, \ r = 0.06, \ n = 12, \ t = 2 \)
\[ P = A \left( 1 + \frac{r}{n} \right)^{-nt} = 100 \left( 1 + \frac{0.06}{12} \right)^{-(12)(2)} = \$88.72 \]

16. \( P = \$75, \ r = 0.08, \ n = 4, \ t = 3 \)
\[ P = A \left( 1 + \frac{r}{n} \right)^{-nt} = 75 \left( 1 + \frac{0.08}{4} \right)^{-4(3)} = \$59.14 \]

17. \( P = \$1000, \ r = 0.06, \ n = 365, \ t = 2.5 \)
\[ P = A \left( 1 + \frac{r}{n} \right)^{-nt} = 1000 \left( 1 + \frac{0.06}{365} \right)^{-(365)(2.5)} = \$860.72 \]
Chapter 5: Exponential and Logarithmic Functions

18. \( A = 800, \ r = 0.07, \ n = 12, \ t = 3.5 \)
\[ P = A \left(1 + \frac{r}{n}\right)^{nt} \]
\[ = 800 \left(1 + \frac{0.07}{12}\right)^{(-12)(3.5)} \approx 626.61 \]

19. \( A = 600, \ r = 0.04, \ n = 4, \ t = 2 \)
\[ P = A \left(1 + \frac{r}{n}\right)^{nt} \]
\[ = 600 \left(1 + \frac{0.04}{4}\right)^{(-4)(2)} \approx 554.09 \]

20. \( A = 300, \ r = 0.03, \ n = 365, \ t = 4 \)
\[ P = A \left(1 + \frac{r}{n}\right)^{nt} \]
\[ = 300 \left(1 + \frac{0.03}{365}\right)^{(-365)(4)} \approx 266.08 \]

21. \( A = 80, \ r = 0.09, \ t = 3.25 \)
\[ P = Ae^{-rt} = 80e^{-0.09(3.25)} \approx 59.71 \]

22. \( A = 800, \ r = 0.08, \ t = 2.5 \)
\[ P = Ae^{-rt} = 800e^{-0.08(2.5)} \approx 654.98 \]

23. Suppose \( P \) dollars are invested for 1 year at 5%.
Compounded quarterly yields:
\[ A = P \left(1 + \frac{0.05}{4}\right)^{4(1)} \approx 1.05095P \]
The interest earned is
\[ I = 1.05095P - P = 0.05095P \]
Thus, \( I = Prt \)
\[ 0.05095 = r \]
The effective interest rate is 5.095%.

24. Suppose \( P \) dollars are invested for 1 year at 6%.
Compounded monthly yields:
\[ A = P \left(1 + \frac{0.06}{12}\right)^{12(1)} \approx 1.06168P \]
The interest earned is
\[ I = 1.06168P - P = 0.06168P \]
Thus, \( I = Prt \)
\[ 0.06168 = r \]
The effective interest rate is 6.168%.

25. Suppose \( P \) dollars are invested for 1 year at 5%.
Compounded continuously yields:
\[ A = Pe^{0.05(1)} = 1.05127P \]
The interest earned is
\[ I = 1.05127P - P = 0.05127P \]
Thus, \( I = Prt \)
\[ 0.05127 = r \]
The effective interest rate is 5.127%.

26. Suppose \( P \) dollars are invested for 1 year at 6%.
Compounded continuously yields:
\[ A = Pe^{0.06(1)} = 1.06184P \]
The interest earned is
\[ I = 1.06184P - P = 0.06184P \]
Thus, \( I = Prt \)
\[ 0.06184 = r \]
The effective interest rate is 6.184%.

27. 6% compounded quarterly:
\[ A = 10,000 \left(1 + \frac{0.06}{4}\right)^{4(1)} \approx 10,613.64 \]

6\( \frac{1}{4} \)% compounded annually:
\[ A = 10,000 \left(1 + \frac{0.0625}{1}\right)^{1} \approx 10,625 \]
6\( \frac{1}{4} \)% compounded annually is the better deal.

28. 9% compounded quarterly:
\[ A = 10,000 \left(1 + \frac{0.09}{4}\right)^{4(1)} \approx 10,930.83 \]

9\( \frac{1}{2} \)% compounded annually:
\[ A = 10,000 \left(1 + \frac{0.0925}{1}\right)^{1} \approx 10,925 \]
9\( \frac{1}{2} \)% compounded quarterly is the better deal.

29. 9% compounded monthly:
\[ A = 10,000 \left(1 + \frac{0.09}{12}\right)^{12(1)} \approx 10,938.07 \]
8.8% compounded daily:
\[ A = 10,000 \left(1 + \frac{0.088}{365}\right)^{365} \approx 10,919.77 \]
9% compounded monthly is the better deal.

30. 8% compounded semiannually:
\[ A = 10,000 \left(1 + \frac{0.08}{2}\right)^{2(1)} \approx 10,816 \]
7.9% compounded daily:
\[
A = 10,000 \left(1 + \frac{0.079}{365}\right)^{365} = $10,821.95
\]
7.9% compounded daily is the better deal.

31. \(2P = P \left(1 + \frac{r}{12}\right)^{3(1)}\)
\[
2 = (1 + r)^3
\]
\[
\sqrt[3]{2} = 1 + r
\]
\[
r = \sqrt[3]{2} - 1 = 0.25992
\]
The required rate is 25.992%.

32. \(2P = P \left(1 + \frac{r}{12}\right)^{6(1)}\)
\[
2 = (1 + r)^6
\]
\[
\sqrt[6]{2} = 1 + r
\]
\[
r = \sqrt[6]{2} - 1 = 0.12246
\]
The required rate is 12.246%.

33. \(3P = P \left(1 + \frac{r}{12}\right)^{5(1)}\)
\[
3 = (1 + r)^5
\]
\[
\sqrt[5]{3} = 1 + r
\]
\[
r = \sqrt[5]{3} - 1 = 0.24573
\]
The required rate is 24.573%.

34. \(3P = P \left(1 + \frac{r}{12}\right)^{10(1)}\)
\[
3 = (1 + r)^{10}
\]
\[
\sqrt[10]{3} = 1 + r
\]
\[
r = \sqrt[10]{3} - 1 = 0.11612
\]
The required rate is 11.612%.

35. a. \(2P = P \left(1 + \frac{0.08}{12}\right)^{12t}\)
\[
2 = \left(1 + \frac{0.08}{12}\right)^{12t}
\]
\[
\ln 2 = \ln \left(1 + \frac{0.08}{12}\right)^{12t}
\]
\[
\ln 2 = 12t \ln \left(1 + \frac{0.08}{12}\right)
\]
\[
t = \frac{\ln 2}{12 \ln \left(1 + \frac{0.08}{12}\right)} \approx 8.69
\]
It will take about 8.69 years to double.

b. \(2P = Pe^{0.08t}\)
\[
2 = e^{0.08t}
\]
\[
\ln 2 = 0.08t
\]
\[
t = \frac{\ln 2}{0.08} \approx 8.66
\]
It will take about 8.66 years to double.

36. a. \(3P = P \left(1 + \frac{0.06}{12}\right)^{12t}\)
\[
3 = \left(1 + \frac{0.06}{12}\right)^{12t}
\]
\[
\ln 3 = \ln \left(1 + \frac{0.06}{12}\right)^{12t}
\]
\[
\ln 3 = 12t \ln \left(1 + \frac{0.06}{12}\right)
\]
\[
t = \frac{\ln 3}{12 \ln \left(1 + \frac{0.06}{12}\right)} \approx 18.36
\]
It will take about 18.36 years to triple.

b. \(3P = Pe^{0.06t}\)
\[
3 = e^{0.06t}
\]
\[
\ln 3 = 0.06t
\]
\[
t = \frac{\ln 3}{0.06} \approx 18.31
\]
It will take about 18.31 years to triple.

37. Since the effective interest rate is 7%, we have:
\[
I = Prt
\]
\[
I = P \cdot 0.07 \cdot 1
\]
\[
I = 0.07P
\]
Thus, the amount in the account is
\[
A = P + 0.07P = 1.07P
\]
Let \( x \) be the required interest rate. Then,
\[
1.07 P = P \left(1 + \frac{r}{4}\right)^{4(1)}
\]
\[
1.07 = \left(1 + \frac{r}{4}\right)^{4}
\]
\[
\sqrt[4]{1.07} = 1 + \frac{r}{4}
\]
\[
\sqrt[4]{1.07} - 1 = \frac{r}{4}
\]
\[
r = 4 \left(\sqrt[4]{1.07} - 1\right) = 0.06823
\]
Thus, an interest rate of 6.823% compounded quarterly has an effective interest rate of 7%.

38. Since the effective interest rate is 6%, we have:
\[
I = Prt
\]
\[
I = P \cdot 0.06 \cdot 1
\]
\[
I = 0.06 P
\]
Thus, the amount in the account is
\[
A = P + 0.06 P = 1.06 P
\]
Let \( x \) be the required interest rate. Then,
\[
1.06 P = Pe^{r(1)}
\]
\[
1.06 = e^r
\]
\[
r = \ln(1.06) = 0.05827
\]
Thus, an interest rate of 5.827% compounded continuously has an effective interest rate of 6%.

39. \[
150 = 100 \left(1 + \frac{0.08}{12}\right)^{12t}
\]
\[
1.5 = (1.006667)^{12t}
\]
\[
\ln 1.5 = 12t \ln (1.006667)
\]
\[
t = \frac{\ln 1.5}{12 \ln (1.006667)} = 5.09
\]
Compounded monthly, it will take about 5.09 years (or 61.02 months).
\[
150 = 100e^{0.08t}
\]
\[
1.5 = e^{0.08t}
\]
\[
\ln 1.5 = 0.08t
\]
\[
t = \frac{\ln 1.5}{0.08} = 5.07
\]
Compounded continuously, it will take about 5.07 years (or 60.82 months).

40. \[
175 = 100 \left(1 + \frac{0.10}{12}\right)^{12t}
\]
\[
1.75 = (1.008333)^{12t}
\]
\[
\ln 1.75 = 12t \ln (1.008333)
\]
\[
t = \frac{\ln 1.75}{12 \ln (1.008333)} = 5.62
\]
Compounded monthly, it will take about 5.62 years (or 67.43 months).
\[
175 = 100e^{0.10t}
\]
\[
1.75 = e^{0.10t}
\]
\[
\ln 1.75 = 0.10t
\]
\[
t = \frac{\ln 1.75}{0.10} = 5.60
\]
Compounded continuously, it will take about 5.60 years (or 67.15 months).

41. \[
25,000 = 10,000e^{0.06t}
\]
\[
2.5 = e^{0.06t}
\]
\[
\ln 2.5 = 0.06t
\]
\[
t = \frac{\ln 2.5}{0.06} = 15.27
\]
It will take about 15.27 years (or 15 years, 3 months).

42. \[
80,000 = 25,000e^{0.07t}
\]
\[
3.2 = e^{0.07t}
\]
\[
\ln 3.2 = 0.07t
\]
\[
t = \frac{\ln 3.2}{0.07} = 16.62
\]
It will take about 16.62 years (or 16 years, 7 months).

43. \[
A = 90,000(1 + 0.03)^5 = $104,335
\]
The house will cost $104,335 in five years.

44. \[
A = 200(1 + 0.0125)^6 = $215.48
\]
Her bill will be $215.48 after 6 months.

45. \[
P = 15,000e^{(-0.05)(3)} = $12,910.62
\]
Jerome should ask for $12,910.62.

46. \[
P = 3,000 \left(1 + \frac{0.03}{12}\right)^{(-12)(0.5)} = $2955.39
\]
John should save $2955.39.
47. \( A = 15(1 + 0.15)^5 = 15(1.15)^5 = \$30.17 \) per share for a total of about \( \$3017 \).

48. \( 850,000 = 650,000(1 + r)^3 \)
\[
\frac{85}{65} = (1 + r)^3 \\
\sqrt[3]{\frac{85}{65}} = 1 + r \\
r = \sqrt[3]{1.3077} - 1 = 0.0935
\]
The annual return is approximately 9.35%.

49. \( 5.6\% \) compounded continuously:
\[
A = 1000e^{(0.056)(1)} = \$1057.60
\]
Jim will not have enough money to buy the computer.

50. \( 6.8\% \) compounded continuously for 3 months:
Amount on April 1:
\[
A = 1000e^{(0.068)(0.25)} = \$1017.15
\]
5.25% compounded monthly for 1 month:
Amount on May 1
\[
A = 1017.15 \left(1 + \frac{0.0525}{12}\right)^{1(1/12)} = \$1021.60
\]
The second bank offers the better deal.

51. Will: \( 9\% \) compounded semiannually:
\[
A = 2000 \left(1 + \frac{0.09}{2}\right)^{2(20)} = \$11,632.73
\]
Henry: \( 8.5\% \) compounded continuously:
\[
A = 2000e^{(0.085)(20)} = \$10,947.89
\]
Will has more money after 20 years.

52. Value of \( \$1000 \) compounded continuously at 10% for 3 years:
\[
A = 1000e^{(0.10)(3)} = \$1349.86
\]
April will have more money if she takes the \( \$1000 \) now and invests it.

53. a. Let \( x \) be the year, then the average annual cost \( C \) of a 4-year private college is by the function \( C(x) = 25,143(1.059)^{x-2008} \).

\[
C(2028) = 25,143(1.059)^{2028-2008} \\
= 25,143(1.059)^{20} \\
\approx 79,129
\]
In 2028, the average annual cost at a 4-year private college will be about \( \$79,129 \).

b. \( A = Pe^{rt} \)
\[
79,129 = Pe^{0.04(18)} \\
P = \frac{79,129}{e^{0.04(18)}} = \$38,516
\]
An investment of \( \$38,516 \) in 2010 would pay for the cost of college at a 4-year private college in 2028.

54. \( P = 100,000; \ t = 5 \)
a. Simple interest at 12% per annum:
\[
A = 100,000 + 100,000(0.12)(5) = \$160,000 \\
I = 160,000 - 100,000 = \$60,000
\]
b. 11.5% compounded monthly:
\[
A = 100,000 \left[1 + \frac{0.115}{12}\right]^{(12)(5)} = \$177,227 \\
I = 177,227 - 100,000 = \$77,227
\]
c. 11.25% compounded continuously:
\[
A = 100,000e^{(0.1125)(5)} = \$175,505 \\
I = 175,505 - 100,000 = \$75,505
\]
Thus, simple interest at 12% is the best option since it results in the least interest.

55. \( A = P\left(1 + \frac{r}{n}\right)^{nt} \)
\[
A = 787 \left(1 + \frac{0.013787}{2}\right)^{2(20)} = 787(1.0065)^{40} = 1019
\]
The government would have to pay back approximately \( \$1019 \) billion in 2029. The amount of interest would be \( 1019 - 787 = \$232 \) billion.

56. From 2008 to 2020 would be 12 years, so \( t = 12 \).
The federal debt (in millions) would be:
\[
F = 10000(1 + 0.078)^{12} = 10000(1.078)^{12}. \quad \text{For } t = 12: \quad F = 10000(1.078)^{12} = 24627.7233.
\]
The U.S. population (in millions) would be:
\[
P = 304(1 + 0.009)^{t} = 304(1.009)^{t}. \quad \text{For } t = 12: \quad P = 304(1.009)^{12} = 338.5069412.\]
Chapter 5: Exponential and Logarithmic Functions

The per capita debt in 2020 will be $246,277,723.33.  
\[ \frac{246,277,723.33}{338,506,9412} = 72.75 \text{,} \]

57.  \( P = 1000 \), \( r = 0.03 \), \( n = 2 \)  
\[ A = 1000(1 - 0.03)^2 = 940.90 \]

58.  \( P = 1000 \), \( r = 0.02 \), \( n = 3 \)  
\[ A = 1000(1 - 0.02)^3 = 941.19 \]

59.  \( P = 1000 \), \( A = 950 \), \( n = 2 \)  
950 = 1000(1 - \( r \))^2  
\[ 0.95 = (1 - \( r \))^2 \]  
\[ \pm \sqrt{0.95} = 1 - r \]  
\[ r = 1 \pm \sqrt{0.95} \]  
\[ r = 0.0253 \text{ or } r = 1.9747 \]  
Disregard \( r = 1.9747 \). The inflation rate was 2.53%.

60.  \( P = 1000 \), \( A = 930 \), \( n = 2 \)  
930 = 1000(1 - \( r \))^2  
\[ 0.93 = (1 - \( r \))^2 \]  
\[ \pm \sqrt{0.93} = 1 - r \]  
\[ r = 1 \pm \sqrt{0.93} \]  
\[ r = 0.0356 \text{ or } r = 1.9644 \]  
Disregard \( r = 1.9644 \). The inflation rate was 3.56%.

61.  \( r = 0.02 \)  
\[ \frac{1}{2} P = P(1 - 0.02)^t \]  
\[ 0.5P = P(0.98)^t \]  
\[ 0.5 = (0.98)^t \]  
\[ t = \log_{0.98}(0.5) \]  
\[ t = \frac{\ln 0.5}{\ln 0.98} = 34.31 \]  
The purchasing power will be half in 34.31 years.

62.  \( r = 0.04 \)  
\[ \frac{1}{2} P = P(1 - 0.04)^t \]  
\[ 0.5P = P(0.96)^t \]  
\[ 0.5 = (0.96)^t \]  
\[ t = \log_{0.96}(0.5) \]  
\[ t = \frac{\ln 0.5}{\ln 0.96} = 16.98 \]  
The purchasing power will be half in 16.98 years.

