Chapter 11
Systems of Equations and Inequalities

Section 11.1

1. \[3x + 4 = 8 - x\]
   \[4x = 4\]
   \[x = 1\]
   The solution set is \{1\}.

2. a. \[3x + 4y = 12\]
   \[x\)-intercept: \[3x + 4(0) = 12\]
   \[3x = 12\]
   \[x = 4\]
   y-intercept: \[3(0) + 4y = 12\]
   \[4y = 12\]
   \[y = 3\]
   A parallel line would have slope \(-\frac{3}{4}\).

   b. \[3x + 4y = 12\]
   \[4y = -3x + 12\]
   \[y = -\frac{3}{4}x + 3\]

3. inconsistent
4. consistent; independent
5. \((3, -2)\)
6. consistent; dependent
7. \[\begin{align*}
2x - y &= 5 \\
5x + 2y &= 8
\end{align*}\]
   Substituting the values of the variables:
   \[\begin{align*}
2(2) - (-1) &= 4 + 1 = 5 \\
5(2) + 2(-1) &= 10 - 2 = 8
\end{align*}\]
   Each equation is satisfied, so \(x = 2, y = -1\), or \((2, -1)\), is a solution of the system of equations.

8. \[\begin{align*}
3x + 2y &= 2 \\
x - 7y &= -30
\end{align*}\]
   Substituting the values of the variables:
   \[\begin{align*}
3(-2) + 2(4) &= -6 + 8 = 2 \\
(-2) - 7(4) &= -2 - 28 = -30
\end{align*}\]
   Each equation is satisfied, so \(x = -2, y = 4\), or \((-2, 4)\), is a solution of the system of equations.

9. \[\begin{align*}
3x - 4y &= 4 \\
\frac{1}{2}x - 3y &= -\frac{1}{2}
\end{align*}\]
   Substituting the values of the variables:
   \[\begin{align*}
3(2) - 4\left(\frac{1}{2}\right) &= 6 - 2 = 4 \\
\frac{1}{2}(2) - 3\left(\frac{1}{2}\right) &= 1 - \frac{3}{2} = -\frac{1}{2}
\end{align*}\]
   Each equation is satisfied, so \(x = 2, y = \frac{1}{2}\), or \((2, \frac{1}{2})\), is a solution of the system of equations.

10. \[\begin{align*}
2x + \frac{1}{2}y &= 0 \\
3x - 4y &= -\frac{19}{2}
\end{align*}\]
    Substituting the values of the variables, we obtain:
    \[\begin{align*}
2\left(-\frac{1}{2}\right) + \frac{1}{2}(2) &= -1 + 1 = 0 \\
3\left(-\frac{1}{2}\right) - 4(2) &= -\frac{3}{2} - 8 = -\frac{19}{2}
\end{align*}\]
    Each equation is satisfied, so \(x = -\frac{1}{2}, y = 2\), or \((-\frac{1}{2}, 2)\), is a solution of the system of equations.

11. \[\begin{align*}
x - y &= 3 \\
\frac{1}{2}x + y &= 3
\end{align*}\]
    Substituting the values of the variables, we obtain:
12. \[ \begin{align*}
4 - 1 &= 3 \\
\frac{1}{2} (4) + 1 &= 2 + 1 = 3
\end{align*} \]
Each equation is satisfied, so \( x = 4, \ y = 1, \) or (4, 1) , is a solution of the system of equations.

13. \[ \begin{align*}
x - y &= 3 \\
-3x + y &= 1
\end{align*} \]
Substituting the values of the variables:
\[ \begin{align*}
(2) - (-5) &= -2 + 5 = 3 \\
-3(2) + (-5) &= 6 - 5 = 1
\end{align*} \]
Each equation is satisfied, so \( x = -2, \ y = -5, \) or \((-2, -5)\), is a solution of the system of equations.

14. \[ \begin{align*}
3x + 3y + 2z &= 4 \\
x - y - z &= 0 \\
2y - 3z &= -8
\end{align*} \]
Substituting the values of the variables:
\[ \begin{align*}
3(1) + 3(-1) + 2(2) &= 3 - 3 + 4 = 4 \\
1 - (-1) - 2 &= 1 + 1 - 2 = 0 \\
2(-1) - 3(2) &= -2 - 6 = -8
\end{align*} \]
Each equation is satisfied, so \( x = 1, \ y = -1, \ z = 2, \) or \((1, -1, 2)\), is a solution of the system of equations.

15. \[ \begin{align*}
4x - z &= 7 \\
8x + 5y - z &= 0 \\
x - y + 5z &= 6
\end{align*} \]
Substituting the values of the variables:
\[ \begin{align*}
4(2) - 1 &= 8 - 1 = 7 \\
8(2) + 5(-3) - 1 &= 16 - 15 - 1 = 0 \\
-2 - (-3) + 5(1) &= -2 + 3 + 5 = 6
\end{align*} \]
Each equation is satisfied, so \( x = 2, \ y = -3, \ z = 1, \) or \((2, -3, 1)\), is a solution of the system of equations.

16. \[ \begin{align*}
4x - 5z &= 6 \\
5y - z &= -17 \\
x - 6y + 5z &= 24
\end{align*} \]
Substituting the values of the variables:
\[ \begin{align*}
4(4) - 5(2) &= 16 - 10 = 6 \\
5(-3) - (2) &= -15 - 2 = -17 \\
-4 - 6(-3) + 5(2) &= -4 + 18 + 10 = 24
\end{align*} \]
Each equation is satisfied, so \( x = 4, \ y = -3, \ z = 2, \) or \((4, -3, 2)\), is a solution of the system of equations.

17. \[ \begin{align*}
x + y &= 8 \\
x - y &= 4
\end{align*} \]
Solve the first equation for \( y \), substitute into the second equation and solve:
\[ \begin{align*}
y &= 8 - x \\
x - y &= 4
\end{align*} \]
\[ \begin{align*}
x - (8 - x) &= 4 \\
x - 8 + x &= 4 \\
2x &= 12 \\
x &= 6
\end{align*} \]
Since \( x = 6, \ y = 8 - 6 = 2 \). The solution of the system is \( x = 6, \ y = 2 \) or using ordered pairs \((6, 2)\).

18. \[ \begin{align*}
x + 2y &= -7 \\
x + y &= -3
\end{align*} \]
Solve the first equation for \( x \), substitute into the second equation and solve:
\[ \begin{align*}
x &= -7 - 2y \\
x + y &= -3
\end{align*} \]
\[ \begin{align*}
(-7 - 2y) + y &= -3 \\
-7 - y &= -3 \\
-4 &= y
\end{align*} \]
Since \( y = -4 \), \( x = -7 - 2(-4) = 1 \). The solution of the system is \( x = 1, \ y = -4 \) or using an ordered pair, \((1, -4)\).
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19. \[ \begin{cases} 5x - y = 21 \\ 2x + 3y = -12 \end{cases} \]
Multiply each side of the first equation by 3 and add the equations to eliminate \(y\):
\[ \begin{cases} 15x - 3y = 63 \\ 2x + 3y = -12 \end{cases} \]
\[ 17x = 51 \]
\[ x = 3 \]
Substitute and solve for \(y\):
\[ 5(3) - y = 21 \]
\[ 15 - y = 21 \]
\[ -y = 6 \]
\[ y = -6 \]
The solution of the system is \(x = 3, y = -6\) or using ordered pairs \((3, -6)\).

20. \[ \begin{cases} x + 3y = 5 \\ 2x - 3y = -8 \end{cases} \]
Add the equations:
\[ \begin{cases} x + 3y = 5 \\ 2x - 3y = -8 \end{cases} \]
\[ 3x = -3 \]
\[ x = -1 \]
Substitute and solve for \(y\):
\[ -1 + 3y = 5 \]
\[ 3y = 6 \]
\[ y = 2 \]
The solution of the system is \(x = -1, y = 2\) or using ordered pairs \((-1, 2)\).