63. a.  \( A = 10,000 \), \( r = 0.10 \), \( n = 12 \), \( t = 20 \)  
\[ P = 10,000 \left(1 + \frac{0.10}{12}\right)^{12 \times 20} = 1364.62 \]

b.  \( A = 10,000 \), \( r = 0.01 \), \( t = 20 \)  
\[ P = 10,000e^{(-0.01 \times 20)} = 1353.35 \]

64.  \( A = 40,000 \), \( r = 0.08 \), \( n = 1 \), \( t = 17 \)  
\[ P = 40,000 \left(1 + \frac{0.08}{1}\right)^{17} = 10,810.76 \]

65.  \( A = 10,000 \), \( r = 0.08 \), \( n = 1 \), \( t = 10 \)  
\[ P = 10,000 \left(1 + \frac{0.08}{1}\right)^{-10} = 4631.93 \]

66.  \( A = 25,000 \), \( P = 12,485.52 \), \( n = 1 \), \( t = 8 \)  
\[ 25,000 = 12,485.52 (1 + r^8) \]  
\[ r = \left(\frac{25,000}{12,485.52}\right)^{\frac{1}{8}} - 1 \]  
\[ r = 0.090665741 \]  
The annual rate of return is about 9.07%.

67. a.  \( t = \frac{\ln 2}{\ln \left(1 + \frac{0.12}{1}\right)} \)  
\[ = \frac{\ln 2}{\ln (1.12)} = 6.12 \text{ years} \]
Section 5.7: Financial Models

b. \[ t = \frac{\ln 3}{4 \cdot \ln \left(1 + \frac{0.064}{4}\right)} = \frac{\ln 3}{4 \ln (1.015)} = 18.45 \text{ years} \]
c. \[ mP = P \left(1 + \frac{r}{n}\right)^{nt} \]
\[ m = \left(1 + \frac{r}{n}\right)^{nt} \]
\[ \ln m = nt \cdot \ln \left(1 + \frac{r}{n}\right) \]
\[ t = \frac{\ln m}{n \cdot \ln \left(1 + \frac{r}{n}\right)} \]

68. a. \[ t = \frac{\ln 8000 - \ln 1000}{0.10} = 20.79 \text{ years} \]
b. \[ 35 = \frac{\ln 30,000 - \ln 2000}{r} \]
\[ r = \frac{\ln 30,000 - \ln 2000}{35} = 0.0774 \]
\[ r = 7.74\% \]
c. \[ A = Pe^{rt} \]
\[ \frac{A}{P} = e^{rt} \]
\[ \ln \left(\frac{A}{P}\right) = rt \]
\[ \ln A - \ln P = rt \]
\[ t = \frac{\ln A - \ln P}{r} \]

69.a. \[ CPI_0 = 163.0, CPI = 215.3, n = 2008 - 1998 = 10 \]
\[ 215.3 = 163.0 \left(1 + \frac{r}{100}\right)^{10} \]
\[ r = \left(\frac{215.3}{163.0}\right)^{1/10} - 1 \]
\[ \approx 7.74\% \]
b. \[ CPI_0 = 163.0, CPI = 300, r = 2.82 \]
\[ 300 = 215.3 \left(1 + \frac{2.82}{100}\right)^{n} \]
\[ n = \frac{\ln \left(\frac{300}{215.3}\right)}{\ln \left(1 + \frac{2.82}{100}\right)} \]
\[ \approx 12.0 \text{ years} \]
The CPI will reach 300 about 12 years after 2008, or in the year 2020.

70. \[ CPI_0 = 234.2, r = 2.8\%, n = 5 \]
\[ CPI = 234.2 \left(1 + \frac{2.8}{100}\right)^{5} = 268.9 \]
In 5 years, the CPI index will be about 268.9.

71. \[ r = 3.1\% \]
\[ 2 \cdot CPI_0 = CPI_0 \left(1 + \frac{3.1}{100}\right)^{n} \]
\[ 2 = (1.031)^{n} \]
\[ n = \log_{1.031} 2 = \frac{\ln 2}{\ln 1.031} \approx 22.7 \]
It will take about 22.7 years for the CPI index to double.
72. \(CPI_0 = 100, \ CPI = 456.5, \ r = 5.57\)
\[
456.5 = 100 \left(1 + \frac{5.57}{100}\right)^n
\]
\[
456.5 = 100 (1.0557)^n
\]
\[
4.565 = (1.0557)^n
\]
\[
n = \log_{1.0558}(4.565)
\]
\[
= \frac{\ln 4.565}{\ln 1.0558} = 28.0 \text{ years}
\]
The years that was used as the base period for the CPI was about 28 years before 1995, or the year 1967.

73. Answers will vary.

74. Answers will vary.

75. Answers will vary.

### Section 5.8

1. \(P(t) = 500e^{0.02t}\)
   a. \(P(0) = 500e^{(0.02)(0)} = 500\) insects
   b. growth rate: \(k = 0.02 = 2\%\)
   c. \(P(10) = 500e^{(0.02)(10)} = 611\) insects
   d. Find \(t\) when \(P = 800:\)
      \[
      800 = 500e^{0.02t}
      \]
      \[
      1.6 = e^{0.02t}
      \]
      \[
      \ln 1.6 = 0.02t
      \]
      \[
      t = \frac{\ln 1.6}{0.02} = 23.5 \text{ days}
      \]
   e. Find \(t\) when \(P = 1000:\)
      \[
      1000 = 500e^{0.02t}
      \]
      \[
      2 = e^{0.02t}
      \]
      \[
      \ln 2 = 0.02t
      \]
      \[
      t = \frac{\ln 2}{0.02} = 34.7 \text{ days}
      \]

2. \(N(t) = 1000e^{0.01t}\)
   a. \(N(0) = 1000e^{(0.01)(0)} = 1000\) bacteria
   b. growth rate: \(k = 0.01 = 1\%\)
   c. \(N(4) = 1000e^{(0.01)(4)} = 1041\) bacteria
   d. Find \(t\) when \(N = 1700:\)
      \[
      1700 = 1000e^{0.011t}
      \]
      \[
      1.7 = e^{0.011t}
      \]
      \[
      \ln 1.7 = 0.011t
      \]
      \[
      t = \frac{\ln 1.7}{0.011} = 53.1 \text{ hours}
      \]
   e. Find \(t\) when \(N = 2000:\)
      \[
      2000 = 1000e^{0.011t}
      \]
      \[
      2 = e^{0.011t}
      \]
      \[
      \ln 2 = 0.011t
      \]
      \[
      t = \frac{\ln 2}{0.011} = 69.3 \text{ hours}
      \]

3. \(A(t) = A_0e^{-0.0244t} = 500e^{-0.0244t}\)
   a. decay rate: \(k = -0.0244 = -2.44\%\)
   b. \(A(10) = 500e^{(-0.0244)(10)} = 391.7\) grams
   c. Find \(t\) when \(A = 400:\)
      \[
      400 = 500e^{-0.0244t}
      \]
      \[
      0.8 = e^{-0.0244t}
      \]
      \[
      \ln 0.8 = -0.0244t
      \]
      \[
      t = \frac{\ln 0.8}{-0.0244} = 9.1 \text{ years}
      \]
   d. Find \(t\) when \(A = 250:\)
      \[
      250 = 500e^{-0.0244t}
      \]
      \[
      0.5 = e^{-0.0244t}
      \]
      \[
      \ln 0.5 = -0.0244t
      \]
      \[
      t = \frac{\ln 0.5}{-0.0244} = 28.4 \text{ years}
      \]

4. \(A(t) = A_0e^{-0.087t} = 100e^{-0.087t}\)
   a. decay rate: \(k = -0.087 = -8.7\%\)
   b. \(A(9) = 100e^{(-0.087)(9)} = 45.7\) grams
   c. Find \(t\) when \(A = 70:\)
      \[
      70 = 100e^{-0.087t}
      \]
      \[
      0.7 = e^{-0.087t}
      \]
      \[
      \ln 0.7 = -0.087t
      \]
      \[
      t = \frac{\ln 0.7}{-0.087} = 4.1 \text{ days}
      \]
Section 5.8: Exponential Growth and Decay; Newton’s Law; Logistic Growth and Decay Models

d. Find \( t \) when \( A = 50 \):
\[
50 = 100e^{-0.087t} \\
0.5 = e^{-0.087t} \\
\ln 0.5 = -0.087t \\
t = \frac{\ln 0.5}{-0.087} = 7.97 \text{ days}
\]

5. a. \( N(t) = N_0e^{kt} \)
b. If \( N(t) = 1800, N_0 = 1000, \) and \( t = 1, \) then
\[
1800 = 1000e^{k(1)} \\
1.8 = e^k \\
k = \ln 1.8 \\
\text{If } t = 3, \text{ then } N(3) = 1000e^{(\ln 1.8)(3)} = 5832 \text{ mosquitoes.}
\]
c. Find \( t \) when \( N(t) = 10,000 : \)
\[
10,000 = 1000e^{(\ln 1.8)t} \\
10 = e^{(\ln 1.8)t} \\
\ln 10 = (\ln 1.8)t \\
t = \frac{\ln 10}{\ln 1.8} = 3.9 \text{ days}
\]

6. a. \( N(t) = N_0e^{kt} \)
b. If \( N(t) = 800, N_0 = 500, \) and \( t = 1, \) then
\[
800 = 500e^{k(1)} \\
1.6 = e^k \\
k = \ln 1.6 \\
\text{If } t = 5, \text{ then } N(5) = 500e^{(\ln 1.6)(5)} = 5243 \text{ bacteria}
\]
c. Find \( t \) when \( N(t) = 20,000 : \)
\[
20,000 = 500e^{(\ln 1.6)t} \\
40 = e^{(\ln 1.6)t} \\
\ln 40 = (\ln 1.6)t \\
t = \frac{\ln 40}{\ln 1.6} = 7.85 \text{ hours}
\]

7. a. \( N(t) = N_0e^{kt} \)

b. Note that 18 months = 1.5 years, so \( t = 1.5. \)
\[
2N_0 = N_0e^{k(1.5)} \\
2 = e^{1.5k} \\
\ln 2 = 1.5k \\
k = \frac{\ln 2}{1.5} \\
\text{If } N_0 = 10,000 \text{ and } t = 2, \text{ then}
\]
\[
P(2) = 10,000e^{(\frac{\ln 2}{1.5})(2)} = 25,198 \\
The population 2 years from now will be 25,198.
\]

8. a. \( N(t) = N_0e^{kt}, k < 0 \)
b. If \( N(t) = 800,000, N_0 = 900,000, \) and \( t = 2005 - 2003 = 2, \) then
\[
800,000 = 900,000e^{k(2)} \\
\frac{8}{9} = e^{2k} \\
\ln \left(\frac{8}{9}\right) = 2k \\
k = \frac{\ln \left(\frac{8}{9}\right)}{2} \\
\text{If } t = 2007 - 2003 = 4, \text{ then}
\]
\[
P(4) = 900,000e^{(\frac{\ln \left(\frac{8}{9}\right)}{2})(4)} = 711,111 \\
The population in 2007 will be 711,111.
\]

9. Use \( A = A_0e^{kt} \) and solve for \( k : \)
\[
0.5A_0 = A_0e^{k(1690)} \\
0.5 = e^{1690k} \\
\ln 0.5 = 1690k \\
k = \frac{\ln 0.5}{1690} \\
\text{When } A_0 = 10 \text{ and } t = 50 : 
\]
\[
A = 10e^{\left(\frac{\ln 0.5}{1690}\right)(50)} = 9.797 \text{ grams}
\]

10. Use \( A = A_0e^{kt} \) and solve for \( k : \)
\[
0.5A_0 = A_0e^{k(1.3 \times 10^9)} \\
0.5 = e^{(1.3 \times 10^9)k} \\
\ln 0.5 = 1.3 \times 10^9k \\
k = \frac{\ln 0.5}{1.3 \times 10^9}
\]
When \( A_0 = 10 \) and \( t = 100 \):
\[
A = 10e^{\left( \frac{\ln 0.5}{1.3 \times 10^9} \right)(100)} = 9.999999467 \text{ grams}
\]

When \( A_0 = 10 \) and \( t = 1000 \):
\[
A = 10e^{\left( \frac{\ln 0.5}{1.3 \times 10^9} \right)(1000)} = 9.999999468 \text{ grams}
\]

11. Use \( A = A_0e^{kt} \) and solve for \( k \):

half-life = 5600 years
\[
0.5A_0 = A_0e^{k(5600)}
\]
\[
0.5 = e^{5600k}
\]
\[
\ln 0.5 = 5600k
\]
\[
k = \frac{\ln 0.5}{5600}
\]

Solve for \( t \) when \( A = 0.3A_0 \):
\[
0.3A_0 = A_0e^{k(5600)}
\]
\[
0.3 = e^{\left( \frac{\ln 0.5}{5600} \right) t}
\]
\[
\ln 0.3 = \left( \frac{\ln 0.5}{5600} \right) t
\]
\[
t = \frac{5600}{\ln 0.5} (\ln 0.3) = 9727
\]
The tree died approximately 9727 years ago.

12. Use \( A = A_0e^{kt} \) and solve for \( k \):

half-life = 5600 years
\[
0.5A_0 = A_0e^{k(5600)}
\]
\[
0.5 = e^{5600k}
\]
\[
\ln 0.5 = 5600k
\]
\[
k = \frac{\ln 0.5}{5600}
\]

Solve for \( t \) when \( A = 0.7A_0 \):
\[
0.7A_0 = A_0e^{k(5600)}
\]
\[
0.7 = e^{\left( \frac{\ln 0.5}{5600} \right) t}
\]
\[
\ln 0.7 = \left( \frac{\ln 0.5}{5600} \right) t
\]
\[
t = \frac{5600}{\ln 0.5} (\ln 0.7) = 2882
\]
The fossil is about 2882 years old.

13. a. Using \( u = T + (u_0 - T)e^{kt} \) with \( t = 5 \),
\[
T = 70, \ u_0 = 450, \text{ and } u = 300:
300 = 70 + (450 - 70)e^{k(5)}
230 = 380e^{5k}
\]
\[
\ln \left( \frac{23}{38} \right) = 5k
\]
\[
k = \frac{\ln \left( \frac{23}{38} \right)}{5} = -0.1004
\]
\[
T = 70, \ u_0 = 450, \ u = 135:
135 = 70 + (450 - 70)e^{\left( \frac{\ln \left( \frac{23}{38} \right)}{5} \right)t}
65 = 380e^{\left( \frac{\ln \left( \frac{23}{38} \right)}{5} \right)t}
\]
\[
\ln \left( \frac{65}{380} \right) = \ln \left( \frac{23}{38} \right) \cdot \frac{5}{t}
\]
\[
t = \frac{5}{\ln \left( \frac{23}{38} \right)} \cdot \ln \left( \frac{65}{380} \right) = 18 \text{ minutes}
\]
The temperature of the pan will be 135˚F at about 5:18 PM.

b. \( T = 70, \ u_0 = 450, \ u = 160:\
160 = 70 + (450 - 70)e^{\left( \frac{\ln \left( \frac{23}{38} \right)}{5} \right)t}
90 = 380e^{\left( \frac{\ln \left( \frac{23}{38} \right)}{5} \right)t}
\]
\[
\ln \left( \frac{90}{380} \right) = \ln \left( \frac{23}{38} \right) \cdot \frac{5}{t}
\]
\[
t = \frac{5}{\ln \left( \frac{23}{38} \right)} \cdot \ln \left( \frac{90}{380} \right) = 14.3 \text{ minutes}
\]
The pan will be 160˚F after about 14.3 minutes.

c. As time passes, the temperature of the pan approaches 70˚F.
14. a. Using $u = T + (u_0 - T)e^{kt}$ with $t = 2$, $T = 38$, $u_0 = 72$, and $u = 60$:

\[ 60 = 38 + (72 - 38)e^{k(2)} \]
\[ 22 = 34e^{2k} \]
\[ 22 \over 34 = e^{2k} \]
\[ \ln \left( \frac{22}{34} \right) = 2k \]
\[ k = \frac{\ln \left( \frac{22}{34} \right)}{2} \]

Then:
\[ T = 38, \quad u_0 = 72, \quad t = 7 \]
\[ u = 38 + (72 - 38)e^{\frac{\ln \left( \frac{22}{34} \right)}{2}} \]
\[ u = 38 + 34e^{\frac{\ln \left( \frac{22}{34} \right)}{2}} \]
\[ = 45.41^\circ F \]

After 7 minutes the thermometer will read about 45.41˚F.

b. Find $t$ when $u = 39^\circ F$

\[ 39 = 38 + (72 - 38)e^{\frac{\ln \left( \frac{22}{34} \right)}{2}} \]

\[ 1 = 34e^{\frac{\ln \left( \frac{22}{34} \right)}{2}} \]
\[ \ln \left( \frac{1}{34} \right) = \frac{\ln \left( \frac{22}{34} \right)}{2} \]
\[ t = \frac{2}{\ln \left( \frac{22}{34} \right)} \cdot \ln \left( \frac{1}{34} \right) \approx 16.2 \]

The thermometer will read 39 degrees after about 16.2 minutes.

c. $T = 38, \ u_0 = 72, \ u = 45$:

\[ 45 = 38 + (72 - 38)e^{\frac{\ln \left( \frac{22}{34} \right)}{2}} \]
\[ 7 = 34e^{\frac{\ln \left( \frac{22}{34} \right)}{2}} \]
\[ \frac{7}{34} = e^{\frac{\ln \left( \frac{22}{34} \right)}{2}} \]

\[ \ln \left( \frac{7}{34} \right) = \frac{\ln \left( \frac{22}{34} \right)}{2} \]
\[ t = \frac{2}{\ln \left( \frac{22}{34} \right)} \cdot \ln \left( \frac{7}{34} \right) \approx 7.26 \text{ minutes} \]

The thermometer will read 45°F after about 7.26 minutes.

d. As time passes, the temperature gets closer to 38°F.

15. Using $u = T + (u_0 - T)e^{kt}$ with $t = 3$, $T = 35$, $u_0 = 8$, and $u = 15$:

\[ 15 = 35 + (8 - 35)e^{\frac{\ln \left( \frac{20}{27} \right)}{3}} \]
\[ -20 = -27e^{\frac{\ln \left( \frac{20}{27} \right)}{3}} \]
\[ 20 \over 27 = e^{\frac{\ln \left( \frac{20}{27} \right)}{3}} \]
\[ \ln \left( \frac{20}{27} \right) = 3k \]
\[ k = \frac{\ln \left( \frac{20}{27} \right)}{3} \]

At $t = 5$:

\[ u = 35 + (8 - 35)e^{\frac{\ln \left( \frac{20}{27} \right)}{3}} \]
\[ = 18.63^\circ C \]

After 5 minutes, the thermometer will read approximately 18.63°C.

At $t = 10$:

\[ u = 35 + (8 - 35)e^{\frac{\ln \left( \frac{20}{27} \right)}{3}} \]
\[ = 25.1^\circ C \]

After 10 minutes, the thermometer will read approximately 25.1°C

16. Using $u = T + (u_0 - T)e^{kt}$ with $t = 10$, $T = 70$, $u_0 = 28$, and $u = 35$:

\[ 35 = 70 + (28 - 70)e^{10k} \]
\[ -35 = -42e^{10k} \]
\[ 35 \over 42 = e^{10k} \]
\[ \ln \left( \frac{35}{42} \right) = 10k \]
\[ k = \frac{\ln \left( \frac{35}{42} \right)}{10} \]

At $t = 30$:

\[ u = 70 + (28 - 70)e^{\frac{\ln \left( \frac{35}{42} \right)}{10}} \]
\[ = 45.69^\circ F \]
Chapter 5: Exponential and Logarithmic Functions

After 30 minutes, the temperature of the stein will be approximately 45.69°F.

Find the value of \( t \) so that the \( u = 45°F \):

\[
45 = 70 + (28 - 70)e^{\frac{\ln(35/42)}{10}}t
\]

\[
-25 = -42e^{\frac{\ln(35/42)}{10}}t
\]

\[
\frac{25}{42} = e^{\frac{\ln(35/42)}{10}}t
\]

\[
\ln \left( \frac{25}{42} \right) = \left( \frac{\ln (35/42)}{10} \right) t
\]

\[
t = \frac{10}{\ln (35/42)} \cdot \ln \left( \frac{25}{42} \right) = 28.46
\]

The temperature of the stein will be 45°F after about 28.46 minutes.

17. Use \( A = A_0e^{kt} \) and solve for \( k \):

\[
2.2 = 2.5e^{k(24)}
\]

\[
0.88 = e^{24k}
\]

\[
\ln 0.88 = 24k
\]

\[
k = \frac{\ln 0.88}{24}
\]

When \( A_0 = 2.5 \) and \( t = 72 \):

\[
A = 2.5e^{\left( \frac{\ln 0.88}{24} \right)(72)} = 1.70
\]

After 3 days (72 hours), the amount of free chlorine will be 1.70 parts per million.

Find \( t \) when \( A = 1 \):

\[
1 = 2.5e^{\left( \frac{\ln 0.88}{24} \right)t}
\]

\[
0.4 = e^{\left( \frac{\ln 0.88}{24} \right)t}
\]

\[
\ln 0.4 = \left( \frac{\ln 0.88}{24} \right)t
\]

\[
t = \frac{24}{\ln 0.88} \cdot \ln 0.4 = 172
\]

Ben will have to shock his pool again after 172 hours (or 7.17 days) when the level of free chlorine reaches 1.0 parts per million.

18. Use \( A = A_0e^{kt} \) and solve for \( k \):

\[
0.15 = 0.25e^{k(17)}
\]

\[
0.6 = e^{17k}
\]

\[
\ln 0.6 = 17k
\]

\[
k = \frac{\ln 0.6}{17}
\]

When \( A_0 = 0.25 \) and \( t = 30 \):

\[
A = 0.25e^{\left( \frac{\ln 0.6}{17} \right)(30)} = 0.10
\]

After 30 minutes, approximately 0.10 M of dinitrogen pentoxide will remain.

Find \( t \) when \( A = 0.01 \):

\[
0.01 = 0.25e^{\left( \frac{\ln 0.6}{17} \right)t}
\]

\[
0.04 = e^{\left( \frac{\ln 0.6}{17} \right)t}
\]

\[
\ln 0.04 = \left( \frac{\ln 0.6}{17} \right)t
\]

\[
t = \frac{17}{\ln 0.6} \cdot \ln 0.04 = 107
\]

It will take approximately 107 minutes until 0.01 M of dinitrogen pentoxide remains.

19. Use \( A = A_0e^{kt} \) and solve for \( k \):

\[
0.36 = 0.40e^{k(30)}
\]

\[
0.9 = e^{30k}
\]

\[
\ln 0.9 = 30k
\]

\[
k = \frac{\ln 0.9}{30}
\]

Note that 2 hours = 120 minutes.

When \( A_0 = 0.40 \) and \( t = 120 \):

\[
A = 0.40e^{\left( \frac{\ln 0.9}{30} \right)(120)} = 0.26
\]

After 2 hours, approximately 0.26 M of sucrose will remain.

Find \( t \) when \( A = 0.10 \):

\[
0.10 = 0.40e^{\left( \frac{\ln 0.9}{30} \right)t}
\]

\[
0.25 = e^{\left( \frac{\ln 0.9}{30} \right)t}
\]

\[
\ln 0.25 = \left( \frac{\ln 0.9}{30} \right)t
\]

\[
t = \frac{30}{\ln 0.9} \cdot \ln 0.25 = 395
\]

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Section 5.8: Exponential Growth and Decay; Newton’s Law; Logistic Growth and Decay Models

It will take approximately 395 minutes (or 6.58 hours) until 0.10 M of sucrose remains.

20. Use \( A = A_0e^{kt} \) and solve for \( k \):

\[
\begin{align*}
15 &= 25e^{k(10)} \\
0.6 &= e^{10k} \\
\ln 0.6 &= 10k \\
k &= \frac{\ln 0.6}{10}
\end{align*}
\]

When \( A_0 = 25 \) and \( t = 24 \):

\[
A = 25e^{\left[\frac{\ln 0.6}{10}\right](24)} = 7.34
\]

There will be about 7.34 kilograms of salt left after 1 day.

Find \( t \) when \( A = 0.5A_0 \):

\[
\begin{align*}
0.5 &= 25e^{\left[\frac{\ln 0.6}{10}\right]t} \\
0.02 &= e^{\left[\frac{\ln 0.6}{10}\right]t} \\
\ln 0.02 &= \left[\frac{\ln 0.6}{10}\right]t \\
t &= \frac{10}{\ln 0.6} \cdot \ln 0.02 = 76.6
\end{align*}
\]

It will take about 76.6 hours until \( \frac{1}{2} \) kilogram of salt is left.

21. Use \( A = A_0e^{kt} \) and solve for \( k \):

\[
\begin{align*}
0.5A_0 &= A_0e^{k(8)} \\
0.5 &= e^{8k} \\
\ln 0.5 &= 8k \\
k &= \frac{\ln 0.5}{8}
\end{align*}
\]

Find \( t \) when \( A = 0.1A_0 \):

\[
\begin{align*}
0.1A_0 &= A_0e^{\left[\frac{\ln 0.5}{8}\right]t} \\
0.1 &= e^{\left[\frac{\ln 0.5}{8}\right]t} \\
\ln 0.1 &= \left[\frac{\ln 0.5}{8}\right]t \\
t &= \frac{8}{\ln 0.5} \cdot \ln 0.1 = 26.6
\end{align*}
\]

The farmers need to wait about 26.6 days before using the hay.

22. Using \( u = T + (u_0 - T)e^{kt} \) with \( t = 2 \), \( T = 325 \), \( u_0 = 75 \), and \( u = 100 \):

\[
\begin{align*}
100 &= 325 + (75 - 325)e^{k(2)} \\
-225 &= -250e^{2k} \\
0.9 &= e^{2k} \\
2k &= \ln 0.9 \\
k &= \frac{\ln 0.9}{2}
\end{align*}
\]

Find the value of \( t \) so that \( u = 175^\circ F \):

\[
\begin{align*}
175 &= 325 + (75 - 325)e^{\left(\frac{\ln 0.9}{2}\right)} \\
-150 &= -250e^{\left(\frac{\ln 0.9}{2}\right)} \\
0.6 &= e^{\left(\frac{\ln 0.9}{2}\right)} \\
\ln 0.6 &= \left(\frac{\ln 0.9}{2}\right)t \\
t &= \frac{2}{\ln 0.9} \cdot \ln 0.6 = 9.7
\end{align*}
\]

The hotel may serve their guests about 9.7 hours after noon or at about 9:42 PM.

23. a. As \( t \to \infty \), \( e^{-0.439t} \to 0 \). Thus, \( P(t) \to 1000 \). The carrying capacity is 1000 grams of bacteria.

b. Growth rate = 0.439 = 43.9%.

c. \( P(0) = \frac{1000}{1 + 32.33e^{-0.439(0)}} = \frac{1000}{33.33} = 30 \)

The initial population was 30 grams of bacteria.

d. \( P(9) = \frac{1000}{1 + 32.33e^{-0.439(9)}} = 616.6 \)

After 9 hours, the population of bacteria will be about 616.8 grams.

e. We need to find \( t \) such that \( P = 700 \)
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\[ \frac{700}{1 + 32.33e^{-0.439t}} = 1000 \]

\[ 700 \left( 1 + 32.33e^{-0.439t} \right) = 1000 \]

\[ 1 + 32.33e^{-0.439t} = \frac{10}{7} \]

\[ 32.33e^{-0.439t} = \frac{10}{7} - 1 \]

\[ e^{-0.439t} = \frac{3/7}{32.33} \]

\[ -0.439t = \ln \left( \frac{3/7}{32.33} \right) \]

\[ t = 9.85 \]

Thus, \( t \approx 9.85 \). The population of bacteria will be 700 grams after about 9.85 hours.

f. We need to find \( t \) such that

\[ P = \frac{1}{2} (1000) = 500: \]

\[ 500 = \frac{1000}{1 + 32.33e^{-0.439t}} \]

\[ 500 \left( 1 + 32.33e^{-0.439t} \right) = 1000 \]

\[ 1 + 32.33e^{-0.439t} = \frac{10}{5} \]

\[ 32.33e^{-0.439t} = 2 - 1 \]

\[ e^{-0.439t} = \frac{1}{32.33} \]

\[ -0.439t = \ln \left( \frac{1}{32.33} \right) \]

\[ t = 7.9 \]

Thus, \( t \approx 7.9 \). The population of bacteria will reach one-half of its carrying capacity after about 7.9 hours.

24. a. As \( t \to \infty \), \( e^{-0.162t} \to 0 \). Thus, \( P(t) \to 500 \). The carrying capacity is 500 bald eagles.

b. Growth rate = 0.162 = 16.2%.

c. \( P(3) = \frac{500}{1 + 82.33e^{-0.162(3)}} = 9.68 \)

After 3 years, the population is almost 10 bald eagles.

d. We need to find \( t \) such that \( P = 300: \)

\[ 300 = \frac{500}{1 + 82.33e^{-0.162t}} \]

\[ 300 \left( 1 + 82.33e^{-0.162t} \right) = 500 \]

\[ 1 + 82.33e^{-0.162t} = \frac{5}{3} \]

\[ 82.33e^{-0.162t} = \frac{2}{3} - 1 \]

\[ e^{-0.162t} = \frac{2}{82.33} \]

\[ -0.162t = \ln \left( \frac{2}{82.33} \right) \]

\[ t = 29.7 \]

Thus, \( t \approx 29.7 \). The bald eagle population will be 300 in approximately 29.7 years.

e. We need to find \( t \) such that

\[ P = \frac{1}{2} (500) = 250: \]

\[ 250 = \frac{500}{1 + 82.33e^{-0.162t}} \]

\[ 250 \left( 1 + 82.33e^{-0.162t} \right) = 500 \]

\[ 1 + 82.33e^{-0.162t} = 2 \]

\[ 82.33e^{-0.162t} = 2 - 1 \]

\[ e^{-0.162t} = \frac{1}{82.33} \]

\[ -0.162t = \ln \left( \frac{1}{82.33} \right) \]

\[ t = 27.2 \]

Thus, \( t \approx 27.2 \). The bald eagle population will reach one-half of its carrying capacity after about 27.2 years.

25. a. \( y = \frac{6}{1 + e^{-(5.085-0.1156(1000))}} = 0.00923 \)

At 100°F, the predicted number of eroded or leaky primary O-rings will be about 0.

b. \( y = \frac{6}{1 + e^{-(5.085-0.1156(60))}} = 0.81 \)

At 60°F, the predicted number of eroded or leaky primary O-rings will be about 1.

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Section 5.9: Building Exponential, Logarithmic, and Logistic Models from Data

1. a. 

b. Using EXPonential REGression on the data yields: \( y = 0.0903(1.3384)^x \)

c. \( y = 0.0903(1.3384)^x = 0.0903\left(e^{\ln(1.3384)}\right)^x = 0.0903e^{\ln(1.3384)x} \)

\( N(t) = 0.0903e^{0.2915t} \)

2. a. 

b. Using EXPonential REGression on the data yields: \( y = 0.0339(1.9474)^x \)
c. \( y = 0.0339(1.9474)^x \)
   \[= 0.0339 \left(e^{\ln(1.9474)}\right)^x\]
   \[= 0.0339e^{\ln(1.9474)x}\]
   \[N(t) = 0.0339e^{(0.6665)t}\]

d. \( Y_i = 0.0339e^{(0.6665)x}\)

e. \( N(6) = 0.0339e^{(0.6665)6} = 1.85 \) bacteria

f. We need to find \( t \) when \( N = 2.1\):
   \[0.0339e^{(0.6665)t} = 2.1\]
   \[e^{(0.6665)t} = \frac{2.1}{0.0339}\]
   \[0.6665t = \ln \left(\frac{2.1}{0.0339}\right)\]
   \[t = \frac{\ln \left(\frac{2.1}{0.0339}\right)}{0.6665} = 6.19 \text{ hours}\]

4. a. Let \( x = 0 \) correspond to 1995, \( x = 3 \) correspond to 1998, etc.

b. Using EXPonential REGression on the data yields: \( y = 228.4370(0.92301)^x\)

c. \( y = 100.3263 (0.8769)^x\)
   \[= 100.3263 \left(e^{\ln(0.8769)}\right)^x\]
   \[= 100.3263e^{\ln(0.8769)x}\]
   \[A(t) = 100.3263e^{(-0.1314)t}\]

d. \( Y_i = 100.3263e^{(-0.1314)x}\)

e. We need to find \( t \) when \( A(t) = 0.5 \cdot A_0\)
   \[100.3263e^{(-0.1314)t} = (0.5)(100.3263)\]
   \[e^{(-0.1314)t} = 0.5\]
   \[-0.1314t = \ln 0.5\]
   \[t = \frac{\ln 0.5}{-0.1314} = 5.3 \text{ weeks}\]

f. \( A(50) = 100.3263e^{(-0.1314)(50)} = 0.14 \) grams

g. We need to find \( t \) when \( A(t) = 20 \).
   \[100.3263e^{(-0.1314)t} = 20\]
   \[e^{(-0.1314)t} = \frac{20}{100.3263}\]
   \[-0.1314t = \ln \left(\frac{20}{100.3263}\right)\]
   \[t = \frac{\ln \left(\frac{20}{100.3263}\right)}{-0.1314} = 12.3 \text{ weeks}\]

b. Using EXPonential REGression on the data yields: \( y = 228.4370(0.92301)^x\)

c. \( y = 228.4370 \left(e^{(0.92301)x}\right)^x\)
   \[= 228.4370 \left(e^{\ln(0.92301)x}\right)^x\]
   \[A(t) = 228.4370e^{(-0.08012)t}\]

d. \( Y_i = 228.4370e^{(-0.08012)x}\)

e. Note that 2010 is represented by \( t = 15 \).
   \[A(15) = 228.4370e^{(-0.08012)(15)} \approx 68.7 \text{ billion cigarettes}\]

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f. We need to find \( t \) when \( A(t) = 50 \).
\[
228.437e^{-0.08012t} = 50
\]
\[
e^{-0.08012t} = \frac{50}{228.437}
\]
\[
-0.08012t = \ln\left(\frac{50}{228.437}\right)
\]
\[
t = \frac{-\ln \left(\frac{50}{228.437}\right)}{-0.08012} = 19 \text{ years}
\]
Now 1995 + 19 = 2014. The number of cigarettes exported from the U.S. will decrease to 50 billion in the year 2014.

5. a. 

b. Using LnREGression on the data yields:
\[ y = 32,741.02 - 6070.96\ln x \]

c. \[ Y_t = 32,741.02 - 6070.96\ln x \]

d. We need to find \( x \) when \( y = 1650 \):
\[ 1650 = 32,741.02 - 6070.96\ln x \]
\[ -31,091.02 = -6070.96\ln x \]
\[ -31,091.02 = \ln x \]
\[ 5.1213 = \ln x \]
\[ e^{5.1213} = x \]
\[ x = 168 \]
If the price were $1650, then approximately 168 computers would be demanded.