21. \[ \begin{cases} 3x = 24 \\ x + 2y = 0 \end{cases} \]
Solve the first equation for \(x\) and substitute into the second equation:
\[ \begin{cases} x = 8 \\ x + 2y = 0 \end{cases} \]
\[ 8 + 2y = 0 \]
\[ 2y = -8 \]
\[ y = -4 \]
The solution of the system is \(x = 8, y = -4\) or using ordered pairs \((8, -4)\).

22. \[ \begin{cases} 4x + 5y = -3 \\ -2y = -8 \end{cases} \]
Solve the second equation for \(y\) and substitute into the first equation:
\[ \begin{cases} 4x + 5y = -3 \\ y = 4 \end{cases} \]
\[ 4x + 20 = -3 \]
\[ 4x = -23 \]
\[ x = -\frac{23}{4} \]
The solution of the system is \(x = -\frac{23}{4}, y = 4\) or using ordered pairs \((-\frac{23}{4}, 4)\).

23. \[ \begin{cases} 3x - 6y = 2 \\ 5x + 4y = 1 \end{cases} \]
Multiply each side of the first equation by 2 and each side of the second equation by 3, then add to eliminate \(y\):
\[ \begin{cases} 6x - 12y = 4 \\ 15x + 12y = 3 \end{cases} \]
\[ 21x = 7 \]
\[ x = \frac{1}{3} \]
Substitute and solve for \(y\):
\[ 3(\frac{1}{3}) - 6y = 2 \]
\[ 1 - 6y = 2 \]
\[ -6y = 1 \]
\[ y = -\frac{1}{6} \]
The solution of the system is \(x = \frac{1}{3}, y = -\frac{1}{6}\) or using ordered pairs \((\frac{1}{3}, -\frac{1}{6})\).
24. \[ \begin{align*}
2x + 4y &= \frac{2}{3} \\
3x - 5y &= -10
\end{align*} \]

Multiply each side of the first equation by 5 and each side of the second equation by 4, then add to eliminate \( y \):

\[ \begin{align*}
10x + 20y &= \frac{10}{3} \\
12x - 20y &= -40
\end{align*} \]

\[ 22x = \frac{110}{3} \]

\[ x = \frac{-5}{3} \]

Substitute and solve for \( y \):

\[ 3\left(\frac{-5}{3}\right) - 5y = -10 \]

\[ -5 - 5y = -10 \]

\[ -5y = -5 \]

\[ y = 1 \]

The solution of the system is \( x = \frac{-5}{3}, y = 1 \) or using ordered pairs \( \left(\frac{-5}{3}, 1\right) \).

25. \[ \begin{align*}
2x + y &= 1 \\
4x + 2y &= 3
\end{align*} \]

Solve the first equation for \( y \), substitute into the second equation and solve:

\[ y = 1 - 2x \]

\[ 4x + 2y = 3 \]

\[ 4x + 2(1 - 2x) = 3 \]

\[ 4x + 2 - 4x = 3 \]

\[ 0 = 1 \]

This equation is false, so the system is inconsistent.

26. \[ \begin{align*}
x - y &= 5 \\
-3x + 3y &= 2
\end{align*} \]

Solve the first equation for \( x \), substitute into the second equation and solve:

\[ x = y + 5 \]

\[ -3x + 3y = 2 \]

\[ -3(y + 5) + 3y = 2 \]

\[ -3y - 15 + 3y = 2 \]

\[ 0 = 17 \]

This equation is false, so the system is inconsistent.

27. \[ \begin{align*}
2x - y &= 0 \\
4x + 2y &= 12
\end{align*} \]

Solve the first equation for \( y \), substitute into the second equation and solve:

\[ y = 2x \]

\[ 4x + 2(2x) = 12 \]

\[ 4x + 4x = 12 \]

\[ 8x = 12 \]

\[ x = \frac{3}{2} \]

Since \( x = \frac{3}{2} \), \( y = 2\left(\frac{3}{2}\right) = 3 \)

The solution of the system is \( x = \frac{3}{2}, y = 3 \) or using ordered pairs \( \left(\frac{3}{2}, 3\right) \).

28. \[ \begin{align*}
3x + 3y &= -1 \\
4x + y &= \frac{8}{3}
\end{align*} \]

Solve the second equation for \( y \), substitute into the first equation and solve:

\[ y = \frac{8}{3} - 4x \]

\[ 3x + 3\left(\frac{8}{3} - 4x\right) = -1 \]

\[ 3x + 8 - 12x = -1 \]

\[ -9x = -9 \]

\[ x = 1 \]

Since \( x = 1 \), \( y = \frac{8}{3} - 4(1) = \frac{8}{3} - 4 = -\frac{4}{3} \).

The solution of the system is \( x = 1, y = -\frac{4}{3} \) or using ordered pairs \( \left(1, -\frac{4}{3}\right) \).

29. \[ \begin{align*}
x + 2y &= 4 \\
2x + 4y &= 8
\end{align*} \]

Solve the first equation for \( x \), substitute into the second equation and solve:

\[ x = 4 - 2y \]

\[ 2x + 4y = 8 \]
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2(4 − 2y) + 4y = 8
8 − 4y + 4y = 8
0 = 0
These equations are dependent. The solution of the system is either \( x = 4 - 2y \), where \( y \) is any real number or \( y = \frac{4 - x}{2} \), where \( x \) is any real number.

Using ordered pairs, we write the solution as \( \{(x, y) | x = 4 - 2y, \ y \text{ is any real number} \} \) or as \( \{(x, y) | y = \frac{4 - x}{2}, \ x \text{ is any real number} \} \).

30. \( \begin{align*}
3x - y &= 7 \\
9x - 3y &= 21
\end{align*} \)
Solve the first equation for \( y \), substitute into the second equation and solve:
\( \begin{align*}
y &= 3x - 7 \\
9x - 3y &= 21
\end{align*} \)
\( 9x - 3(3x - 7) = 21 \)
\( 9x - 9x + 21 = 21 \)
\( 0 = 0 \)
These equations are dependent. The solution of the system is either \( y = 3x - 7 \), where \( x \) is any real number is \( x = \frac{y + 7}{3} \), where \( y \) is any real number.

Using ordered pairs, we write the solution as \( \{(x, y) | y = 3x - 7, \ x \text{ is any real number} \} \) or as \( \{(x, y) | x = \frac{y + 7}{3}, \ y \text{ is any real number} \} \).

31. \( \begin{align*}
2x - 3y &= -1 \\
10x + y &= 11
\end{align*} \)
Multiply each side of the first equation by \(-5\), and add the equations to eliminate \( x \):
\( \begin{align*}
-10x + 15y &= 5 \\
10x + y &= 11
\end{align*} \)
\( 16y = 16 \)
\( y = 1 \)
Substitute and solve for \( x \):
\( 2x - 3(1) = -1 \)
\( 2x - 3 = -1 \)
\( 2x = 2 \)
\( x = 1 \)
The solution of the system is \( x = 1, \ y = 1 \) or using ordered pairs \((1, 1)\).

32. \( \begin{align*}
3x - 2y &= 0 \\
5x + 10y &= 4
\end{align*} \)
Multiply each side of the first equation by \(5\), and add the equations to eliminate \( y \):
\( \begin{align*}
15x - 10y &= 0 \\
5x + 10y &= 4
\end{align*} \)
\( 20x = 4 \)
\( x = \frac{1}{5} \)
Substitute and solve for \( y \):
\( 5(1/5) + 10y = 4 \)
\( 1 + 10y = 4 \)
\( 10y = 3 \)
\( y = \frac{3}{10} \)
The solution of the system is \( x = \frac{1}{5}, \ y = \frac{3}{10} \) or using ordered pairs \( \left(\frac{1}{5}, \frac{3}{10}\right) \).