6. a. 

b. Using LOGISTIC REGression on the data yields: \[ y = \frac{799,475,916.5}{1 + 9.1968e^{-0.0160x}} \]

c. \[ Y_t = \frac{799,475,916.5}{1 + 9.1968e^{-0.0160x}} \]

d. As \( x \to \infty \), \( 9.1968e^{-0.0160x} \to 0 \), which means \( 1 + 9.1968e^{-0.0160x} \to 1 \), so
\[ y = \frac{799,475,916.5}{1 + 9.1968e^{-0.0160x}} = 799,475,916.5 \]
Therefore, the carrying capacity of the
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United States is approximately 799,475,917 people.

e. The year 2004 corresponds to \( x = 104 \), so
\[
y = \frac{799,475,916.5}{1 + 9.1968e^{-0.0160(104)}}
\]
\[
= 291,599,733 \text{ people}
\]

f. Find \( x \) when \( y = 300,000,000 \)
\[
799,475,916.5 = 300,000,000 \left( 1 + 9.1968e^{-0.0160x} \right)
\]
\[
\frac{799,475,916.5}{300,000,000} - 1 = 9.1968e^{-0.0160x}
\]
\[
\ln \left( \frac{799,475,916.5 - 300,000,000}{9.1968} \right) = -0.0160x
\]
\[
\ln \left( \frac{1.6649}{9.1968} \right) = -0.0160x
\]
\[
x = 106.82
\]
Therefore, the United States population was 300,000,000 in the year 2006.

8. a. Let \( x = 1 \) correspond to 2001, \( x = 2 \) correspond to 2002, etc.

b. Using LOGISTIC REGression on the data yields:
\[
y = \frac{22.56103}{1 + 2.7003e^{-0.0167x}}
\]

c. \( y = \frac{22.56103}{1 + 2.7003e^{-0.0167x}} \)

9. a. Let \( x = 5 \) correspond to 1975, \( x = 10 \) correspond to 1980, \( x = 20 \) correspond to 1990, etc.

b. Using LOGISTIC REGression on the data yields:
\[
y = \frac{67,856.6}{1 + 19.844e^{-0.0202x}}
\]

d. As \( x \to \infty \), \( 2.7003e^{-0.0167x} \to 0 \), which means \( 1 + 2.7003e^{-0.0167x} \to 1 \), so
\[
y = \frac{22.56103}{1 + 2.7003e^{-0.0167x}} \to 22.56103
\]
Therefore, the carrying capacity of the world is approximately 22.56103 billion people.

e. The year 2015 corresponds to \( x = 15 \), so
\[
y = \frac{22.56103}{1 + 2.7003e^{-0.0167(15)}} \approx 7.27
\]
In 2015, the population of the world was approximately 7.27 billion people.

f. We need to find \( x \) when \( y = 7 \):
\[
y = \frac{22.56103}{1 + 2.7003e^{-0.0167x}} = 7
\]
\[
22.56103 = 7(1 + 2.7003e^{-0.0167x})
\]
\[
2.256103 = 1 + 2.7003e^{-0.0167x}
\]
\[
2.256103 - 1 = 2.7003e^{-0.0167x}
\]
\[
1.256103 = 2.7003e^{-0.0167x}
\]
\[
\ln \left( \frac{1.256103}{2.7003} \right) = -0.0167x
\]
\[
x = \frac{\ln \left( \frac{1.256103}{2.7003} \right)}{-0.0167} = 45.8
\]
Therefore, the world population will be 10 billion in approximately the year 2046.
Section 5.9: Building Exponential, Logarithmic, and Logistic Models from Data

c. \( Y_t = \frac{67,856.6}{1+19.844e^{-0.2029x}} \)

d. As \( x \to \infty \), \( 19.844e^{-0.2029x} \to 0 \), which means \( 1+19.844e^{-0.2029x} \to 1 \), so
\[
y = \frac{67,856.6}{1+19.844e^{-0.2029x}} \to 67,856.6
\]
Therefore, the maximum number of cable TV subscribers in the U.S. is about 67,856,600 subscribers.

e. The year 2015 corresponds to \( x = 45 \), so
\[
y = \frac{67,856.6}{1+19.844e^{-0.2029(45)}} = 67,711.
\]
In 2015, cable TV will have approximately 67,711,000 subscribers in the U.S.

10. a. Let \( x = 1 \) correspond to 1985, \( x = 2 \) correspond to 1986, \( x = 3 \) correspond to 1987, etc. Using LOGISTIC REGression on the data yields: \( y = \frac{343.0837}{1+178.4661e^{-0.2694x}} \)

b. \( Y_t = \frac{343.0837}{1+178.4661e^{-0.2694x}} \)

c. As \( x \to \infty \), \( 178.4661e^{-0.2694x} \to 0 \), which means \( 1+178.4661e^{-0.2694x} \to 1 \), so
\[
y = \frac{343.0837}{1+178.4661e^{-0.2694x}} \to 343.0837
\]
Therefore, the carrying capacity of the cellular phone market in the U.S. is about 343,083,700 subscribers.

d. The year 2009 corresponds to \( x = 25 \), so
\[
y = \frac{343.0837}{1+178.4661e^{-0.2694(25)}} = 283.044
\]
For 2009, the function predicts that the number of cell phone subscribers will be approximately 283,044,000 subscribers.

e. The answer in part (e) of Example 1 predicts 869,000,000 cell phone subscribers in the U.S. in 2009. This prediction is more than 3 times our prediction in part (d). The function in Example 1 assumes exponential growth which is unlimited growth. Our logistic function in part (d) assumes that the growth is limited.

11. a.

b. Based on the “upside down U-shape” of the graph, a quadratic model with \( a < 0 \) would best describe the data.

c. Using QUADratic REGression, the quadratic model is \( y = -0.0311x^2 + 3.4444x + 118.2493 \).

d. 

e. \( y = -0.0311(35)^2 + 3.4444(35) + 118.2493 = 201 \)
The model predicts a total cholesterol of 201 for a 35-year-old male.

12. a.

b. Based on the graph, a logarithmic model would best describe the data.

c. Using Logarithmic REGression, the model is \( y = 378.997 - 19.442 \ln x \).

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**Chapter 5: Exponential and Logarithmic Functions**

**d.**

\[ f(x) = \ln(225) \approx 37.897 - 19.442 \ln(55000) = 166.8 \]

The model predicts a rate of 166.8 crimes per 1000 population.

**13. a.**

An exponential model would fit best because depreciation of a car is described by exponential models in the theory of finance.

**c.** Using EXPonential REGression, the model is \( y = 31808.51(0.8474)^x \).

**d.**

\[ f(x) = 3x - 5 \quad g(x) = 1 - 2x^2 \]

**a.** \( (f \circ g)(2) = f(g(2)) \]

\[ = f \left( 1 - 2(2)^2 \right) \]

\[ = f(-7) \]

\[ = 3(-7) - 5 \]

\[ = -26 \]

**b.** \( (g \circ f)(-2) = g(f(-2)) \]

\[ = g \left( 3(-2) - 5 \right) \]

\[ = g(-11) \]

\[ = 1 - 2(-11)^2 \]

\[ = -241 \]

**c.** \( (f \circ f)(4) = f(f(4)) \]

\[ = f \left( 3(4) - 5 \right) \]

\[ = f(7) \]

\[ = 3(7) - 5 \]

\[ = 16 \]

**d.** \( (g \circ g)(-1) = g(g(-1)) \]

\[ = g \left( 1 - 2(-1)^2 \right) \]

\[ = g(-1) \]

\[ = 1 - 2(-1)^2 \]

\[ = -1 \]

2. \( f(x) = 4 - x \quad g(x) = 1 + x^2 \)

**a.** \( (f \circ g)(2) = f(g(2)) \]

\[ = f \left( 1 + 2^2 \right) \]

\[ = f(5) \]

\[ = 4 - 5 \]

\[ = -1 \]

**b.** \( (g \circ f)(-2) = g(f(-2)) \]

\[ = g \left( 4 - (-2) \right) \]

\[ = g(6) \]

\[ = 1 + 6^2 \]

\[ = 37 \]

**c.** \( (f \circ f)(4) = f(f(4)) \]

\[ = f \left( 4 - 4 \right) \]

\[ = f(0) \]

\[ = 4 - 0 \]

\[ = 4 \]

**d.** \( (g \circ g)(-1) = g(g(-1)) \]

\[ = g \left( 1 + (-1)^2 \right) \]

\[ = g(2) \]

\[ = 1 + 2^2 \]

\[ = 5 \]

Chapter 5 Review Exercises

1. \( f(x) = 3x - 5 \quad g(x) = 1 - 2x^2 \)

**a.** \( (f \circ g)(2) = f(g(2)) \]

\[ = f \left( 1 - 2(2)^2 \right) \]

\[ = f(-7) \]

\[ = 3(-7) - 5 \]

\[ = -26 \]

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3. \( f(x) = \sqrt{x + 2} \quad g(x) = 2x^2 + 1 \)
   a. \((f \circ g)(2) = f(g(2)) = f(2(2)^2 + 1)
   = f(9)
   = \sqrt{9 + 2}
   = \sqrt{11} \)
   b. \((g \circ f)(-2) = g(f(-2)) = g(2(-2)^2 + 1)
   = g(0)
   = 2(0)^2 + 1
   = 1 \)
   c. \((f \circ f)(4) = f(f(4)) = f(\sqrt{4 + 2})
   = f(\sqrt{6})
   = \sqrt{6 + 2} \)
   d. \((g \circ g)(-1) = g(g(-1))
   = g(2(-1)^2 + 1)
   = g(3)
   = 2(3)^2 + 1
   = 19 \)

4. \( f(x) = 1 - 3x^2 \quad g(x) = \sqrt{4 - x} \)
   a. \((f \circ g)(2) = f(g(2)) = f(\sqrt{4 - 2})
   = f(\sqrt{2})
   = 1 - 3(\sqrt{2})^2
   = 1 - 3 \cdot 2
   = -5 \)
   b. \((g \circ f)(-2) = g(f(-2)) = g(1 - 3(-2)^2)
   = g(1 - 3(-4))
   = g(-11)
   = \sqrt{4 - (-11)}
   = \sqrt{15} \)

5. \( f(x) = e^x \quad g(x) = 3x - 2 \)
   a. \((f \circ g)(2) = f(g(2)) = f(3(2) - 2)
   = f(4)
   = e^4 \)
   b. \((g \circ f)(-2) = g(f(-2))
   = g(\sqrt{4} - (-2))
   = g(\sqrt{5})
   = \sqrt{4 - \sqrt{5}} \)
   c. \((f \circ f)(4) = f(f(4))
   = f(e^4)
   = e^{e^4} \)
   d. \((g \circ g)(-1) = g(g(-1))
   = g(3(-1) - 2)
   = g(-5)
   = 3(-5) - 2
   = -17 \)

6. \( f(x) = \frac{2}{1 + 2x^2} \quad g(x) = 3x \)
   a. \((f \circ g)(2) = f(g(2)) = f(3(2))
   = f(6)
   = \frac{2}{1 + 2(6)^2} = \frac{2}{73} \)

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Chapter 5: Exponential and Logarithmic Functions

b. \((g \circ f)(-2) = g(f(-2))\)
   \[
g \left( \frac{2}{1+2(-2)^2} \right)
   = g \left( \frac{2}{9} \right)
   = 3 \left( \frac{2}{9} \right) = \frac{2}{3}
   \]

c. \((f \circ f)(4) = f(f(4))\)
   \[
f \left( \frac{2}{1+2(4)^2} \right)
   = f \left( \frac{2}{33} \right)
   = \frac{2}{1+2 \left( \frac{2}{33} \right)^2}
   = \frac{2}{\frac{1097}{1089}}
   = \frac{2178}{1097}
   \]

d. \((g \circ g)(-1) = g(g(-1))\)
   \[
g \left( 3(-1) \right)
   = g(-3)
   = -9
   \]

7. \(f(x) = 2 - x\) \quad \(g(x) = 3x + 1\)
   The domain of \(f\) is \(\{x \mid x \text{ is any real number} \}\).
   The domain of \(g\) is \(\{x \mid x \text{ is any real number} \}\).
   \((f \circ g)(x) = f(g(x))\)
   \[
   = f(3x + 1)
   = 2 - (3x + 1)
   = 2 - 3x - 1
   = -3 - x
   \]
   Domain: \(\{x \mid x \text{ is any real number} \}\).

8. \(f(x) = 2x - 1\) \quad \(g(x) = 2x + 1\)
   The domain of \(f\) is \(\{x \mid x \text{ is any real number} \}\).
   The domain of \(g\) is \(\{x \mid x \text{ is any real number} \}\).
   \((g \circ f)(x) = g(f(x))\)
   \[
   = g(2x - 1)
   = 2(2x - 1) + 1
   = 4x - 2 + 1
   = 4x - 1
   \]
   Domain: \(\{x \mid x \text{ is any real number} \}\).
(g \circ g)(x) = g(g(x))
= g(2x+1)
= 2(2x+1)+1
= 4x+2+1
= 4x+3
Domain: \{x \mid x \text{ is any real number} \}.

9. \quad f(x) = 3x^2 + x + 1 \quad g(x) = \left| 3x \right|
The domain of \ f \ is \ \{x \mid x \text{ is any real number} \}.
The domain of \ g \ is \ \{x \mid x \text{ is any real number} \}.
\begin{align*}
(f \circ g)(x) &= f(g(x)) \\
&= f(\left| 3x \right|) \\
&= 3\left(\left| 3x \right|^2 + \left| 3x \right| \right)+1 \\
&= 27x^2 + 3\left| x \right| + 1 \\
\end{align*}
Domain: \{x \mid x \text{ is any real number} \}.

\begin{align*}
(g \circ f)(x) &= g(f(x)) \\
&= g(3x^2 + x + 1) \\
&= \left| 3\left( 3x^2 + x + 1 \right) \right| \\
&= 3\left| 3x^2 + x + 1 \right| \\
\end{align*}
Domain: \{x \mid x \text{ is any real number} \}.

\begin{align*}
(f \circ f)(x) &= f(f(x)) \\
&= f(3x^2 + x + 1) \\
&= 3\left( 3x^2 + x + 1 \right)^2 + \left( 3x^2 + x + 1 \right)+1 \\
&= 27x^4 + 18x^3 + 24x^2 + 7x + 5 \\
\end{align*}
Domain: \{x \mid x \text{ is any real number} \}.

11. \quad f(x) = \frac{x+1}{x-1} \quad g(x) = \frac{1}{x}
The domain of \ f \ is \ \{x \mid x \neq 1 \}.
The domain of \ g \ is \ \{x \mid x \neq 0 \}.
\begin{align*}
(f \circ g)(x) &= f(g(x)) \\
&= f\left( \frac{1}{x} \right) \\
&= \frac{1+1}{x-1} \\
&= \frac{2}{x-1} \\
&= \frac{1}{x} + \frac{1}{1-x} \\
\end{align*}
Domain: \{x \mid x \neq 0, x \neq 1 \}.  

\text{Chapter 5 Review Exercises}
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\[(g \circ f)(x) = g(f(x)) \]

\[= g\left(\frac{x+1}{x-1}\right) = \frac{1}{\frac{x+1}{x-1}} = \frac{x-1}{x+1}\]

Domain \(\{x|x \neq -1, x \neq 1\}\)

\[(f \circ f)(x) = f(f(x)) \]

\[= f\left(\frac{x+1}{x-1}\right) = \frac{x+1}{x-1} + 1\]

\[= \left(\frac{x+1}{x-1}\right)(x-1)\]

\[= \frac{x+1}{x-1} - 1(x-1)\]

\[= \frac{x+1+x-1}{x+1-(x-1)} = \frac{2x}{2} = x\]

Domain \(\{x|x \neq 1\}\).

\[(g \circ g)(x) = g(g(x)) = \frac{1}{\frac{1}{x}} = x\]

Domain \(\{x|x \neq 0\}\).

12. \(f(x) = \sqrt{x-3} \quad g(x) = \frac{3}{x}\)

The domain of \(f\) is \(\{x|x \geq 3\}\).

The domain of \(g\) is \(\{x|x \neq 0\}\).

\[(f \circ g)(x) = f(g(x)) \]

\[= f\left(\frac{3}{x}\right) = \sqrt{\frac{3}{x}-3} = \sqrt{\frac{3-3x}{x}}\]

To find the domain, we must find where \(p(x) = \frac{3-3x}{x} > 0\). \(p\) is zero or undefined when \(x = 1\) and \(x = 0\).

<table>
<thead>
<tr>
<th>Interval</th>
<th>((-\infty, 0))</th>
<th>((0,1))</th>
<th>((1,\infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Value</td>
<td>-1</td>
<td>(\frac{1}{2})</td>
<td>2</td>
</tr>
<tr>
<td>Value of (p)</td>
<td>-6</td>
<td>3</td>
<td>(-\frac{3}{2})</td>
</tr>
<tr>
<td>Conclusion</td>
<td>negative</td>
<td>positive</td>
<td>negative</td>
</tr>
</tbody>
</table>

Domain \(\{x|0 < x \leq 1\}\).

\[(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x-3}\right) = \frac{3}{\sqrt{x-3}}\]

To find the domain, solve \(x-3 > 0\)

\[x > 3\]

Domain \(\{x|x > 3\}\)

\[(f \circ f)(x) = f(f(x)) = f\left(\sqrt{x-3}\right) = \sqrt{x-3} - 3\]

To find the domain, solve \(\sqrt{x-3} - 3 \geq 0\)

\[x-3 \geq 9\]

\[x \geq 12\]

Domain \(\{x|x \geq 12\}\).

\[(g \circ g)(x) = g(g(x)) = g\left(\frac{3}{x}\right) = \frac{3}{\left(\frac{3}{x}\right)} = \frac{x}{3} = x\]

Domain \(\{x|x \neq 0\}\).

13. a. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

b. The inverse is \(\{(2,1),(5,3),(8,5),(10,6)\}\).

14. a. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

b. The inverse is \(\{(4, -1),(2,0),(5,1),(7,3)\}\).

15. The function \(f\) is one-to-one because every horizontal line intersects the graph at exactly one point.

16. The function \(f\) is one-to-one because every horizontal line intersects the graph at exactly one point.
17. \[ f(x) = \frac{2x+3}{5x-2} \]
\[ y = \frac{2x+3}{5x-2} \]
\[ x = \frac{2y+3}{5y-2} \]

Domain of \( f \) = Range of \( f^{-1} \)

Range of \( f \) = Domain of \( f^{-1} \)

18. \[ f(x) = \frac{2-x}{3+x} \]
\[ y = \frac{2-x}{3+x} \]
\[ x = \frac{2-y}{3+y} \]
\[ x(3+y) = 2-y \]
\[ 3x+y = 2-y \]
\[ xy+y = 2-3x \]
\[ y(x+1) = 2-3x \]
\[ y = \frac{2-3x}{x+1} \]
\[ f^{-1}(x) = \frac{2-3x}{x+1} \]

Domain of \( f \) = Range of \( f^{-1} \)

Range of \( f \) = Domain of \( f^{-1} \)

19. \[ f(x) = \frac{1}{x-1} \]
\[ y = \frac{1}{x-1} \]
\[ x = \frac{1}{y-1} \]

Domain of \( f \) = Range of \( f^{-1} \)

Range of \( f \) = Domain of \( f^{-1} \)

20. \[ f(x) = \sqrt{x-2} \]
\[ y = \sqrt{x-2} \]
\[ x = \sqrt{y-2} \]

Domain of \( f \) = Range of \( f^{-1} \)

Range of \( f \) = Domain of \( f^{-1} \)
Chapter 5: Exponential and Logarithmic Functions

Domain of \( f \) = Range of \( f^{-1} = \{ x \mid x \geq 2 \} \) or \( [2, \infty) \)

Range of \( f \) = Domain of \( f^{-1} = \{ x \mid x \geq 0 \} \) or \( [0, \infty) \)

21. \( f(x) = \frac{3}{x^{1/3}} \)

\[ y = \frac{3}{x^{1/3}} \]

\[ x = \frac{3}{y^{1/3}} \text{ Inverse} \]

\[ xy^{1/3} = 3 \]

\[ y^{1/3} = \frac{3}{x} \]

\[ y = \left( \frac{3}{x} \right)^3 = \frac{27}{x^3} \]

\[ f^{-1}(x) = \frac{27}{x^3} \]

Domain of \( f \) = Range of \( f^{-1} \)

All real numbers except 0

Range of \( f \) = Domain of \( f^{-1} \)

All real numbers except 0

22. \( f(x) = x^{1/3} + 1 \)

\[ y = x^{1/3} + 1 \]

\[ x = y^{1/3} + 1 \text{ Inverse} \]

\[ y^{1/3} = x - 1 \]

\[ y = (x - 1)^3 \]

\[ f^{-1}(x) = (x - 1)^3 \]

Domain of \( f \) = Range of \( f^{-1} \)

All real numbers or \((-\infty, \infty)\)

Range of \( f \) = Domain of \( f^{-1} \)

All real numbers or \((-\infty, \infty)\)

23. a. \( f(4) = 3^4 = 81 \)

b. \( g(9) = \log_3 (9) = \log_3 (3^2) = 2 \)

c. \( f(-2) = 3^{-2} = \frac{1}{9} \)

d. \( g\left(\frac{1}{27}\right) = \log_3 \left(\frac{1}{27}\right) = \log_3 (3^{-3}) = -3 \)

24. a. \( f(1) = 3^1 = 3 \)

b. \( g(81) = \log_3 (81) = \log_3 (3^4) = 4 \)

c. \( f(-4) = 3^{-4} = \frac{1}{81} \)

d. \( g\left(\frac{1}{243}\right) = \log_3 \left(\frac{1}{243}\right) = \log_3 (3^{-5}) = -5 \)

25. \( 5^2 = z \) is equivalent to \( 2 = \log_5 z \)

26. \( a^5 = m \) is equivalent to \( 5 = \log_a m \)

27. \( \log_5 u = 13 \) is equivalent to \( 5^{13} = u \)

28. \( \log_a 4 = 3 \) is equivalent to \( a^3 = 4 \)

29. \( f(x) = \log(3x - 2) \) requires:

\[ 3x - 2 > 0 \]

\[ x > \frac{2}{3} \]

Domain: \( \left\{ x \mid x > \frac{2}{3} \right\} \) or \( \left(\frac{2}{3}, \infty\right) \)

30. \( F(x) = \log_5 (2x+1) \) requires:

\[ 2x + 1 > 0 \]

\[ x > -\frac{1}{2} \]

Domain: \( \left\{ x \mid x > -\frac{1}{2} \right\} \) or \( \left(-\frac{1}{2}, \infty\right) \)

31. \( H(x) = \log_2 \left(x^2 - 3x + 2\right) \) requires

\[ p(x) = x^2 - 3x + 2 > 0 \]

\[ (x - 2)(x - 1) > 0 \]

\( x = 2 \) and \( x = 1 \) are the zeros of \( p \).

<table>
<thead>
<tr>
<th>Interval</th>
<th>((-\infty, 1))</th>
<th>(1, 2)</th>
<th>(2, \infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Value</td>
<td>0</td>
<td>(\frac{3}{2})</td>
<td>3</td>
</tr>
<tr>
<td>Value of ( p )</td>
<td>2</td>
<td>(-\frac{1}{4})</td>
<td>2</td>
</tr>
<tr>
<td>Conclusion</td>
<td>positive</td>
<td>negative</td>
<td>positive</td>
</tr>
</tbody>
</table>

Thus, the domain of \( H(x) = \log_2 \left(x^2 - 3x + 2\right) \)

is \( \{ x \mid x < 1 \text{ or } x > 2 \} \) or \( (-\infty, 1) \cup (2, \infty) \).

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32. \( F(x) = \ln(x^2 - 9) \) requires
\[ p(x) = x^2 - 9 > 0 \]
\[ (x + 3)(x - 3) > 0 \]
x = -3 and \( x = 3 \) are the zeros of \( p \).

<table>
<thead>
<tr>
<th>Interval</th>
<th>((-\infty,-3))</th>
<th>(-3,3)</th>
<th>(3,( \infty ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Value</td>
<td>-4</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Value of ( p )</td>
<td>7</td>
<td>-9</td>
<td>7</td>
</tr>
<tr>
<td>Conclusion</td>
<td>positive</td>
<td>negative</td>
<td>positive</td>
</tr>
</tbody>
</table>

Thus, the domain of \( F(x) = \ln(x^2 - 9) \) is
\( \{x \mid x < -3 \text{ or } x > 3\} \) or \( (-\infty,3) \cup (3,\infty) \).

33. \[ \log_2 \left( \frac{1}{8} \right) = \log_2 2^{-3} = -3 \log_2 2 = -3 \]

34. \[ \log_3 81 = \log_3 3^4 = 4 \log_3 3 = 4 \]

35. \[ \ln e^{\frac{\sqrt{2}}{\sqrt{3}}} = \sqrt{2} \]

36. \[ e^{\ln 0.1} = 0.1 \]

37. \[ 2^{\log_2 0.4} = 0.4 \]

38. \[ \log_2 2^{\sqrt{3}} = \sqrt{3} \log_2 2 = \sqrt{3} \]

39. \[ \log_3 \left( \frac{uv^2}{w} \right) = \log_3 uv^2 - \log_3 w \]
\[ = \log_3 u + \log_3 v^2 - \log_3 w \]
\[ = \log_3 u + 2 \log_3 v - \log_3 w \]

40. \[ \log_2 \left( a^2 \sqrt{b} \right)^2 = 4 \log_2 \left( a^2 \sqrt{b} \right) \]
\[ = 4 \left( \log_2 a^2 + \log_2 b^{1/2} \right) \]
\[ = 4 \left( 2 \log_2 a + \frac{1}{2} \log_2 b \right) \]
\[ = 8 \log_2 a + 2 \log_2 b \]

41. \[ \log_2 \left( x^2 + x^3 + 1 \right) = \log_2 x^2 + \log_2 \left( x^3 + 1 \right)^{1/2} \]
\[ = 2 \log_2 x + \frac{1}{2} \log_2 \left( x^3 + 1 \right) \]

42. \[ \log_5 \left( \frac{x^2 + 2x + 1}{x^3} \right) = \log_5 (x+1)^2 - \log_5 x^3 \]
\[ = 2 \log_5 (x+1) - 2 \log_5 x \]

43. \[ \ln \left( \frac{x\sqrt{x^2 + 1}}{x-3} \right) = \ln(x\sqrt{x^2 + 1}) - \ln(x-3) \]
\[ = \ln x + \ln \left( x^2 + 1 \right)^{1/3} - \ln(x-3) \]
\[ = \ln x + \frac{1}{3} \ln \left( x^2 + 1 \right) - \ln(x-3) \]

44. \[ \ln \left( \frac{2x+3}{x^2 - 3x + 2} \right)^2 \]
\[ = 2 \ln \left( \frac{2x+3}{x^2 - 3x + 2} \right) \]
\[ = 2 \left( \ln(2x+3) - \ln((x-1)(x-2)) \right) \]
\[ = 2 \left( \ln(2x+3) - \ln(x-1) - \ln(x-2) \right) \]
\[ = 2 \ln(2x+3) - 2 \ln(x-1) - 2 \ln(x-2) \]

45. \[ 3 \log_4 x^2 + \frac{1}{2} \log_4 \sqrt{x} = \log_4 \left( x^2 \right)^{3} + \log_4 \left( x^{1/2} \right)^{1/2} \]
\[ = \log_4 x^6 + \log_4 x^{1/4} \]
\[ = \log_4 \left( x^6 \cdot x^{1/4} \right) \]
\[ = \log_4 x^{25/4} \]
\[ = \frac{25}{4} \log_4 x \]

46. \[ -2 \log_3 \left( \frac{1}{x} \right) + \frac{1}{3} \log_3 \sqrt{x} \]
\[ = \log_3 \left( x^{-1} \right)^2 + \log_3 \left( x^{1/2} \right)^{1/3} \]
\[ = \log_3 x^2 + \log_3 x^{1/6} \]
\[ = \log_3 \left( x^2 \cdot x^{1/6} \right) \]
\[ = \log_3 x^{13/6} \]
\[ = \frac{13}{6} \log_3 x \]
47. \( \ln \left( \frac{x-1}{x} \right) + \ln \left( \frac{x}{x+1} \right) - \ln (x^2 - 1) \)
   \[ = \ln \left( \frac{x-1}{x(x+1)} \right) - \ln (x^2 - 1) \]
   \[ = \ln \left( \frac{x-1}{x^2 - 1} \right) \]
   \[ = \ln \left( \frac{x-1}{(x+1)(x-1)} \right) \]
   \[ = \ln \frac{1}{(x+1)^2} \]
   \[ = \ln(1) \]
   \[ = -2 \ln(x+1) \]

48. \( \log (x^2 - 9) - \log (x^2 + 7x + 12) \)
   \[ = \log \left( \frac{(x-3)(x+3)}{(x+3)(x+4)} \right) \]
   \[ = \log (x-3) \]

49. \( 2 \log 2 + 3 \log x - \frac{1}{2} [\log(x+3) + \log(x-2)] \)
   \[ = \log 2^2 + \log x^3 - \frac{1}{2} \log [(x+3)(x-2)] \]
   \[ = \log \left( \frac{4x^3}{(x+3)(x-2)} \right) \]

50. \( \frac{1}{2} \ln (x^2 + 1) - 4 \ln \frac{1}{2} \left[ \ln (x-4) + \ln x \right] \)
   \[ = \ln \left( \frac{x^2 + 1}{2} \right) - \ln \left( \frac{1}{2} \right)^4 - \ln (x-4)^{1/2} \]
   \[ = \ln \left( \frac{1}{16} \left[ \frac{x(x-4)}{x} \right]^{1/2} \right) \]
   \[ = \ln \left( \frac{16x^2 + 1}{\sqrt{x(x-4)}} \right) \]

51. \( \log_4 19 = \frac{\ln 19}{\ln 4} = 2.124 \)

52. \( \log_2 21 = \frac{\ln 21}{\ln 2} = 4.392 \)

53. \( Y_t = \log_3 x = \frac{\ln x}{\ln 3} \)

54. \( Y_t = \log_7 x = \frac{\ln x}{\ln 7} \)

55. \( f(x) = 2^{x^3} \)
   a. Domain: \((-\infty, \infty)\)
   b. Using the graph of \( y = 2^x \), shift the graph horizontally 3 units to the right.
   c. Range: \((0, \infty)\)
   Horizontal Asymptote: \( y = 0 \)
   d. \( f(x) = 2^{x^3} \)
   \( y = 2^{x^3} \)
   \( x = 2^{y^3} \)\quad \text{Inverse}
   \( y - 3 = \log_2 x \)
   \( y = 3 + \log_2 x \)
   \( f^{-1}(x) = 3 + \log_2 x \)
   e. Range of \( f = \text{Domain} f^{-1} : (0, \infty) \)
Domain of \( f = \) Range of \( f^{-1} : (\infty, \infty) \)

f. Using the graph of \( y = \log_2 x \), shift the graph vertically 3 units up.

\[
\begin{align*}
(2, 4) & \quad (1, 3) \\
-5 & \quad 5 \\
-5 & \quad 5 \\
\end{align*}
\]

56. \( f(x) = -2^x + 3 \)

a. Domain: \((\infty, \infty)\)

b. Using the graph of \( y = 2^x \), reflect the graph about the \( x \)-axis, and shift vertically 3 units up.

c. Range: \((\infty, 3)\)
   Horizontal Asymptote: \( y = 3 \)

d. \[
\begin{align*}
f(x) &= -2^x + 3 \\
y &= -2^x + 3 \\
x &= -2^y + 3 & \text{Inverse} \\
x - 3 &= -2^y \\
3 - x &= 2^y \\
y &= \log_2 (3 - x) \\
f^{-1}(x) &= \log_2 (3 - x)
\end{align*}
\]

e. \[
\begin{align*}
3 - x &> 0 \\
x &< 3 \\
-x &> -3 \\
x &< 3
\end{align*}
\]
   Range of \( f = \) Domain \( f^{-1} : (\infty, 3) \)

57. \( f(x) = \frac{1}{2}(3^{-x}) \)

a. Domain: \((\infty, \infty)\)

b. Using the graph of \( y = 3^x \), reflect the graph about the \( y \)-axis, and compress vertically by a factor of \( \frac{1}{2} \).

c. Range: \((0, \infty)\)
   Horizontal Asymptote: \( y = 0 \)

d. \[
\begin{align*}
f(x) &= \frac{1}{2}(3^{-x}) \\
y &= \frac{1}{2}(3^{-x}) \\
x &= \frac{1}{2}(3^{-y}) & \text{Inverse} \\
2x &= 3^{-y} \\
-y &= \log_3 (2x) \\
y &= -\log_3 (2x) \\
f^{-1}(x) &= -\log_3 (2x)
\end{align*}
\]
Chapter 5: Exponential and Logarithmic Functions

e. \(2x > 0\)
   \(x > 0\)

   Range of \(f = \text{Domain } f^{-1}: (0, \infty)\)
   Domain of \(f = \text{Range of } f^{-1}: (-\infty, \infty)\)

f. Using the graph of \(y = \log_3 x\), compress the graph horizontally by a factor of \(\frac{1}{2}\), and reflect about the \(x\)-axis.

\[f(x) = 1 + 3^{-x}\]

a. Domain: \((-\infty, \infty)\)

b. Using the graph of \(y = 3^x\), reflect the graph about the \(y\)-axis, and shift vertically 1 unit up.

c. Range: \((1, \infty)\)
   Horizontal Asymptote: \(y = 1\)

d. \(f(x) = 1 + 3^{-x}\)
\(y = 1 + 3^{-x}\)
\(x = 1 + 3^{-y}\) \(\text{Inverse}\)
\(x - 1 = 3^{-y}\)
\(-y = \log_3 (x - 1)\)
\(y = -\log_3 (x - 1)\)
\(f^{-1}(x) = -\log_3 (x - 1)\)

e. \(x - 1 > 0\)
   \(x > 1\)

   Range of \(f = \text{Domain } f^{-1}: (1, \infty)\)
   Domain of \(f = \text{Range of } f^{-1}: (-\infty, \infty)\)

f. Using the graph of \(y = \log_3 x\), shift the graph horizontally to the right 1 unit, and reflect vertically about the \(x\)-axis.

\[f(x) = 1 - e^{-x}\]

a. Domain: \((-\infty, \infty)\)

b. Using the graph of \(y = e^x\), reflect about the \(y\)-axis, reflect about the \(x\)-axis, and shift up 1 unit.

c. Range: \((-\infty, 1)\)
   Horizontal Asymptote: \(y = 1\)

d. \(f(x) = 1 - e^{-x}\)
\(y = 1 - e^{-x}\)
\(x = 1 - e^{-y}\) \(\text{Inverse}\)
\(x - 1 = -\ln(1 - x)\)
\(1 - x = e^{-y}\)
\(-y = \ln(1 - x)\)
\(y = -\ln(1 - x)\)
\(f^{-1}(x) = -\ln(1 - x)\)

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e. $1 - x > 0$
   $-x > -1$
   $x < 1$

Range of $f = \text{Domain } f^{-1}: (-\infty, 1)$
Domain of $f = \text{Range of } f^{-1}: (-\infty, \infty)$

f. Using the graph of $y = \ln x$, reflect the graph about the $y$-axis, shift to the right 1 unit, and reflect about the $x$-axis.