33. \( \begin{align*}
2x + 3y &= 6 \\
x - y &= \frac{1}{2}
\end{align*} \)
Solve the second equation for \( x \), substitute into the first equation and solve:
\( \begin{align*}
2x + 3y &= 6 \\
x &= y + \frac{1}{2}
\end{align*} \)
\( 2 \left( y + \frac{1}{2} \right) + 3y = 6 \)
\( 2y + 1 + 3y = 6 \)
\( 5y = 5 \)
\( y = 1 \)
Since \( y = 1 \), \( x = 1 + \frac{1}{2} = \frac{3}{2} \). The solution of the system is \( x = \frac{3}{2}, \ y = 1 \) or using ordered pairs \( \left(\frac{3}{2}, 1\right) \).
34. \[
\begin{align*}
\frac{1}{2}x + y &= -2 \\
-x - 2y &= 8 \\
\end{align*}
\]
Solve the second equation for \(x\), substitute into the first equation and solve:
\[
\begin{align*}
\frac{1}{2}x + y &= -2 \\
x &= 2y + 8 \\
\frac{1}{2}(2y + 8) + y &= -2 \\
y + 4 + y &= -2 \\
2y &= -6 \\
y &= -3 \\
Since \ y = -3, \ x = 2(-3) + 8 = -6 + 8 = 2. \ The \\
solution of the system is \(x = 2, \ y = -3\) or using ordered pairs \((2, -3)\).
\]

35. \[
\begin{align*}
\frac{1}{2}x + \frac{1}{3}y &= -3 \\
\frac{1}{4}x - \frac{2}{3}y &= -1 \\
\end{align*}
\]
Multiply each side of the first equation by \(-6\) and each side of the second equation by \(12\), then add to eliminate \(x\):
\[
\begin{align*}
-3x - 2y &= -18 \\
3x - 8y &= -12 \\
-10y &= -30 \\
y &= 3 \\
\end{align*}
\]
Substitute and solve for \(x\):
\[
\begin{align*}
\frac{1}{2}x + \frac{1}{3}(3) &= 3 \\
\frac{1}{2}x + 1 &= 3 \\
\frac{1}{2}x &= 2 \\
x &= 4 \\
\end{align*}
\]
The solution of the system is \(x = 4, \ y = 3\) or using ordered pairs \((4, 3)\).

36. \[
\begin{align*}
\frac{1}{3}x - \frac{3}{2}y &= -5 \\
\frac{3}{4}x + \frac{1}{3}y &= 11 \\
\end{align*}
\]
Multiply each side of the first equation by \(-54\) and each side of the second equation by \(24\), then add to eliminate \(x\):
\[
\begin{align*}
18x + 81y &= 270 \\
18x + 8y &= 264 \\
89y &= 534 \\
y &= 6 \\
\end{align*}
\]
Substitute and solve for \(x\):
\[
\begin{align*}
\frac{3}{4}x + \frac{1}{3}(6) &= 11 \\
\frac{3}{4}x + 2 &= 11 \\
\frac{3}{4}x &= 9 \\
x &= 12 \\
\end{align*}
\]
The solution of the system is \(x = 12, \ y = 6\) or using ordered pairs \((12, 6)\).

37. \[
\begin{align*}
3x - 5y &= 3 \\
15x + 5y &= 21 \\
\end{align*}
\]
Add the equations to eliminate \(y\):
\[
\begin{align*}
3x - 5y &= 3 \\
15x + 5y &= 21 \\
18x &= 24 \\
x &= \frac{4}{3} \\
\end{align*}
\]
Substitute and solve for \(y\):
\[
\begin{align*}
3(\frac{4}{3}) - 5y &= 3 \\
4 - 5y &= 3 \\
-5y &= -1 \\
y &= \frac{1}{5} \\
\end{align*}
\]
The solution of the system is \(x = \frac{4}{3}, \ y = \frac{1}{5}\) or using ordered pairs \(\left(\frac{4}{3}, \frac{1}{5}\right)\).
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38. \[
\begin{align*}
2x - y &= -1 \\
x + \frac{1}{2}y &= \frac{3}{2}
\end{align*}
\]
Multiply each side of the second equation by 2, and add the equations to eliminate \( y \):
\[
\begin{align*}
2x - y &= -1 \\
2x + y &= 3 \\
4x &= 2
\end{align*}
\] \( x = \frac{1}{2} \)

Substitute and solve for \( y \):
\[
2\left(\frac{1}{2}\right) - y = -1
\]
\[
1 - y = -1 \\
y = 2
\]

The solution of the system is \( x = \frac{1}{2}, y = 2 \) or using ordered pairs \( \left(\frac{1}{2}, 2\right) \).

39. \[
\begin{align*}
\frac{1}{x} + \frac{1}{y} &= 8 \\
\frac{3}{x} - \frac{5}{y} &= 0 \quad \text{or using ordered pairs} \quad \left(\frac{1}{2}, 2\right)
\end{align*}
\]
Rewrite letting \( u = \frac{1}{x}, v = \frac{1}{y} \):
\[
\begin{align*}
u + v &= 8 \\
3u - 5v &= 0
\end{align*}
\]
Solve the first equation for \( u \), substitute into the second equation and solve:
\[
\begin{align*}
u &= 8 - v \\
3u - 5v &= 0
\end{align*}
\]
\[
3(8 - v) - 5v = 0 \\
24 - 3v - 5v = 0 \\
-8v = -24 \\
v = 3
\]

Since \( v = 3 \), \( u = 8 - 3 = 5 \). Thus, \( x = \frac{1}{u} = \frac{1}{5} \).
\[
\frac{1}{v} = \frac{1}{3} \quad \text{or using ordered pairs} \quad \left(\frac{1}{5}, \frac{1}{3}\right)
\]

40. \[
\begin{align*}
4 - \frac{3}{x} &= 0 \\
6 + \frac{3}{2}y &= 2 \\
x &= \frac{1}{y}
\end{align*}
\]
Rewrite letting \( u = \frac{1}{x}, v = \frac{1}{y} \):
\[
\begin{align*}
4u - 3v &= 0 \\
6u + \frac{3}{2}v &= 2
\end{align*}
\]
Multiply each side of the second equation by 2, and add the equations to eliminate \( v \):
\[
\begin{align*}
4u - 3v &= 0 \\
12u + 3v &= 4 \\
16u &= 4
\end{align*}
\]
\[
\frac{u}{16} = \frac{1}{4}
\]
Substitute and solve for \( v \):
\[
4\left(\frac{1}{4}\right) - 3v = 0 \\
1 - 3v = 0 \\
-3v = -1 \\
v = \frac{1}{3}
\]

Thus, \( x = \frac{1}{u} = 4, \ y = \frac{1}{v} = 3 \). The solution of the system is \( x = 4, \ y = 3 \) or using ordered pairs (4, 3).

41. \[
\begin{align*}
x - y &= 6 \\
2x - 3z &= 16 \\
2y + z &= 4
\end{align*}
\]
Multiply each side of the first equation by –2 and add to the second equation to eliminate \( x \):
\[
\begin{align*}
-2x + 2y &= -12 \\
2x - 3z &= 16
\end{align*}
\]
\[
2y - 3z = 4
\]
Multiply each side of the result by –1 and add to the original third equation to eliminate \( y \):
\[
\begin{align*}
-2y + 3z &= -4 \\
2y + z &= 4
\end{align*}
\]
\[
4z = 0 \\
z = 0
\]
Substituting and solving for the other variables:
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2y + 0 = 4 
2y = 4 
y = 2 

2x − 3(0) = 16 
2x = 16 
x = 8 

The solution is \( x = 8, \ y = 2, \ z = 0 \) or using ordered triples (8, 2, 0).

42. \[
\begin{align*}
2x + y & = -4 \\
-2y + 4z & = 0 \\
3x - 2z & = -11 \\
\end{align*}
\]

Multiply each side of the first equation by 2 and add to the second equation to eliminate \( y \): 
\[ 4x + 2y = -8 \]
\[ -2y + 4z = 0 \]
\[ 4x + 4z = -8 \]

Multiply each side of the result by \( \frac{1}{2} \) and add to the original third equation to eliminate \( z \): 
\[ 2x + 2z = -4 \]
\[ 3x - 2z = -11 \]
\[ 5x = -15 \]
\[ x = -3 \]

Substituting and solving for the other variables: 
\[ 2(-3) + y = -4 \]
\[ 3(-3) - 2z = -11 \]
\[ -6 + y = -4 \]
\[ -9 - 2z = -11 \]
\[ y = 2 \]
\[ -2z = -2 \]
\[ z = 1 \]

The solution is \( x = -3, \ y = 2, \ z = 1 \) or using ordered triples \((-3, 2, 1)\).