60. $f(x) = 3e^{x-2}$

a. Domain: $(-\infty, \infty)$

b. Using the graph of $y = e^x$, shift the graph two units horizontally to the right, and stretch vertically by a factor of 3.

c. Range: $(0, \infty)$
   Horizontal Asymptote: $y = 0$

d. $f(x) = 3e^{x-2}$
   $y = 3e^{x-2}$
   $x = 3e^{y-2}$
   Inverse
   \[
   \frac{x}{3} = e^{y-2} \\
   y - 2 = \ln \left( \frac{x}{3} \right) \\
   y = 2 + \ln \left( \frac{x}{3} \right) \\
   f^{-1}(x) = 2 + \ln \left( \frac{x}{3} \right)
   \]

61. $f(x) = \frac{1}{2} \ln(x + 3)$

a. Domain: $(-3, \infty)$

b. Using the graph of $y = \ln x$, shift the graph to the left 3 units and compress vertically by a factor of $\frac{1}{2}$.

c. Range: $(-\infty, \infty)$
   Vertical Asymptote: $x = -3$
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d. \( f(x) = \frac{1}{2} \ln(x + 3) \)
\[
\begin{align*}
y &= \frac{1}{2} \ln(x + 3) \\
x &= \frac{1}{2} \ln(y + 3) \quad \text{Inverse} \\
2x &= \ln(y + 3) \\
y + 3 &= e^{2x} \\
y &= e^{2x} - 3 \\
f^{-1}(x) &= e^{2x} - 3
\end{align*}
\]
e. Range of \( f = \) Domain \( f^{-1} : (-\infty, \infty) \)
Domain of \( f = \) Range of \( f^{-1} : (-3, \infty) \)

f. Using the graph of \( y = e^x \), compress horizontally by a factor of \( \frac{1}{2} \), and shift down 3 units.

![Graph of \( y = e^x \) compressed horizontally and shifted down 3 units]

62. \( f(x) = 3 + \ln(2x) \)

a. Domain: \( (0, \infty) \)

b. Using the graph of \( y = \ln x \), compress the graph horizontally by a factor of \( \frac{1}{2} \), and shift up 3 units.

![Graph of \( y = \ln x \) compressed horizontally and shifted up 3 units]

c. Range: \( (-\infty, \infty) \)
Vertical Asymptote: \( x = 0 \)

d. \( f(x) = 3 + \ln(2x) \)
\[
\begin{align*}
y &= 3 + \ln(2x) \\
x &= 3 + \ln(2y) \quad \text{Inverse} \\
x - 3 &= \ln(2y) \\
2y &= e^{x-3} \\
y &= \frac{1}{2} e^{x-3} \\
f^{-1}(x) &= \frac{1}{2} e^{x-3}
\end{align*}
\]
e. Range of \( f = \) Domain \( f^{-1} : (-\infty, \infty) \)
Domain of \( f = \) Range of \( f^{-1} : (0, \infty) \)

f. Using the graph of \( y = e^x \), shift to horizontally to the right 3 units, and compress vertically by a factor of \( \frac{1}{2} \).

![Graph of \( y = e^x \) shifted horizontally and compressed vertically]

63. \( 4^{1-2x} = 2 \)
\[
\begin{align*}
(2^2)^{1-2x} &= 2 \\
2^{2-4x} &= 2^1 \\
2 - 4x &= 1 \\
-4x &= -1 \\
x &= \frac{1}{4}
\end{align*}
\]
The solution set is \( \left\{ \frac{1}{4} \right\} \).
64. \[ 8^{6+3x} = 4 \]
\[ (2^3)^{6+3x} = 2^2 \]
\[ 2^{18+9x} = 2^2 \]
\[ 18 + 9x = 2 \]
\[ 9x = -16 \]
\[ x = \frac{-16}{9} \]

The solution set is \( \left\{ \frac{-16}{9} \right\} \).

65. \[ 3^{2^x + x} = \sqrt{3} \]
\[ 3^{2^x + x} = 3^{1/2} \]
\[ x^2 + x = \frac{1}{2} \]
\[ 2x^2 + 2x - 1 = 0 \]
\[ x = \frac{-2 \pm \sqrt{4 - 4(2)(-1)}}{2(2)} \]
\[ = \frac{-2 \pm \sqrt{4 + 8}}{4} = \frac{-2 \pm \sqrt{12}}{4} \]
\[ = -1 \pm \sqrt{3} \]

The solution is \( \left\{ \frac{-1 - \sqrt{3}}{2}, \frac{-1 + \sqrt{3}}{2} \right\} = \{-1.366, 0.366\} \).

66. \[ 4^{x^2 - x^2} = \frac{1}{2} \]
\[ 2^{2(x^2 - x^2)} = 2^{-1} \]
\[ 2x^2 - 2x = -1 \]
\[ 2x^2 - 2x - 1 = 0 \]
\[ x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)} \]
\[ = \frac{2 \pm \sqrt{4 + 8}}{4} = \frac{2 \pm \sqrt{12}}{4} \]
\[ = 1 \pm \sqrt{3} \]

The solution is \( \left\{ \frac{1 - \sqrt{3}}{2}, \frac{1 + \sqrt{3}}{2} \right\} = \{-0.366, 1.366\} \).

67. \[ \log_4 64 = -3 \]
\[ x^{-3} = 64 \]
\[ (x^{-3})^{-1/3} = 64^{-1/3} \]
\[ x = \frac{1}{\sqrt[3]{64}} = \frac{1}{4} \]

The solution set is \( \left\{ \frac{1}{4} \right\} \).

68. \[ \log_{\sqrt{2}} x = -6 \]

\[ x = \left(\sqrt{2}\right)^{-6} \]
\[ = \left(2^{1/2}\right)^{-6} \]
\[ = 2^{-3} = \frac{1}{8} \]

The solution set is \( \left\{ \frac{1}{8} \right\} \).

69. \[ 5^x = 3^{x+2} \]
\[ \ln (5^x) = \ln (3^{x+2}) \]
\[ x \ln 5 = (x + 2) \ln 3 \]
\[ x \ln 5 = x \ln 3 + 2 \ln 3 \]
\[ x \ln 5 - x \ln 3 = 2 \ln 3 \]
\[ x \ln (5 - 3) = 2 \ln 3 \]
\[ x = \frac{2 \ln 3}{\ln 5 - \ln 3} = 4.301 \]

The solution set is \( \left\{ \frac{2 \ln 3}{\ln 5 - \ln 3} \right\} = \{4.301\} \).

70. \[ 5^{x+2} = 7^{x-2} \]
\[ \ln (5^{x+2}) = \ln (7^{x-2}) \]
\[ (x + 2) \ln 5 = (x - 2) \ln 7 \]
\[ x \ln 5 + 2 \ln 5 = x \ln 7 - 2 \ln 7 \]
\[ x \ln 5 - x \ln 7 = -2 \ln 7 - 2 \ln 5 \]
\[ x \ln (5 - 7) = -2 \ln 7 - 2 \ln 5 \]
\[ x = \frac{-2 \ln 7 - 2 \ln 5}{\ln 5 - \ln 7} \]
\[ = \frac{-2 \ln (7 + 5)}{\ln 5 - \ln 7} \]
\[ = \frac{2 \ln 7 + 2 \ln 5}{\ln 7 - \ln 5} = 21.133 \]

The solution set is \( \left\{ \frac{2 \ln (7 + 5)}{\ln 7 - \ln 5} \right\} = \{21.133\} \).
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71. \[9^{2x} = 27^{3x-4}\]
\[
(3^2)^{2x} = (3^3)^{3x-4}
\]
\[
3^{4x} = 3^9 x^{-12}
\]
\[
4x = 9x - 12
\]
\[
x = \frac{12}{5}
\]
The solution set is \[\left\{\frac{12}{5}\right\}\].

72. \[25^{2x} = 5^{x^2-12}\]
\[
(5^2)^{2x} = 5^{x^2-12}
\]
\[
5^{4x} = 5^{x^2-12}
\]
\[
x^2 - 4x - 12 = 0
\]
\[
(x - 6)(x + 2) = 0
\]
\[
x = 6 \quad \text{or} \quad x = -2
\]
The solution set is \[-2, 6\].

73. \[\log_3 \sqrt{x-2} = 2\]
\[
\sqrt{x-2} = 3^2
\]
\[
x - 2 = 9
\]
\[
x - 2 = 81
\]
\[
x = 83
\]
Check: \[\log_3 \sqrt{83-2} = \log_3 \sqrt{81}\]
\[= \log_3 9 = 2\]
The solution set is \[\{83\}\].

74. \[2^{x+1} - 8^{-x} = 4\]
\[
2^{x+1} \cdot (2^3)^{-x} = 2^2
\]
\[
2^{x+1} \cdot 2^{-3x} = 2^2
\]
\[
2^{-2x+1} = 2^2
\]
\[
-2x + 1 = 2
\]
\[
-2x = 1
\]
\[
x = -\frac{1}{2}
\]
The solution set is \[-\frac{1}{2}\].

75. \[8 = 4^{x^2} \cdot 2^{5x}\]
\[
2^3 = (2^2)^{\frac{x^2}{2}} \cdot 2^{5x}
\]
\[
2^3 = 2^{2x^2 + 5x}
\]
\[
3 = 2x^2 + 5x
\]
\[
0 = 2x^2 + 5x - 3
\]
\[
0 = (2x - 1)(x + 3)
\]
\[
x = \frac{1}{2} \quad \text{or} \quad x = -3
\]
The solution set is \[-3, \frac{1}{2}\].

76. \[2^x \cdot 5 = 10^x\]
\[
\ln(2^x \cdot 5) = \ln 10^x
\]
\[
\ln 2^x + \ln 5 = x \ln 10
\]
\[
x \ln 2 + \ln 5 = x \ln 10 - x \ln 2
\]
\[
\ln 5 = x(\ln 10 - \ln 2)
\]
\[
\frac{\ln 5}{\ln 10 - \ln 2} = x
\]
\[
x = \frac{\ln 5}{\ln 10 - \ln 2} = \frac{\ln 5}{\ln \frac{10}{2}} = 1
\]
The solution set is \[\{1\}\].

77. \[\log_6 (x+3) + \log_6 (x+4) = 1\]
\[
\log_6 ((x+3)(x+4)) = 1
\]
\[x^2 + 7x + 12 = 6
\]
\[x^2 + 7x + 6 = 0
\]
\[(x + 6)(x + 1) = 0
\]
\[x = -6 \quad \text{or} \quad x = -1
\]
Since \[\log_6 (-6 + 3) = \log_6 (-3)\] is undefined, the solution set is \[-1\].

78. \[\log(7x - 12) = 2 \log x\]
\[
\log(7x - 12) = \log x^2
\]
\[
7x - 12 = x^2
\]
\[
x^2 - 7x + 12 = 0
\]
\[(x - 4)(x - 3) = 0
\]
\[x = 4 \quad \text{or} \quad x = 3
\]
Since each original logarithm is defined, for \[x = 3\] and \[x = 4\], the solution set is \[\{3, 4\}\].

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79. \[ e^{1-x} = 5 \]
\[ 1 - x = \ln 5 \]
\[ -x = -1 + \ln 5 \]
\[ x = 1 - \ln 5 = -0.609 \]
The solution set is \( \{1 - \ln 5\} = \{-0.609\} \).

80. \[ e^{1-2x} = 4 \]
\[ 1 - 2x = \ln 4 \]
\[ -2x = -1 + \ln 4 \]
\[ x = \frac{1 - \ln 4}{2} = -0.193 \]
The solution set is \( \left\{\frac{1 - \ln 4}{2}\right\} = \{-0.193\} \).

81. \[ 9^x + 4 \cdot 3^x - 3 = 0 \]
\[ (3^2)^x + 4 \cdot 3^x - 3 = 0 \]
\[ (3^2)^x + 4 \cdot 3^x - 3 = 0 \]
Let \( u = 3^x \).
\[ u^2 + 4u - 3 = 0 \]
\[ a = 1, b = 4, c = -3 \]
\[ u = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-3)}}{2(1)} \]
\[ u = \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7} \]
\[ 3^x = -2 + \sqrt{7} \] or \( 3^x = -2 - \sqrt{7} \)
\[ 3^x \) can't be negative \( x = \log_3 (-2 + \sqrt{7}) \]
The solution set is \( \left\{ \log_3 (-2 + \sqrt{7}) \right\} = \{-0.398\} \).

82. \[ 4^x - 14 \cdot 4^{-x} = 5 \]
Multiply both sides of the equation by \( 4^x \).
\[ (4^x)^2 - 14 \cdot 4^{-x} \cdot 4^x = 5 \cdot 4^x \]
\[ (4^x)^2 - 14 = 5 \cdot 4^x \]
\[ (4^x)^2 - 5 \cdot 4^x - 14 = 0 \]
\[ (4^x - 7)(4^x + 2) = 0 \]
\[ 4^x - 7 = 0 \text{ or } 4^x + 2 = 0 \]
\[ 4^x = 7 \text{ or } 4^x = -2 \]
\[ x = \log_4 7 \text{ or } x \text{ can't be negative} \]
The solution set is \( \left\{ \log_4 7 \right\} = \{1.404\} \).

83. a. \[ f(x) = \log_2 (x-2) + 1 \]
Using the graph of \( y = \log_2 x \), shift the graph right 2 units and up 1 unit.

b. \[ f(6) = \log_2 (6-2) + 1 \]
\[ = \log_2 (4) + 1 = 2 + 1 = 3 \]
The point \((6, 3)\) is on the graph of \( f \).

c. \[ f(x) = 4 \]
\[ \log_2 (x-2) + 1 = 4 \]
\[ \log_2 (x-2) = 3 \]
\[ x - 2 = 2^3 \]
\[ x = 10 \]
The solution set is \( \{10\} \). The point \((10, 4)\) is on the graph of \( f \).

d. \[ f(x) = 0 \]
\[ \log_2 (x-2) + 1 = 0 \]
\[ \log_2 (x-2) = -1 \]
\[ x - 2 = 2^{-1} \]
\[ x = \frac{1}{2} \]
Based on the graph drawn in part (a), \( f(x) > 0 \) when \( x > \frac{5}{2} \). The solution set is \( \left\{ x \mid x > \frac{5}{2} \right\} \) or \( \left(\frac{5}{2}, \infty\right) \).
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e. \( f(x) = \log_2(x-2) + 1 \)
\[ y = \log_2(x-2) + 1 \]
\[ x = \log_2(y-2) + 1 \] \text{Inverse}
\[ x - 1 = \log_2(y-2) \]
\[ y - 2 = 2^{x-1} \]
\[ y = 2^{x-1} + 2 \]
\[ f^{-1}(x) = 2^{x-1} + 2 \]

84. a. \( f(x) = \log_3(x+1) - 4 \)
Using the graph of \( \log_3 x \), shift the graph left 1 unit and down 4 units.

b. \( f(8) = \log_3(8+1) - 4 \)
\[ = \log_3(9) - 4 = 2 - 4 = -2 \]
The point \((8, -2)\) is on the graph of \( f \).

c. \( f(x) = -3 \)
\[ \log_3(x+1) - 4 = -3 \]
\[ \log_3(x+1) = 1 \]
\[ x + 1 = 3^1 \]
\[ x + 1 = 3 \]
\[ x = 2 \]
The solution set is \( \{2\} \). The point \((2, -3)\) is on the graph of \( f \).

d. \( f(x) = 0 \)
\[ \log_3(x+1) - 4 = 0 \]
\[ \log_3(x+1) = 4 \]
\[ x + 1 = 3^4 \]
\[ x + 1 = 81 \]
\[ x = 80 \]
Based on the graph in part (a), \( f(x) < 0 \) for \(-1 < x < 80 \). The solution set is \( \{x \mid -1 < x < 80 \} \) or \((-1, 80)\).

e. \( f(x) = \log_3(x+1) - 4 \)
\[ y = \log_3(x+1) - 4 \]
\[ x = \log_3(y+1) \]
\[ x + 4 = \log_3(y+1) \]
\[ y + 1 = 3^{x+4} \]
\[ y = 3^{x+4} - 1 \]
\[ f^{-1}(x) = 3^{x+4} - 1 \]

85. \( h(300) = (30(0) + 8000) \log \left( \frac{760}{300} \right) \)
\[ = 3229.5 \text{ meters} \]

86. \( h(500) = (30(5) + 8000) \log \left( \frac{760}{500} \right) \)
\[ = 1482 \text{ meters} \]

87. \( P = 25e^{0.1d} \)
a. \( P = 25e^{0.1(4)} = 25e^{0.4} = 37.3 \text{ watts} \)
b. \( 50 = 25e^{0.1d} \)
\[ 2 = e^{0.1d} \]
\[ \ln 2 = 0.1d \]
\[ d = \frac{\ln 2}{0.1} = 6.9 \text{ decibels} \)
Chapter 5 Review Exercises

88. $L = 9 + (5.1) \log d$
   a. $L = 9 + (5.1) \log 3.5 = 11.77$
   b. $14 = 9 + (5.1) \log d$
      
      $5 = (5.1) \log d$
      
      $\log d = \frac{5}{5.1} = 0.9804$
      
      $d = 10^{0.9804} = 9.56$ inches

89. a. $n = \frac{\log 10,000 - \log 90,000}{\log(1 - 0.20)} = 9.85$ years
   b. $n = \frac{\log (0.5i) - \log(i)}{\log(1 - 0.15)}$
      
      $\log (0.5i) \over \log 0.85 = \log 0.5 \over \log 0.85 = 4.27$ years

90. In 18 years, $A = 10,000 \left(1 + \frac{0.04}{2}\right)^{(2)(18)}$
    
    $= 10,000(1.02)^{36}$
    
    $= $20,398.87

   The effective interest rate is computed as follows:
   When $t = 1$, $A = 10,000 \left(1 + \frac{0.04}{2}\right)^{(2)(1)}$
    
    $= 10,000(1.02)^2$
    
    $= $10,404

Note, $\frac{10,404 - 10,000}{10,000} = \frac{404}{10,000} = 0.0404$, so the effective interest rate is 4.04%.

In order for the bond to double in value, we have the equation: $A = 2P$.

$10,000 \left(1 + \frac{0.04}{2}\right)^{2t} = 20,000$

$(1.02)^{2t} = 2$

$2t \ln 1.02 = \ln 2$

$t = \frac{\ln 2}{2 \ln 1.02} = 17.5$ years

91. $P = A \left(1 + \frac{r}{n}\right)^{nt} = 85,000 \left(1 + \frac{0.04}{2}\right)^{(2)(18)}$
    
    $= $41,668.97

92. a. $5000 = 620.17 e^{r(20)}$
    
    $\frac{5000}{620.17} = e^{20r}$
    
    $\ln \left(\frac{5000}{620.17}\right) = 20r$
    
    $r = \frac{\ln \left(\frac{5000}{620.17}\right)}{20} = 0.10436$
    
    $r = 10.436%$

   b. $A = 4000 e^{0.10436(20)} = $32,249.24

   The bank’s claim is correct.

93. $A = A_0 e^{tr}$

   $0.5A_0 = A_0 e^{t(5600)}$
   
   $0.5 = e^{5600k}$
   
   $\ln 0.5 = 5600k$
   
   $k = \frac{\ln 0.5}{5600}$
   
   $0.05A_0 = A_0 e^{\left(\frac{\ln 0.5}{5600}\right)t}$
   
   $0.05 = e^{\left(\frac{\ln 0.5}{5600}\right)t}$
   
   $\ln 0.05 = \left(\frac{\ln 0.5}{5600}\right)t$
   
   $t = \frac{\ln 0.05}{\left(\frac{\ln 0.5}{5600}\right)} = 24,203$

   The man died approximately 24,203 years ago.

94. Using $u = T + (u_0 - T)e^{kt}$, with $t = 5$, $T = 70$, $u_0 = 450$, and $u = 400$.

   $400 = 70 + (450 - 70)e^{k(5)}$
   
   $330 = 380 e^{5k}$
   
   $330 = 380 e^{5k}$
   
   $\ln \left(\frac{330}{380}\right) = 5k$
   
   $k = \frac{\ln \left(\frac{330}{380}\right)}{5}$

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Find time for temperature of 150°F:

\[ 150 = 70 + (450 - 70)e^{\frac{\ln(330/380)}{5}}t \]

\[ 80 = 380e^{\frac{\ln(330/380)}{5}}t \]

\[ \ln \left( \frac{80}{380} \right) = \ln \left( \frac{330}{380} \right) \cdot \frac{t}{5} \]

\[ t = \frac{\ln \left( \frac{80}{380} \right)}{\ln \left( \frac{330}{380} \right)/5} = 55.22 \]

The temperature of the skillet will be 150°F after approximately 55.22 minutes (or 55 minutes, 13 seconds).

95. \( P_0 = 6,451,058,790 \), \( k = 0.0115 \), and \( t = 2015 - 2005 = 10 \)

\[ P = P_0e^{kt} = 6,451,058,790e^{0.0115 \cdot 10} = 7,237,271,501 \text{ people} \]

96. \( A = A_0e^{kt} \)

\[ 0.5A_0 = A_0e^{k \cdot 0.5} \]

\[ 0.5 = e^{\ln 0.5 \cdot k} \]

\[ k = \frac{\ln 0.5}{0.5} = 5.27 \]

In 20 years: \( A = 100e^{50.27} = 7.204 \text{ grams} \)

In 40 years: \( A = 100e^{100.54} = 0.519 \text{ grams} \)

97. \( A = P \left( 1 + \frac{r}{n} \right)^{nt} \)

\[ A = 319 \left( 1 + \frac{0.0425}{1} \right)^{10} \]

\[ = 319(1.0425)^{10} = 483.67 \]

The government would have to pay back approximately $483.67 billion in 2015.

98. a. \( P(0) = \frac{0.8}{1 + 1.67e^{-0.16 \cdot 0}} = \frac{0.8}{1 + 1.67} = 0.3 \)

In 2006, about 30% of cars had a GPS.

b. The maximum proportion is the carrying capacity, \( c = 0.8 = 80\% \).

c. \( Y_t = \frac{0.8}{1 + 1.67e^{-0.16 \cdot t}} \)

\( t \) such that \( P(t) = 0.75 \).

\[ \frac{0.8}{1 + 1.67e^{-0.16 \cdot t}} = 0.75 \]

\[ 0.8 = 0.75 \left( 1 + 1.67e^{-0.16 \cdot t} \right) \]

\[ 0.8 - 1 = 1.67e^{-0.16 \cdot t} \]

\[ 0.8 - 0.75 = e^{-0.16 \cdot t} \]

\[ \ln \left( \frac{0.8}{0.75} \right) = -0.16t \]

\[ t = \frac{\ln \left( \frac{0.8}{0.75} \right)}{-0.16} = 20.13 \]

Note that 2006 + 20.13 = 2026.13, so 75% of new cars will have GPS in 2026.
Chapter 5 Review Exercises

99. a.

b. Using EXPonential REGression on the data yields: \( y = (165.73)(0.9951)^x \)

c. \( Y_i = (165.73)(0.9951)^x \)

d. Find \( x \) when \( y = 110 \).

\[
(165.73)(0.9951)^x = 110
\]

\[
(0.9951)^x = \frac{110}{165.73}
\]

\[
x \ln 0.9951 = \ln \left( \frac{110}{165.73} \right)
\]

\[
x = \frac{\ln 0.9951}{\ln 0.9951} = 83
\]

Therefore, it will take approximately 83 seconds for the probe to reach a temperature of 110°F.

100. a.

b. Using LnREGression on the data yields: \( y = 18.9028 - 7.0963 \ln x \) where \( y \) = wind chill and \( x \) = wind speed.

c. \( Y_i = 18.9028 - 7.0963 \ln x \)

d. If \( x = 23 \), then

\[
y = 18.9028 - 7.0963 \ln 23 = -3 \degree F.
\]

101. a.

b. Using LOGISTIC REGression on the data yields:

\[
C = \frac{46.93}{1 + 21.273e^{-0.7306t}}
\]

c. \( Y_i = \frac{46.93}{1 + 21.273e^{-0.7306t}} \)

d. As \( t \to \infty \), \( 21.2733e^{-0.7306t} \to 0 \), which means \( 1 + 21.2733e^{-0.7306t} \to 1 \), so

\[
C = \frac{46.9292}{1 + 21.2733e^{-0.7306t}} \to 46.9292
\]

Therefore, according to the function, a maximum of about 47 people can catch the cold.

In reality, all 50 people living in the town might catch the cold.
Chapter 5: Exponential and Logarithmic Functions

e. Find \( t \) when \( C = 10 \).

\[
\begin{align*}
46.9292 & = 10\left(1 + 21.2733e^{-0.7306t}\right) \\
46.9292 & = 10 + 212.733e^{-0.7306t} \\
46.9292 & = 1 + 21.2733e^{-0.7306t} \\
46.9292 & = 10 + 212.733e^{-0.7306t} \\
46.9292 & = 1 + 21.2733e^{-0.7306t} \\
46.9292 & = 10 + 212.733e^{-0.7306t} \\
\ln\left(\frac{46.9292}{21.2733}\right) & = -0.7306t \\
\ln\left(\frac{46.9292}{21.2733}\right) & = t \\
-0.7306 & = t
\end{align*}
\]

Therefore, after approximately 2.4 days (during the 10th hour on the 3rd day), 10 people had caught the cold.

f. Find \( t \) when \( C = 46 \).

\[
\begin{align*}
46.9292 & = 46\left(1 + 21.2733e^{-0.7306t}\right) \\
46.9292 & = 46 + 212.733e^{-0.7306t} \\
46.9292 & = 46 + 212.733e^{-0.7306t} \\
46.9292 & = 46 + 212.733e^{-0.7306t} \\
46.9292 & = 46 + 212.733e^{-0.7306t} \\
\ln\left(\frac{46.9292}{21.2733}\right) & = -0.7306t \\
\ln\left(\frac{46.9292}{21.2733}\right) & = t \\
-0.7306 & = t
\end{align*}
\]

Therefore, after approximately 9.5 days (during the 12th hour on the 10th day), 46 people had caught the cold.

Chapter 5 Test

1. \( f(x) = \frac{x+2}{x-2} \quad g(x) = 2x+5 \)

The domain of \( f \) is \( \{x \mid x \neq 2\} \).

The domain of \( g \) is all real numbers.

a. \( (f \circ g)(x) = f(g(x)) \)

\[
\begin{align*}
(f \circ g)(x) & = f(2x+5) \\
& = (2x+5)+2 \\
& = 2x+7 \\
\text{Domain} & = \left\{x \mid x \neq -\frac{3}{2}\right\}
\end{align*}
\]

b. \( (g \circ f)(-2) = g(f(-2)) \)

\[
\begin{align*}
(g \circ f)(-2) & = g(1+2) \\
& = g(3) \\
& = 2(0)+5 \\
& = 5
\end{align*}
\]

c. \( (f \circ g)(-2) = f(g(-2)) = f(2(-2)+5) \)

\[
\begin{align*}
(f \circ g)(-2) & = f(1+2) \\
& = 3 \\
\end{align*}
\]

2. a. Graph \( y = 4x^2 + 3 \):

![Graph of y = 4x^2 + 3](image)

The function is not one-to-one because it fails the horizontal line test. A horizontal line (for example, \( y = 4 \)) intersects the graph twice.
b. Graph \( y = \sqrt{x+3} - 5 \):

The function is one-to-one because it passes the horizontal line test. Every horizontal line intersects the graph at most once.

3. \( f(x) = \frac{2}{3x-5} \)
   \[ y = \frac{2}{3x-5} \]
   \[ x = \frac{2}{3y-5} \text{ Inverse} \]
   \[ x(3y-5) = 2 \]
   \[ 3xy - 5x = 2 \]
   \[ 3xy = 5x + 2 \]
   \[ y = \frac{5x + 2}{3x} \]
   \[ f^{-1}(x) = \frac{5x + 2}{3x} \]
   Domain of \( f \) = Range of \( f^{-1} \)
   \[ = \{ x \mid x \neq \frac{5}{3} \} \].
   Range of \( f \) = Domain of \( f^{-1} \)
   \[ = \{ x \mid x \neq 0 \} \]

4. If the point \((3, -5)\) is on the graph of \( f \), then the point \((-5, 3)\) must be on the graph of \( f^{-1} \).

5. \( 3^4 = 243 \)
   \[ 3^4 = 3^4 \]
   \[ x = 5 \]

6. \( \log_b 16 = 2 \)
   \[ b^2 = 16 \]
   \[ b = \pm \sqrt{16} = \pm 4 \]
   Since the base of a logarithm must be positive, the only viable solution is \( b = 4 \).

7. \( \log_5 x = 4 \)
   \[ x = 5^4 \]
   \[ x = 625 \]

8. \( e^3 + 2 = 22.086 \)

9. \( \log 20 = 1.301 \)

10. \( \log_3 21 = \frac{\ln 21}{\ln 3} = 2.771 \)

11. \( \ln 133 = 4.890 \)

12. \( f(x) = 4^{x+1} - 2 \)
   a. Domain: \((-\infty, \infty)\)
   b. Using the graph of \( y = 4^x \), shift the graph 1 unit to the left, and shift 2 units down.
   c. Range: \((-2, \infty)\)
      Horizontal Asymptote: \( y = -2 \)
   d. \( f(x) = 4^{x+1} - 2 \)
      \[ y = 4^{x+1} - 2 \]
      \[ x = 4^{x+1} - 2 \text{ Inverse} \]
      \[ x + 2 = 4^{y+1} \]
      \[ y + 1 = \log_4 (x + 2) \]
      \[ y = \log_4 (x + 2) - 1 \]
   e. Range of \( f \) = Domain \( f^{-1} \): \((-2, \infty)\)
      Domain of \( f \) = Range of \( f^{-1} \): \((-\infty, \infty)\)
   f. Using the graph of \( y = \log_4 x \), shift the graph 2 units to the left, and shift down 1
Chapter 5: Exponential and Logarithmic Functions

13. \( f(x) = 1 - \log_5 (x-2) \)
   a. Domain: \((2, \infty)\)
   b. Using the graph of \( y = \log_5 x \), shift the graph to the right 2 units, reflect vertically about the y-axis, and shift up 1 unit.
   c. Range: \((-\infty, \infty)\)
   Vertical Asymptote: \( x = 2 \)
   d. \( f(x) = 1 - \log_5 (x-2) \)
   \( y = 1 - \log_5 (x-2) \)
   \( x = 1 - \log_5 (y-2) \) \( \text{Inverse} \)
   \( x-1 = -\log_5 (y-2) \)
   \( 1-x = \log_5 (y-2) \)
   \( y-2 = 5^{x-1} \)
   \( y = 5^{x-1} + 2 \)
   \( f^{-1}(x) = 5^{x-1} + 2 \)
   e. Range of \( f = \text{Domain of} \ f^{-1} : (-\infty, \infty) \)
   Domain of \( f = \text{Range of} \ f^{-1} : (2, \infty) \)
   f. Using the graph of \( y = 5^x \), reflect the graph horizontally about the y-axis, shift to the right 1 unit, and shift up 2 units.

14. \( 5^{x+2} = 125 \)
   \( 5^{x+2} = 5^3 \)
   \( x+2 = 3 \)
   \( x = 1 \)
   The solution set is \( \{1\} \).

15. \( \log(x+9) = 2 \)
   \( x+9 = 10^2 \)
   \( x+9 = 100 \)
   \( x = 91 \)
   The solution set is \( \{91\} \).

16. \( 8 - 2e^{-x} = 4 \)
   \( -2e^{-x} = -4 \)
   \( e^{-x} = 2 \)
   \( -x = \ln 2 \)
   \( x = -\ln 2 = -0.693 \)
   The solution set is \( \{-\ln 2\} = \{-0.693\} \).

17. \( \log(x^2 + 3) = \log(x+6) \)
   \( x^2 + 3 = x + 6 \)
   \( x^2 - x - 3 = 0 \)
   \( x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)} = \frac{1 \pm \sqrt{13}}{2} \)
   The solution set is \( \left\{ \frac{1 - \sqrt{13}}{2}, \frac{1 + \sqrt{13}}{2} \right\} = \{-1.303, 2.303\} \).
Chapter 5 Chapter Test

18. \[7^{x+3} = e^x\]
   \[\ln 7^{x+3} = \ln e^x\]
   \[(x+3) \ln 7 = x\]
   \[x \ln 7 + 3 \ln 7 = x\]
   \[x \ln 7 - x = -3 \ln 7\]
   \[x(\ln 7 - 1) = -3 \ln 7\]
   \[x = \frac{3 \ln 7}{\ln 7 - 1} = \frac{3 \ln 7}{1 - \ln 7} = -6.172\]
   The solution set is \(\{\frac{3 \ln 7}{1 - \ln 7}\} = \{-6.172\}\).

19. \[\log_2(x-4) + \log_2(x+4) = 3\]
   \[\log_2[(x-4)(x+4)] = 3\]
   \[\log_2(x^2 - 16) = 3\]
   \[x^2 - 16 = 2^3\]
   \[x^2 - 16 = 8\]
   \[x^2 = 24\]
   \[x = \pm \sqrt{24} = \pm 2\sqrt{6}\]
   Because \(x = -2\sqrt{6}\) results in a negative arguments for the original logarithms, the only viable solution is \(x = 2\sqrt{6}\). That is, the solution set is \(\{2\sqrt{6}\} = \{4.899\}\).