43. \[
\begin{align*}
x - 2y + 3z & = 7 \\
2x + y + z & = 4 \\
-3x + 2y - 2z & = -10 \\
\end{align*}
\]

Multiply each side of the first equation by \(-2\) and add to the second equation to eliminate \( x \); and multiply each side of the first equation by 3 and add to the third equation to eliminate \( x \): 
\[ -2x + 4y - 6z = -14 \]
\[ 2x + y + z = 4 \]
\[ 5y - 5z = -10 \]
\[ 3x - 6y + 9z = 21 \]
\[ -3x + 2y - 2z = -10 \]
\[ -4y + 7z = 11 \]

Multiply each side of the result by \( \frac{4}{5} \) and add to the second result to eliminate \( y \):

44. \[
\begin{align*}
4y - 4z & = -8 \\
-4y + 7z & = 11 \\
3z & = 3 \\
z & = 1 \\
\end{align*}
\]

Substituting and solving for the other variables: 
\[ y - 1 = -2 \]
\[ x - 2(-1) + 3(1) = 7 \]
\[ y = -1 \]
\[ x + 2 + 3 = 7 \]
\[ x = 2 \]

The solution is \( x = 2, \ y = -1, \ z = 1 \) or using ordered triples \((2, -1, 1)\).
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\[
2\left(\frac{56}{13}\right) + y - 3\left(\frac{35}{13}\right) = 0
\]
\[
\frac{112}{13} + y - \frac{105}{13} = 0
\]
\[
y = \frac{7}{13}
\]

The solution is \(x = \frac{56}{13}, y = \frac{7}{13}, z = \frac{35}{13}\) or using ordered triples \(\left(\frac{56}{13}, \frac{7}{13}, \frac{35}{13}\right)\).

45. \[
\begin{align*}
x - y - z &= 1 \\
2x + 3y + z &= 2 \\
3x + 2y &= 0
\end{align*}
\]
Add the first and second equations to eliminate \(z\):
\[
x - y - z = 1
\]
\[
2x + 3y + z = 2
\]
\[
3x + 2y = 0
\]
Multiply each side of the result by \(-1\) and add to the original third equation to eliminate \(y\):
\[
-3x - 2y = -3
\]
\[
3x + 2y = 0
\]
\[
0 = -3
\]
This equation is false, so the system is inconsistent.

46. \[
\begin{align*}
2x - 3y - z &= 0 \\
-x + 2y + z &= 5 \\
3x - 4y - z &= 1
\end{align*}
\]
Add the first and second equations to eliminate \(z\); then add the second and third equations to eliminate \(z\):
\[
2x - 3y - z = 0
\]
\[
-x + 2y + z = 5
\]
\[
x - y = 5
\]
\[
-x + 2y + z = 5
\]
\[
3x - 4y - z = 1
\]
\[
2x - 2y = 6
\]
Multiply each side of the first result by \(-2\) and add to the second result to eliminate \(y\):
\[
-2x + 2y = -10
\]
\[
2x - 2y = 6
\]
\[
0 = -2
\]
This equation is false, so the system is inconsistent.

47. \[
\begin{align*}
x - y - z &= 1 \\
-x + 2y - 3z &= -4 \\
3x - 2y - 7z &= 0
\end{align*}
\]
Add the first and second equations to eliminate \(x\); multiply the first equation by \(-3\) and add to the third equation to eliminate \(x\):
\[
x - y - z = 1
\]
\[
-x + 2y - 3z = -4
\]
\[
y - 4z = -3
\]
\[
-3x + 3y + 3z = -3
\]
\[
3x - 2y - 7z = 0
\]
\[
y - 4z = -3
\]
Multiply each side of the first result by \(-1\) and add to the second result to eliminate \(y\):
\[
-y + 4z = 3
\]
\[
y - 4z = -3
\]
\[
0 = 0
\]
The system is dependent. If \(z\) is any real number, then \(y = 4z - 3\).

Solving for \(x\) in terms of \(z\) in the first equation:
\[
x - (4z - 3) - z = 1
\]
\[
x - 4z + 3 - z = 1
\]
\[
x - 5z + 3 = 1
\]
\[
x = 5z - 2
\]
The solution is \(\{(x, y, z) \mid x = 5z - 2, y = 4z - 3, z\text{ is any real number}\}\).

48. \[
\begin{align*}
2x - 3y - z &= 0 \\
3x + 2y + 2z &= 2 \\
x + 5y + 3z &= 2
\end{align*}
\]
Multiply the first equation by \(2\) and add to the second equation to eliminate \(z\); multiply the first equation by \(3\) and add to the third equation to eliminate \(z\):
\[
4x - 6y - 2z = 0
\]
\[
3x + 2y + 2z = 2
\]
\[
7x - 4y = 2
\]
\[
6x - 9y - 3z = 0
\]
\[
x + 5y + 3z = 2
\]
\[
7x - 4y = 2
\]
Multiply each side of the first result by \(-1\) and add to the second result to eliminate \(y\):
Section 11.1: Systems of Linear Equations: Substitution and Elimination

−7x + 4y = −2
7x − 4y = 2
0 = 0
The system is dependent. If y is any real number, then \( x = \frac{4}{7} y + \frac{2}{7} \).

Solving for \( z \) in terms of \( x \) in the first equation:
\[ z = 2x − 3y \]
\[ = 2 \left( \frac{4y + 2}{7} \right) − 3y \]
\[ = \frac{8y + 4 - 21y}{7} \]
\[ = \frac{-13y + 4}{7} \]
The solution is \( \left\{ (x, y, z) \middle| x = \frac{4}{7} y + \frac{2}{7}, \quad z = \frac{-13}{7} y + \frac{4}{7}, \text{ y is any real number} \right\} \).

49.
\[
\begin{bmatrix}
2x − 2y + 3z &= 6 \\
x − 3y + 2z &= 0 \\
−2x + 3y − 7z &= 1
\end{bmatrix}
\]
Multiply the first equation by −2 and add to the second equation to eliminate \( x \); add the first and third equations to eliminate \( x \):
\[ −4x + 4y − 6z = −12 \]
\[ 4x − 3y + 2z = 0 \]
\[ y − 4z = −12 \]
\[ 2x − 2y + 3z = 6 \]
\[ −2x + 3y − 7z = 1 \]
\[ y − 4z = 7 \]
Multiply each side of the first result by −1 and add to the second result to eliminate \( y \):
\[ −y + 4z = 12 \]
\[ y − 4z = 7 \]
\[ 0 = 19 \]
This result is false, so the system is inconsistent.

50.
\[
\begin{bmatrix}
3x − 2y + 2z &= 6 \\
7x − 3y + 2z &= −1 \\
2x − 3y + 4z &= 0
\end{bmatrix}
\]
Multiply the first equation by −1 and add to the second equation to eliminate \( z \); multiply the first equation by −2 and add to the third equation to eliminate \( z \):
\[ −3x + 2y − 2z = −6 \]
\[ 7x − 3y + 2z = −1 \]
\[ 4x − y = −7 \]
\[ −6x + 4y − 4z = −12 \]
\[ 2x − 3y + 4z = 0 \]
\[ −4x + y = −12 \]
Add the first result to the second result to eliminate \( y \):
\[ 4x − y = −7 \]
\[ −4x + y = −12 \]
\[ 0 = −19 \]
This result is false, so the system is inconsistent.

51.
\[
\begin{bmatrix}
x + y − z &= 6 \\
3x − 2y + z &= −5 \\
x + 3y − 2z &= 14
\end{bmatrix}
\]
Add the first and second equations to eliminate \( z \); multiply the second equation by 2 and add to the third equation to eliminate \( z \):
\[ x + y − z = 6 \]
\[ 3x − 2y + z = −5 \]
\[ 4x − y = 1 \]
\[ 6x − 4y + 2z = −10 \]
\[ x + 3y − 2z = 14 \]
\[ 7x − y = 4 \]
Multiply each side of the first result by −1 and add to the second result to eliminate \( y \):
\[ −4x + y = −1 \]
\[ 7x − y = 4 \]
\[ 3x = 3 \]
\[ x = 1 \]
Substituting and solving for the other variables:
\[ 4(1) − y = 1 \]
\[ 3(1) − 2(3) + z = −5 \]
\[ −y = −3 \]
\[ 3 − 6 + z = −5 \]
\[ y = 3 \]
\[ z = −2 \]
The solution is \( x = 1, \ y = 3, \ z = −2 \) or using ordered triplets \( (1, 3, −2) \).
Chapter 11: Systems of Equations and Inequalities

52. \[
\begin{align*}
\begin{cases}
x - y + z &= -4 \\
2x - 3y + 4z &= -15 \\
5x + y - 2z &= 12 
\end{cases}
\end{align*}
\]
Multiply the first equation by \(-3\) and add to the second equation to eliminate \(y\); add the first and third equations to eliminate \(y\):
\[
\begin{align*}
-3x + 3y - 3z &= 12 \\
2x - 3y + 4z &= -15 \\
-x + z &= -3 \\
x - y + z &= -4 \\
5x + y - 2z &= 12 \\
6x - z &= 8 \\
\end{align*}
\]
Substitute and solve:
\[
\begin{align*}
6x - (x - 3) &= 8 \\
6x - x + 3 &= 8 \\
5x &= 5 \\
x &= 1 \\
z &= x - 3 = 1 - 3 = -2 \\
y &= 12 - 5x + 2z = 12 - 5(1) + 2(-2) = 3 \\
\end{align*}
\]
The solution is \(x = 1, y = 3, z = -2\) or using ordered triplets \((1, 3, -2)\).

53. \[
\begin{align*}
\begin{cases}
x + 2y - z &= -3 \\
2x - 4y + z &= -7 \\
-2x + 2y - 3z &= 4 
\end{cases}
\end{align*}
\]
Add the first and second equations to eliminate \(z\); multiply the second equation by 3 and add to the third equation to eliminate \(z\):
\[
\begin{align*}
x + 2y &- z = -3 \\
2x - 4y + z &= -7 \\
3x - 2y &= -10 \\
6x - 12y + 3z &= -21 \\
-2x + 2y - 3z &= 4 \\
4x - 10y &= -17 \\
\end{align*}
\]
Multiply each side of the first result by \(-5\) and add to the second result to eliminate \(y\):
\[
\begin{align*}
-15x + 10y &= 50 \\
4x - 10y &= -17 \\
-11x &= 33 \\
x &= -3 \\
\end{align*}
\]
Substituting and solving for the other variables:
\[
\begin{align*}
3(-3) - 2y &= -10 \\
-9 - 2y &= -10 \\
-2y &= -1 \\
y &= \frac{1}{2} \\
\end{align*}
\]

54. \[
\begin{align*}
\begin{cases}
x + 4y - 3z &= -8 \\
3x - y + 3z &= 12 \\
x + y + 6z &= 1 
\end{cases}
\end{align*}
\]
Add the first and second equations to eliminate \(z\); multiply the first equation by 2 and add to the third equation to eliminate \(z\):
\[
\begin{align*}
x + 4y - 3z &= -8 \\
3x - y + 3z &= 12 \\
4x + 3y &= 4 \\
2x + 8y - 6z &= -16 \\
x + y + 6z &= 1 \\
3x + 9y &= -15 \\
\end{align*}
\]
Multiply each side of the second result by \(-1/3\) and add to the first result to eliminate \(y\):
\[
\begin{align*}
4x + 3y &= 4 \\
-3x - 3y &= 5 \\
3x &= 9 \\
\end{align*}
\]
Substituting and solving for the other variables:
\[
\begin{align*}
3 + 3y &= -5 \\
3y &= -8 \\
y &= -\frac{8}{3} \\
3 + \left(-\frac{8}{3}\right) + 6z &= 1 \\
6z &= \frac{2}{3} \\
z &= \frac{1}{9} \\
\end{align*}
\]
The solution is \(x = 3, y = -\frac{8}{3}, z = \frac{1}{9}\) or using ordered triplets \(\left(3, \frac{-8}{3}, \frac{1}{9}\right)\).
55. Let \( l \) be the length of the rectangle and \( w \) be the width of the rectangle. Then:
\[
2lw = 90
\]
Solve by substitution:
\[
4w + 2w = 90
\]
\[
w = 15 \text{ feet}
\]
\[
l = 2(15) = 30 \text{ feet}
\]
The floor is 15 feet by 30 feet.

56. Let \( l \) be the length of the rectangle and \( w \) be the width of the rectangle. Then:
\[
50lw = 3000
\]
Solve by substitution:
\[
2(25) + 2w = 3000
\]
\[
w = 725 \text{ meters}
\]
\[
l = 775 \text{ meters}
\]
The dimensions of the field are 775 meters by 725 meters.

57. Let \( x \) = the number of commercial launches and \( y \) = the number of noncommercial launches. Then:
\[
x + y = 55 \text{ and } y = 2x + 1
\]
Solve by substitution:
\[
x + (2x + 1) = 55
\]
\[
3x = 54
\]
\[
x = 18
\]
\[
y = 37
\]
In 2005 there were 18 commercial launches and 37 noncommercial launches.

58. Let \( x \) = the number of adult tickets sold and \( y \) = the number of senior tickets sold. Then:
\[
\begin{align*}
x + y &= 325 \\
9x + 7y &= 2495
\end{align*}
\]
Solve the first equation for \( y \):
\[
y = 325 - x
\]
Solve by substitution:
\[
9x + 7(325 - x) = 2495
\]
\[
9x + 2275 - 7x = 2495
\]
\[
2x = 220
\]
\[
x = 110
\]
\[
y = 325 - 110 = 215
\]
There were 110 adult tickets sold and 215 senior citizen tickets sold.

59. Let \( x \) = the number of pounds of cashews. Let \( y \) = is the number of pounds in the mixture. The value of the cashews is \( 5x \).
The value of the peanuts is \( 1.50(30) = 45 \).
The value of the mixture is \( 3y \).
Then \( x + 30 = y \) represents the amount of mixture.
\[
5x + 45 = 3y
\]
Solve by substitution:
\[
5x + 45 = 3(x + 30)
\]
\[
2x = 45
\]
\[
x = 22.5
\]
So, 22.5 pounds of cashews should be used in the mixture.

60. Let \( x \) = the amount invested in AA bonds. Let \( y \) = the amount invested in the Bank Certificate.
a. Then \( x + y = 150,000 \) represents the total investment.
\[
0.10x + 0.05y = 12,000
\]
Solve by substitution:
\[
0.10(150,000 - y) + 0.05y = 12,000
\]
\[
15,000 - 0.10y + 0.05y = 12,000
\]
\[
-0.05y = -3000
\]
\[
y = 60,000
\]
\[
x = 150,000 - 60,000 = 90,000
\]
Thus, $90,000 should be invested in AA Bonds and $60,000 in a Bank Certificate.
b. Then \( x + y = 150,000 \) represents the total investment.
\[
0.10x + 0.05y = 14,000
\]
Solve by substitution:
\[
0.10(150,000 - y) + 0.05y = 14,000
\]
\[
15,000 - 0.10y + 0.05y = 14,000
\]
\[
-0.05y = -1000
\]
\[
y = 20,000
\]
\[
x = 150,000 - 20,000 = 130,000
\]
Thus, $130,000 should be invested in AA Bonds and $20,000 in a Bank Certificate.
61. Let \( x \) = the plane’s average airspeed and \( y \) = the average wind speed.

<table>
<thead>
<tr>
<th>Rate Time Distance</th>
<th>( x + y )</th>
<th>3</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>Against</td>
<td>( x - y )</td>
<td>4</td>
<td>600</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
(x + y)(3) &= 600 \\
(x - y)(4) &= 600
\end{align*}
\]

Multiply each side of the first equation by \( \frac{1}{3} \),
multiply each side of the second equation by \( \frac{1}{4} \),
and add the result to eliminate \( y \)

\[
\begin{align*}
x + y &= 200 \\
x - y &= 150 \\
2x &= 350 \\
x &= 175 \\
175 + y &= 200 \\
y &= 25
\end{align*}
\]

The average airspeed of the plane is 175 mph, and the average wind speed is 25 mph.