20. \[\log_2\left(\frac{4x^3}{x^2 - 3x - 18}\right)\]
   \[= \log_2\left(\frac{2^2x^3}{(x+3)(x-6)}\right)\]
   \[= \log_2\left(2^2x^3\right) - \log_2\left[(x-6)(x+3)\right]\]
   \[= \log_2 2^2 + \log_2 x^3 - \left[\log_2(x-6) + \log_2(x+3)\right]\]
   \[= 2 + 3 \log_2 x - \log_2(x-6) - \log_2(x+3)\]

21. \[A = A_0e^{rt}\]
   \[34 = 50e^{30k}\]
   \[0.68 = e^{30k}\]
   \[\ln 0.68 = 30k\]
   \[k = \frac{\ln 0.68}{30}\]
   Thus, the decay model is \(A = 50e^{-\left(\frac{\ln 0.68}{30}\right)t}\).
   We need to find \(t\) when \(A = 2:\)

22. a. Note that 8 months = \(\frac{2}{3}\) year. Thus,
   \[P = 1000, r = 0.05, n = 12,\] and \(t = \frac{2}{3}\).
   So, \[A = 1000\left(1 + \frac{0.05}{12}\right)^{\left(\frac{12}{3}\right)}\]
   \[= 1000\left(1 + \frac{0.05}{12}\right)^{4}\]
   \[= 1033.82\]

b. Note that 9 months = \(\frac{3}{4}\) year. Thus,
   \[A = 1000, r = 0.05, n = 4,\] and \(t = \frac{3}{4}\).
   So, \[A = 1000\left(1 + \frac{0.05}{4}\right)^{\left(\frac{4}{3}\right)}\]
   \[= 1000\left(1 + \frac{0.05}{4}\right)^{3}\]
   \[= 963.42\]

c. \(r = 0.06\) and \(n = 1\). So,
   \[2A_0 = A_0\left(1 + \frac{0.06}{1}\right)^t\]
   \[2A_0 = A_0(1.06)^t\]
   \[2 = (1.06)^t\]
   \[t = \log_{1.06} 2 = \frac{\ln 2}{\ln 1.06} = 11.9\]
   It will take about 11.9 years to double your money under these conditions.
23. a. \(80 = 10 \log \left( \frac{I}{10^{-12}} \right)\)
\[8 = \log \left( \frac{I}{10^{-12}} \right)\]
\[8 = \log I - \log 10^{-12}\]
\[8 = \log I - (-12)\]
\[8 = \log I + 12\]
\[-4 = \log I\]
\[I = 10^{-4} = 0.0001\]
If one person shouts, the intensity is \(10^{-4}\) watts per square meter. Thus, if two people shout at the same time, the intensity will be \(2 \times 10^{-4}\) watts per square meter. Thus, the loudness will be
\[D = 10 \log \left( \frac{2 \times 10^{-4}}{10^{-12}} \right) = 10 \log \left( 2 \times 10^8 \right) = 83\]

b. Let \(n\) represent the number of people who must shout. Then the intensity will be \(n \times 10^{-4}\). If \(D = 125\), then
\[125 = 10 \log \left( \frac{n \times 10^{-4}}{10^{-12}} \right)\]
\[125 = 10 \log \left( n \times 10^8 \right)\]
\[12.5 = \log \left( n \times 10^8 \right)\]
\[n \times 10^8 = 10^{12.5}\]
\[n = 10^{4.5} = 31,623\]
About 31,623 people would have to shout at the same time in order for the resulting sound level to meet the pain threshold.

c. \(f(x + h) = 2(x + h)^2 - 3(x + h) + 1\)
\[= 2(x^2 + 2xh + h^2) - 3x - 3h + 1\]
\[= 2x^2 + 4xh + 2h^2 - 3x - 3h + 1\]

3. \(x^2 + y^2 = 1\)

a. \(\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1; \left(\frac{1}{2}, \frac{1}{2}\right)\) is not on the graph.

b. \(\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1; \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\) is on the graph.

4. \(3(x - 2) = 4(x + 5)\)
\[3x - 6 = 4x + 20\]
\[-26 = x\]
The solution set is \(\{-26\}\).

5. \(2x - 4y = 16\)
\[x\text{-intercept: } 2x - 4(0) = 16\]
\[2x = 16\]
\[x = 8\]

\[y\text{-intercept: } 2(0) - 4y = 16\]
\[-4y = 16\]
\[y = -4\]

6. a. \(f(x) = -x^2 + 2x - 3\); \(a = -1, b = 2, c = -3\).
Since \(a = -1 < 0\), the graph opens down.
The \(x\)-coordinate of the vertex is
\[x = -\frac{b}{2a} = -\frac{2}{2(-1)} = -\frac{2}{-2} = 1\].

Chapter 5 Cumulative Review

1. The graph represents a function since it passes the Vertical Line Test.
The function is not a one-to-one function since the graph fails the Horizontal Line Test.

2. \(f(x) = 2x^2 - 3x + 1\)
   a. \(f(3) = 2(3)^2 - 3(3) + 1 = 18 - 9 + 1 = 10\)
   b. \(f(-x) = 2(-x)^2 - 3(-x) + 1 = 2x^2 + 3x + 1\)
Chapter 5 Cumulative Review

The y-coordinate of the vertex is
\[ f\left(-\frac{b}{2a}\right) = f(1) \]
\[ = -1^2 + 2(1) - 3 \]
\[ = -1 + 2 - 3 \]
\[ = -2 \]
Thus, the vertex is \((1, -2)\).
The axis of symmetry is the line \(x = 1\).
The discriminant is:
\[ b^2 - 4ac = 2^2 - 4(-1)(-3) = 4 - 12 = -8 < 0 \]
The graph has no \(x\)-intercepts.
The y-intercept is \(f(0) = -0^2 + 2(0) - 3 = -3\).

b. The graph of \(f(x) = -x^2 + 2x - 3\) indicates that \(f(x) \leq 0\) for all values of \(x\). Thus, the solution to \(f(x) \leq 0\) is \((-\infty, \infty)\).

7. Given that the graph of \(f(x) = ax^2 + bx + c\) has vertex \((4, -8)\) and passes through the point \((0, 24)\), we can conclude \(-\frac{b}{2a} = 4\), \(f(4) = -8\), and \(f(0) = 24\). Notice that
\[ f(0) = 24 \]
\[ a(0)^2 + b(0) + c = 24 \]
\[ c = 24 \]
Therefore, \(f(x) = ax^2 + bx + c = ax^2 + bx + 24\).
Furthermore, \(-\frac{b}{2a} = 4\), so that \(b = -8a\), and

\[ f(4) = -8 \]
\[ a(4)^2 + b(4) + 24 = -8 \]
\[ 16a + 4b + 24 = -8 \]
\[ 16a + 4b = -32 \]
\[ 4a + b = -8 \]
Replacing \(b\) with \(-8a\) in this equation yields
\[ 4a - 8a = -8 \]
\[ -4a = -8 \]
\[ a = 2 \]
So \(b = -8a = -8(2) = -16\).
Therefore, we have the function \(f(x) = 2x^2 - 16x + 24\).

8. \(f(x) = 3(x+1)^3 - 2\)
Using the graph of \(y = x^3\), shift the graph 1 unit to the left, stretch vertically by a factor of 3, and shift 2 units down.

9. \(f(x) = x^2 + 2\) \(g(x) = \frac{2}{x-3}\)
\[ f(g(x)) = f\left(\frac{2}{x-3}\right) \]
\[ = \left(\frac{2}{x-3}\right)^2 + 2 \]
\[ = \frac{4}{(x-3)^2} + 2 \]
The domain of \(f\) is \(\{x| x \text{ is any real number}\}\).
The domain of \(g\) is \(\{x| x \neq 3\}\).
So, the domain of \(f(g(x))\) is \(\{x| x \neq 3\}\).
\[ f(g(5)) = \frac{4}{(5-3)^2} + 2 = \frac{4}{2^2} + 2 = \frac{4}{4} + 2 = 3 \]

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10. \( f(x) = 4x^3 + 9x^2 - 30x - 8 \)

a. The graph of \( Y_1 = 4x^3 + 9x^2 - 30x - 8 \) appears to indicate zeros at \( x = -4 \) and \( x = 2 \).

\[
\begin{align*}
75 & \quad 3 \\
-50 & \quad -6
\end{align*}
\]

\[
f(-4) &= 4(-4)^3 + 9(-4)^2 - 30(-4) - 8 \\
      &= -256 + 144 + 120 - 8 \\
      &= 0
\]

\[
f(2) &= 4(2)^3 + 9(2)^2 - 30(2) - 8 \\
     &= 32 + 36 - 60 - 8 \\
     &= 0
\]

Therefore, \( x = -4 \) and \( x = 2 \) are real zeros for \( f \).

Using synthetic division:

\[
\begin{array}{ccc|c}
4 & 9 & -30 & -8 \\
   & 8 & 34 & 8 \\
\end{array}
\]

\[
f(x) &= 4x^3 + 9x^2 - 30x - 8 \\
     &= (x-2)(4x^2 + 17x + 4) \\
     &= (x-2)(x+4)(4x+1)
\]

Therefore, \( x = 2, x = -\frac{1}{4} \) and \( x = -4 \) are real zeros of \( f \).

b. \( f \) has \( x \)-intercepts at \( x = 2, x = -\frac{1}{4} \) and \( x = -4 \).

\( f \) has \( y \)-intercept at \( f(0) = 4 \cdot 0^3 + 9 \cdot 0^2 - 30 \cdot 0 - 8 = -8 \)

c. Use MAXIMUM to determine that \( f \) has a local maximum at the point \((-2.5, 60.75)\).

\[
\begin{align*}
75 & \quad 3 \\
-50 & \quad -6
\end{align*}
\]

11. a. \( g(x) = 3^x + 2 \)

Using the graph of \( y = 3^x \), shift up 2 units.

\[
\begin{align*}
(0, 3) & \quad (3, 0) \\
(0, 2) & \quad (2, 0)
\end{align*}
\]

Use MINIMUM to determine that \( f \) has a local minimum at the point \((1, -25)\).

Thus, \( f \) has a local maximum of 60.75 that occurs at \( x = -2.5 \), and \( f \) has a local minimum of \(-25 \) that occurs at \( x = 1 \).

d. Graphing by hand:

The graph of \( f \) is above the \( x \)-axis for \(-4, -\frac{1}{4}\) and \((2, \infty)\).

The graph of \( f \) is below the \( x \)-axis for \((-\infty, -4)\).
Domain of \( g \): \((-\infty, \infty)\)
Range of \( g \): \((2, \infty)\)
Horizontal Asymptote for \( g \): \( y = 2 \)

b. \( g(x) = 3^x + 2 \)
\[ y = 3^x + 2 \]
\[ x = 3^y + 2 \quad \text{Inverse} \]
\[ x - 2 = 3^y \]
\[ y = \log_3(x - 2) \]
\[ g^{-1}(x) = \log_3(x - 2) \]
Domain of \( g^{-1} \): \((2, \infty)\)
Range of \( g^{-1} \): \((-\infty, \infty)\)
Vertical Asymptote for \( g^{-1} \): \( x = 2 \)

c. 

12. \( 4^{x-3} = 8^{2x} \)
\[ (2^2)^{x-3} = (2^3)^{2x} \]
\[ 2^{2x-6} = 2^{6x} \]
\[ 2x - 6 = 6x \]
\[ -6 = 4x \]
\[ x = -\frac{6}{4} = -\frac{3}{2} \]
The solution set is \( \left\{ -\frac{3}{2} \right\} \).

13. \( \log_3(x + 1) + \log_3(2x - 3) = \log_9 9 \)
\[ \log_3 \left( \frac{(x + 1)(2x - 3)}{1} \right) = 1 \]
\[ (x + 1)(2x - 3) = 3^1 \]
\[ 2x^2 - x - 3 = 3 \]
\[ 2x^2 - x - 6 = 0 \]
\[ (2x + 3)(x - 2) = 0 \]
\[ x = -\frac{3}{2} \quad \text{or} \quad x = 2 \]

Since \( \log_3 \left( -\frac{3}{2} + 1 \right) = \log_3 \left( -\frac{1}{2} \right) \) is undefined
the solution set is \( \{2\} \).

14. a. \( \log_3 (x + 2) = 0 \)
\[ x + 2 = 3^0 \]
\[ x + 2 = 1 \]
\[ x = -1 \]
The solution set is \( \{-1\} \).

b. \( \log_3 (x + 2) > 0 \)
\[ x + 2 > 3^0 \]
\[ x + 2 > 1 \]
\[ x > -1 \]
The solution set is \( \{x | x > -1\} \) or \((-1, \infty)\).

c. \( \log_3 (x + 2) = 3 \)
\[ x + 2 = 3^3 \]
\[ x + 2 = 27 \]
\[ x = 25 \]
The solution set is \( \{25\} \).

15. a. 

b. Logarithmic: \( y = 49.293 - 10.563 \ln x \)

c. Answers will vary.
Chapter 5: Exponential and Logarithmic Functions

Chapter 5 Projects

Project I – Internet-based Project - Ans will vary

Project II

a. Newton’s Law of Cooling:
   \[ u(t) = T + (u_0 - T)e^{kt} \], \( k < 0 \)

   Container 1: \( u_0 = 200^\circ F, T = 70^\circ F, u(30)=100^\circ F, t = 30 \text{ mins.} \)
   \[ 100 = 70 + (200 - 70)e^{30k} \]
   \[ 30 = 130e^{30k} \]
   \[ \frac{30}{130} = e^{30k} \]
   \[ 30k = \ln\left(\frac{30}{130}\right) \]
   \[ k = \frac{\ln\left(\frac{30}{130}\right)}{30} = -0.04888 \]
   \[ u_1(t) = 70 + 130e^{-0.04888t} \]

   Container 2: \( u_0 = 200^\circ F, T = 60^\circ F, u(25)=110^\circ F, t = 25 \text{ mins.} \)
   \[ 100 = 60 + (200 - 60)e^{25k} \]
   \[ 50 = 140e^{25k} \]
   \[ \frac{50}{140} = e^{25k} \]
   \[ 25k = \ln\left(\frac{50}{140}\right) \]
   \[ k = \frac{\ln\left(\frac{50}{140}\right)}{25} = -0.04118 \]
   \[ u_2(t) = 60 + 140e^{-0.04118t} \]

   Container 3: \( u_0 = 200^\circ F, T = 65^\circ F, u(20)=120^\circ F, t = 20 \text{ mins.} \)

b. We need time for each of the problems, so solve for \( t \) first then substitute the specific values for each container:
   \[ u = T + (u_0 - T)e^{kt} \]
   \[ u - T = (u_0 - T)e^{kt} \quad u - T = u_0 - T \]
   \[ kt = \ln\left(\frac{u - T}{u_0 - T}\right) \Rightarrow t = \frac{\ln\left(\frac{u - T}{u_0 - T}\right)}{k} \]

   Container 1:
   \[ t = \frac{\ln\left(\frac{130 - 70}{200 - 70}\right)}{-0.04888} = 15.82 \text{ minutes} \]

   Container 2:
   \[ t = \frac{\ln\left(\frac{130 - 60}{200 - 60}\right)}{-0.04118} = 16.83 \text{ minutes} \]

   Container 3:
   \[ t = \frac{\ln\left(\frac{130 - 65}{200 - 65}\right)}{-0.04490} = 16.28 \text{ minutes} \]

c. Container 1:
   \[ t = \frac{\ln\left(\frac{110 - 70}{130 - 70}\right)}{-0.04888} = 8.295 \]
   It will remain between 110\(^\circ\) and 130\(^\circ\) for about 8.3 minutes.
Container 2:
\[
\ln \left( \frac{110 - 60}{130 - 60} \right) \approx -0.04118
\]
It will remain between 110º and 130º for about 8.17 minutes.

Container 3:
\[
\ln \left( \frac{110 - 65}{130 - 65} \right) \approx -0.04490
\]
It will remain between 110º and 130º for about 8.19 minutes.

d. All three graphs basically lie on top of each other.

e. Container 1 would be the best. It cools off the quickest but it stays in a warm beverage range the longest.

f. Since all three containers are within seconds of each other in cooling and staying warm, the cost would have an effect. The cheaper one would be the best recommendation.

Project III

<table>
<thead>
<tr>
<th>Solder Joint Strain, εp</th>
<th>X=ln(εp)</th>
<th>Fatigue Cycles, Nf</th>
<th>Y=ln(Nf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>-4.605</td>
<td>10,000</td>
<td>9.210</td>
</tr>
<tr>
<td>0.035</td>
<td>-3.352</td>
<td>1000</td>
<td>6.908</td>
</tr>
<tr>
<td>0.1</td>
<td>-2.303</td>
<td>100</td>
<td>4.605</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.916</td>
<td>10</td>
<td>2.303</td>
</tr>
<tr>
<td>1.5</td>
<td>0.405</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

1. The shape becomes exponential.

2. The shape became linear.

3. \[ Y = -1.84X + 0.63 \]

5. \[ Y = -1.84X + 0.63 \]
   \[ \ln(Nf) = -1.84\ln(\varepsilon p) + 0.63 \]
   \[ \ln(Nf) = \ln((\varepsilon p)^{-1.84}) + \ln(e^{0.63}) \]
   \[ Nf = ((\varepsilon p)^{-1.84})(e^{0.63}) \]
   \[ Nf = e^{0.63}(\varepsilon p)^{-1.84} \]

6. \[ Nf = e^{0.63}(0.02)^{-1.84} \]
   \[ Nf = 2510.21 cycles \]
   \[ Nf = e^{0.63}(\varepsilon p)^{-1.84} \]
   \[ 3000 = e^{0.63}(\varepsilon p)^{-1.84} \]
   \[ 3000 = (\varepsilon p)^{0.63} \]
   \[ \varepsilon p = \left( \frac{3000}{e^{0.63}} \right)^{\frac{1}{1.84}} \]
   \[ \varepsilon p = 0.018 \]

7. \[ Nf = e^{0.63}(\varepsilon p)^{-1.84} \quad \varepsilon p = 1.41(Nf)^{-543} \]
   \[ Nf = 1.88(\varepsilon p)^{-1.84} \quad \varepsilon p = 1.41(3000)^{-543} \]
   \[ Nf = (\varepsilon p)^{-1.84} \quad \varepsilon p = 0.018 \]
   \[ \varepsilon p = (0.53Nf)^{\frac{1}{1.84}} \]
   \[ \varepsilon p = (0.53Nf)^{-543} \]
   \[ \varepsilon p = 1.41(Nf)^{-543} \]