62. Let \( x \) = the average wind speed and \( y \) = the distance.

<table>
<thead>
<tr>
<th>Rate Time Distance</th>
<th>( 150 + x )</th>
<th>2</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Against</td>
<td>( 150 - x )</td>
<td>3</td>
<td>( y )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
(150 + x)(2) &= y \\
(150 - x)(3) &= y
\end{align*}
\]

Solve by substitution:

\[
\begin{align*}
(150 + x)(2) &= (150 - x)(3) \\
300 + 2x &= 450 - 3x \\
5x &= 150 \\
x &= 30
\end{align*}
\]

Thus, the average wind speed is 30 mph.

63. Let \( x \) = the number of $25-design.

Let \( y \) = the number of $45-design.

Then \( x + y \) = the total number of sets of dishes.
\( 25x + 45y \) = the cost of the dishes.

Setting up the equations and solving by substitution:

\[
\begin{align*}
x + y &= 200 \\
25x + 45y &= 7400
\end{align*}
\]

Solve the first equation for \( y \), the solve by substitution: \( y = 200 - x \)

\[
\begin{align*}
25x + 45(200 - x) &= 7400 \\
25x + 9000 - 45x &= 7400 \\
-20x &= -1600 \\
x &= 80 \\
y &= 200 - 80 = 120
\end{align*}
\]

Thus, 80 sets of the $25 dishes and 120 sets of the $45 dishes should be ordered.

64. Let \( x \) = the cost of a hot dog.

Let \( y \) = the cost of a soft drink.

Setting up the equations and solving by substitution:

\[
\begin{align*}
10x + 5y &= 35.00 \\
7x + 4y &= 25.25 \\
10x + 5y &= 35.00 \\
2x + y &= 7 \\
y &= 7 - 2x \\
7x + 4(7 - 2x) &= 25.25 \\
7x + 28 - 8x &= 25.25 \\
-x &= -2.75 \\
x &= 2.75 \\
y &= 7 - 2(2.75) = 1.50
\end{align*}
\]

A single hot dog costs $2.75 and a single soft drink costs $1.50.

65. Let \( x \) = the cost per package of bacon.

Let \( y \) = the cost of a carton of eggs.

Set up a system of equations for the problem:

\[
\begin{align*}
3x + 2y &= 13.45 \\
2x + 3y &= 11.45
\end{align*}
\]

Multiply each side of the first equation by 3 and each side of the second equation by \(-2\) and solve by elimination:

\[
\begin{align*}
9x + 6y &= 40.35 \\
-4x - 6y &= -22.90 \\
5x &= 17.45 \\
x &= 3.49 \\
3(3.49) + 2y &= 13.45 \\
10.47 + 2y &= 13.45 \\
2y &= 3.98 \\
y &= 1.99
\end{align*}
\]

A package of bacon costs $3.49 and a carton of eggs cost $1.49. The refund for 2 packages of bacon and 2 cartons of eggs will be

\[
2(3.49) + 2(1.49) = 9.96.
\]
66. Let \( x \) = Pamela’s average speed in still water. Let \( y \) = the speed of the current.

<table>
<thead>
<tr>
<th></th>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstream</td>
<td>( x + y )</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>Upstream</td>
<td>( x - y )</td>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

Set up a system of equations for the problem:

\[
\begin{align*}
3(x + y) & = 15 \\
5(x - y) & = 15
\end{align*}
\]

Multiply each side of the first equation by \( \frac{1}{3} \), multiply each side of the second equation by \( \frac{1}{5} \), and add the result to eliminate \( y \):

\[
\begin{align*}
5 \cdot 3(x + y) + 3 \cdot 5(x - y) & = 15 \cdot 3 + 15 \cdot 5 \\
15x + 15y + 15x - 15y & = 45 + 75 \\
2(15x) & = 120 \\
15x & = 60 \\
x & = 4
\end{align*}
\]

\[
\begin{align*}
4 + y & = 5 \\
y & = 1
\end{align*}
\]

Pamela’s average speed is 4 miles per hour and the speed of the current is 1 mile per hour.

67. Let \( x \) = the # of mg of compound 1. Let \( y \) = the # of mg of compound 2.

Setting up the equations and solving by substitution:

\[
\begin{align*}
0.2x + 0.4y & = 40 & \text{vitamin C} \\
0.3x + 0.2y & = 30 & \text{vitamin D}
\end{align*}
\]

Multiplying each equation by 10 yields

\[
\begin{align*}
2x + 4y & = 400 \\
6x + 4y & = 600
\end{align*}
\]

Subtracting the bottom equation from the top equation yields

\[
\begin{align*}
2x + 4y - (6x + 4y) & = 400 - 600 \\
2x - 6x & = -200 \\
-4x & = -200 \\
x & = 50
\end{align*}
\]

\[
\begin{align*}
2(50) + 4y & = 400 \\
100 + 4y & = 400 \\
4y & = 300 \\
y & = \frac{300}{4} = 75
\end{align*}
\]

So 50 mg of compound 1 should be mixed with 75 mg of compound 2.

68. Let \( x \) = the # of units of powder 1. Let \( y \) = the # of units of powder 2.

Setting up the equations and solving by substitution:

\[
\begin{align*}
0.2x + 0.4y & = 12 & \text{vitamin B12} \\
0.3x + 0.2y & = 12 & \text{vitamin E}
\end{align*}
\]

Multiplying each equation by 10 yields

\[
\begin{align*}
2x + 4y & = 120 \\
6x + 4y & = 240
\end{align*}
\]

Subtracting the bottom equation from the top equation yields

\[
\begin{align*}
2x + 4y - (6x + 4y) & = 120 - 240 \\
-4x & = -120 \\
x & = 30
\end{align*}
\]

\[
\begin{align*}
2(30) + 4y & = 120 \\
60 + 4y & = 120 \\
4y & = 60 \\
y & = \frac{60}{4} = 15
\end{align*}
\]

So 30 units of powder 1 should be mixed with 15 units of powder 2.

69. \( y = ax^2 + bx + c \)

At \((-1, 4)\) the equation becomes:

\[
4 = a(-1)^2 + b(-1) + c \\
4 = a - b + c
\]

At \((2, 3)\) the equation becomes:

\[
3 = a(2)^2 + b(2) + c \\
3 = 4a + 2b + c
\]

At \((0, 1)\) the equation becomes:

\[
1 = a(0)^2 + b(0) + c \\
1 = c
\]

The system of equations is:

\[
\begin{align*}
\text{a} - b + c & = 4 \\
4a + 2b + c & = 3
\end{align*}
\]

Substitute \( c = 1 \) into the first and second equations and simplify:

\[
\begin{align*}
a - b + 1 & = 4 \\
4a + 2b + 1 & = 3
\end{align*}
\]

\[
\begin{align*}
a - b & = 3 \\
4a + 2b & = 2
\end{align*}
\]

\[
\begin{align*}
a + b & = 3
\end{align*}
\]

Solve the first result for \( a \), substitute into the second result and solve:
Chapter 11: Systems of Equations and Inequalities

4(b + 3) + 2b = 2
4b + 12 + 2b = 2

6b = -10
b = -5/3

a = -5/3 + 3 = 4/3

The solution is \( a = \frac{4}{3}, b = \frac{-5}{3}, c = 1 \). The equation is \( y = \frac{4}{3}x^2 - \frac{5}{3}x + 1 \).

70. \( y = ax^2 + bx + c \)

At (-1, -2) the equation becomes:
\[-2 = a(-1)^2 + b(-1) + c \]
\[a - b + c = -2 \]

At (1, -4) the equation becomes:
\[-4 = a(1)^2 + b(1) + c \]
\[a + b + c = -4 \]

At (2, 4) the equation becomes:
\[4 = a(2)^2 + b(2) + c \]
\[4a + 2b + c = 4 \]

The system of equations is:
\[
\begin{align*}
a - b + c &= -2 \\
a + b + c &= -4 \\
4a + 2b + c &= 4
\end{align*}
\]

Multiply the first equation by -1 and add to the second equation; multiply the first equation by -1 and add to the third equation to eliminate \( c \):
\[
\begin{align*}
-a + b - c &= 2 \\
a + b + c &= -4 \\
2b &= -2 \\
b &= -1
\end{align*}
\]

Substitute and solve:
\[
\begin{align*}
a + (-1) &= 2 \\
a &= 3 \\
\end{align*}
\]
\[
\begin{align*}
c &= -a - b - 4 \\
c &= -3 - (-1) - 4 \\
c &= -6
\end{align*}
\]

The solution is \( a = 3, b = -1, c = -6 \). The equation is \( y = 3x^2 - x - 6 \).

71. \[
\begin{align*}
0.06Y - 5000r &= 240 \\
0.06Y + 6000r &= 900 \\
10000r &= 660 \\
r &= 0.06
\end{align*}
\]

Multiply the first equation by -1, the add the result to the second equation to eliminate \( Y \):
\[
-0.06Y + 5000r = -240
\]
\[
0.06Y + 6000r = 900
\]

Substitute this result into the first equation to find \( Y \):
\[
0.06Y - 5000(0.06) = 240
\]
\[
0.06Y - 300 = 240
\]
\[
0.06Y = 540
\]
\[
Y = 9000
\]

The equilibrium level of income and interest rates is $9000 million and 6%.

72. \[
\begin{align*}
0.05Y - 1000r &= 10 \\
0.05Y + 800r &= 100 \\
1800r &= 90 \\
r &= 0.05
\end{align*}
\]

Multiply the first equation by -1, the add the result to the second equation to eliminate \( Y \):
\[
-0.05Y + 1000r = -10
\]
\[
0.05Y + 800r = 100
\]

Substitute this result into the first equation to find \( Y \):
\[
0.05Y - 1000(0.05) = 10
\]
\[
0.05Y - 50 = 10
\]
\[
0.05Y = 60
\]
\[
Y = 1200
\]

The equilibrium level of income and interest rates is $1200 million and 5%.

73. \[
\begin{align*}
5 - 3I_1 - 5I_2 &= 0 \\
10 - 5I_2 - 7I_3 &= 0
\end{align*}
\]

Substitute the expression for \( I_2 \) into the second and third equations and simplify:
\[
5 - 3I_1 - 5(I_1 + I_3) = 0
\]
\[
-8I_1 - 5I_3 = -5
\]
\[
10 - 5(I_1 + I_3) - 7I_3 = 0
\]
\[
-5I_1 - 12I_3 = -10
\]

Multiply both sides of the first result by 5 and multiply both sides of the second result by -8 to eliminate \( I_1 \):
Section 11.1: Systems of Linear Equations: Substitution and Elimination

\[ -40I_1 - 25I_3 = -25 \\
40I_1 + 96I_3 = 80 \\
71I_3 = 55 \\
I_3 = \frac{55}{71} \]

Substituting and solving for the other variables:
\[ -8I_1 - 5\left(\frac{55}{71}\right) = -5 \\
-8I_1 - 275 = -5 \\
-8I_1 = \frac{80}{71} \\
I_1 = \frac{10}{71} \]
\[ I_2 = \left(\frac{10}{71}\right) + \frac{55}{71} = \frac{65}{71} \]

The solution is \( I_1 = \frac{10}{71}, \ I_2 = \frac{65}{71}, \ I_3 = \frac{55}{71} \).

**74.**
\[
\begin{align*}
I_3 &= I_1 + I_2 \\
8 &= 4I_3 + 6I_2 \\
8I_1 &= 4 + 6I_2
\end{align*}
\]
Substitute the expression for \( I_3 \) into the second equation and simplify:
\[
\begin{align*}
8 &= 4(I_1 + I_2) + 6I_2 \\
8 &= 4I_1 + 10I_2 \\
8I_1 - 6I_2 &= 4 \\
4I_1 + 10I_2 &= 8
\end{align*}
\]
Multiply both sides of the first result by \(-2\) and add to the second result to eliminate \( I_1 \):
\[
\begin{align*}
-8I_1 - 20I_2 &= -16 \\
8I_1 - 6I_2 &= 4 \\
-26I_2 &= -12 \\
I_2 &= \frac{6}{13}
\end{align*}
\]
\[
\begin{align*}
4I_1 + 10\left(\frac{6}{13}\right) &= 8 \\
4I_1 + \frac{60}{13} &= 8 \\
4I_1 &= \frac{44}{13} \\
I_1 &= \frac{11}{13}
\end{align*}
\]

\[ I_3 = I_1 + I_2 = \frac{11}{13} + \frac{6}{13} = \frac{17}{13} \]

The solution is \( I_1 = \frac{11}{13}, \ I_2 = \frac{6}{13}, \ I_3 = \frac{17}{13} \).

**75.** Let \( x \) = the number of orchestra seats.
Let \( y \) = the number of main seats.
Let \( z \) = the number of balcony seats.
Since the total number of seats is 500, \( x + y + z = 500 \).
Since the total revenue is \$17,100 if all seats are sold, \( 50x + 35y + 25z = 17,100 \).
If only half of the orchestra seats are sold, the revenue is \$14,600.
So, \( 50\left(\frac{1}{2}x\right) + 35y + 25z = 14,600 \).
Thus, we have the following system:
\[
\begin{align*}
x + y + z &= 500 \\
50x + 35y + 25z &= 17,100 \\
25x + 35y + 25z &= 14,600
\end{align*}
\]
Multiply each side of the first equation by \(-25\) and add to the second equation to eliminate \( z \);
multiply each side of the third equation by \(-1\) and add to the second equation to eliminate \( z \):
\[
\begin{align*}
-25x - 25y - 25z &= -12,500 \\
50x + 35y + 25z &= 17,100 \\
25x + 10y &= 4600 \\
-25x - 35y - 25z &= -14,600 \\
25x &= 2500 \\
x &= 100
\end{align*}
\]
Substituting and solving for the other variables:
\[
\begin{align*}
25(100) + 10y &= 4600 \\
10y &= 2100 \\
y &= 210 \\
10y &= 2100 \\
z &= 190 \\
y &= 210
\end{align*}
\]
There are 100 orchestra seats, 210 main seats, and 190 balcony seats.
76. Let \( x \) = the number of adult tickets. 
   Let \( y \) = the number of child tickets. 
   Let \( z \) = the number of senior citizen tickets. 
   Since the total number of tickets is 405, 
   \( x + y + z = 405 \). 
   Since the total revenue is $2320, 
   \( 8x + 4.50y + 6z = 2320 \). 
   Twice as many children's tickets as adult tickets 
   are sold. So, \( 2x = y \). 
   Thus, we have the following system:  
   \[
   \begin{align*}
   x + y + z &= 405 \\
   8x + 4.50y + 6z &= 2320 \\
   y &= 2x 
   \end{align*}
   \] 
   Substitute for \( y \) in the first two equations and 
   simplify: 
   \[
   \begin{align*}
   x + (2x) + z &= 405 \\
   3x + z &= 405 \\
   8x + 4.50(2x) + 6z &= 2320 \\
   17x + 6z &= 2320 
   \end{align*}
   \] 
   Multiply the first result by \(-6\) and add to the 
   second result to eliminate \( z \):  
   \[
   \begin{align*}
   -18x - 6z &= -2430 \\
   17x + 6z &= 2320 \\
   -x &= -110 \\
   x &= 110 
   \end{align*}
   \] 
   \[
   \begin{align*}
   y &= 2x \\
   &= 2(110) \\
   &= 220 \\
   3x + z &= 405 \\
   &= 3(110) + z = 405 \\
   &= 330 + z = 405 \\
   z &= 75 
   \end{align*}
   \] 
   There were 110 adults, 220 children, and 75 
   senior citizens that bought tickets.

77. Let \( x \) = the number of servings of chicken. 
   Let \( y \) = the number of servings of corn. 
   Let \( z \) = the number of servings of 2% milk. 
   Protein equation: \( 30x + 3y + 9z = 66 \) 
   Carbohydrate equation: \( 35x + 16y + 13z = 94.5 \) 
   Calcium equation: \( 200x + 10y + 300z = 910 \) 
   Multiply each side of the first equation by \(-16\) 
   and multiply each side of the second equation by \(3\) and add them to eliminate \( y \); multiply each 
   side of the second equation by \(-5\) and multiply each 
   side of the third equation by \(8\) and add to 
   eliminate \( y \):  
   \[
   \begin{align*}
   -480x - 48y - 144z &= -1056 \\
   105x + 48y + 39z &= 283.5 \\
   -375x - 105z &= -772.5 \\
   -175x - 80y - 65z &= -472.5 \] 
   \] 
   \[
   \begin{align*}
   1600x + 80y + 2400z &= 7280 \\
   1425x + 2335z &= 6807.5 
   \end{align*}
   \] 
   Multiply each side of the first result by \(19\) and 
   multiply each side of the second result by \(5\) to 
   eliminate \( x \):  
   \[
   \begin{align*}
   -7125x - 1995z &= -14,677.5 \\
   7125x + 11,675z &= 34,037.5 \\
   9680z &= 19,360 \\
   z &= 2 \] 
   \] 
   Substituting and solving for the other variables:  
   \[
   \begin{align*}
   -375x - 105(2) &= -772.5 \\
   -375x - 210 &= -772.5 \\
   -375x &= -562.5 \\
   x &= 1.5 \\
   30(1.5) + 3y + 9(2) &= 66 \\
   45 + 3y + 18 &= 66 \\
   3y &= 3 \\
   y &= 1 
   \end{align*}
   \] 
   The dietitian should serve 1.5 servings of 
   chicken, 1 serving of corn, and 2 servings of 2% 
   milk.

78. Let \( x \) = the amount in Treasury bills. 
   Let \( y \) = the amount in Treasury bonds. 
   Let \( z \) = the amount in corporate bonds. 
   Since the total investment is \$20,000, 
   \( x + y + z = 20,000 \). 
   Since the total income is to be \$1390, 
   \( 0.05x + 0.07y + 0.10z = 1390 \) 
   The investment in Treasury bills is to be \$3000 
   more than the investment in corporate bonds. 
   So, \( x = 3000 + z \) 
   Substitute for \( x \) in the first two equations and 
   simplify: 
   \[
   \begin{align*}
   (3000 + z) + y + z &= 20,000 \\
   y + 2z &= 17,000 \\
   5(3000 + z) + 7y + 10z &= 139,000 \\
   7y + 15z &= 124,000 
   \end{align*}
   \]
Multiply each side of the first result by \(-7\) and add to the second result to eliminate \(y\):

\[
-7y - 14z = -119,000 \\
7y + 15z = 124,000 \\
\hline
z = 5,000 \\
x = 3000 + z = 3000 + 5000 = 8000 \\
y + 2z = 17,000 \\
y + 2(5000) = 17,000 \\
y + 10,000 = 17,000 \\
y = 7000
\]

Kelly should invest $8000 in Treasury bills, $7000 in Treasury bonds, and $5000 in corporate bonds.

Let \(x\) = the price of 1 hamburger.
Let \(y\) = the price of 1 order of fries.
Let \(z\) = the price of 1 drink.

We can construct the system

\[
\begin{align*}
8x + 6y + 6z &= 26.10 \\
10x + 6y + 8z &= 31.60
\end{align*}
\]

A system involving only 2 equations that contain 3 or more unknowns cannot be solved uniquely.

Multiply the first equation by \(-\frac{1}{2}\) and the second equation by \(\frac{1}{2}\), then add to eliminate \(y\):

\[
\begin{align*}
-4x - 3y - 3z &= -13.05 \\
5x + 3y + 4z &= 15.80
\end{align*}
\]

\[
\begin{align*}
x + z &= 2.75 \\
x &= 2.75 - z
\end{align*}
\]

Substitute and solve for \(y\) in terms of \(z\):

\[
\begin{align*}
5(2.75 - z) + 3y + 4z &= 15.80 \\
13.75 + 3y - z &= 15.80 \\
3y &= z + 2.05 \\
y &= \frac{1}{3}z + \frac{41}{60}
\end{align*}
\]

Solutions of the system are: \(x = 2.75 - z\), \(y = \frac{1}{3}z + \frac{41}{60}\).

Since we are given that \(0.60 \leq z \leq 0.90\), we choose values of \(z\) that give two-decimal-place values of \(x\) and \(y\) with \(1.75 \leq x \leq 2.25\) and \(0.75 \leq y \leq 1.00\).

The possible values of \(x\), \(y\), and \(z\) are shown in the table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.13</td>
<td>0.89</td>
<td>0.62</td>
</tr>
<tr>
<td>2.10</td>
<td>0.90</td>
<td>0.65</td>
</tr>
<tr>
<td>2.07</td>
<td>0.91</td>
<td>0.68</td>
</tr>
<tr>
<td>2.04</td>
<td>0.92</td>
<td>0.71</td>
</tr>
<tr>
<td>2.01</td>
<td>0.93</td>
<td>0.74</td>
</tr>
<tr>
<td>1.98</td>
<td>0.94</td>
<td>0.77</td>
</tr>
<tr>
<td>1.95</td>
<td>0.95</td>
<td>0.80</td>
</tr>
<tr>
<td>1.92</td>
<td>0.96</td>
<td>0.83</td>
</tr>
<tr>
<td>1.89</td>
<td>0.97</td>
<td>0.86</td>
</tr>
<tr>
<td>1.86</td>
<td>0.98</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Therefore, one hamburger costs $1.95, one order of fries costs $0.95, and one drink costs $0.80.
Let \( x \) = Beth’s time working alone.
Let \( y \) = Bill’s time working alone.
Let \( z \) = Edie’s time working alone.

We can use the following tables to organize our work:

<table>
<thead>
<tr>
<th></th>
<th>Beth</th>
<th>Bill</th>
<th>Edie</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours to do job</td>
<td>( x )</td>
<td>( y )</td>
<td>( z )</td>
</tr>
<tr>
<td>Part of job done</td>
<td>( \frac{1}{x} )</td>
<td>( \frac{1}{y} )</td>
<td>( \frac{1}{z} )</td>
</tr>
<tr>
<td>in 1 hour</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In 10 hours they complete 1 entire job, so
\[
10 \left( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) = 1
\]
\[
\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{10}
\]

<table>
<thead>
<tr>
<th></th>
<th>Bill</th>
<th>Edie</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours to do job</td>
<td>( y )</td>
<td>( z )</td>
</tr>
<tr>
<td>Part of job done</td>
<td>( \frac{1}{y} )</td>
<td>( \frac{1}{z} )</td>
</tr>
<tr>
<td>in 1 hour</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In 15 hours they complete 1 entire job, so
\[
15 \left( \frac{1}{y} + \frac{1}{z} \right) = 1.
\]
\[
\frac{1}{y} + \frac{1}{z} = \frac{1}{15}
\]

<table>
<thead>
<tr>
<th></th>
<th>Beth</th>
<th>Bill</th>
<th>Edie</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours to do job</td>
<td>( x )</td>
<td>( y )</td>
<td>( z )</td>
</tr>
<tr>
<td>Part of job done</td>
<td>( \frac{1}{x} )</td>
<td>( \frac{1}{y} )</td>
<td>( \frac{1}{z} )</td>
</tr>
<tr>
<td>in 1 hour</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

With all 3 working for 4 hours and Beth and Bill working for an additional 8 hours, they complete 1 entire job, so
\[
4 \left( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) + 8 \left( \frac{1}{x} + \frac{1}{y} \right) = 1
\]
\[
\frac{12}{x} + \frac{12}{y} + \frac{4}{z} = 1
\]

We have the system
\[
\begin{align*}
\frac{1}{x} + \frac{1}{y} + \frac{1}{z} &= \frac{1}{10} \\
\frac{1}{y} + \frac{1}{z} &= \frac{1}{15} \\
\frac{12}{x} + \frac{12}{y} + \frac{4}{z} &= 1
\end{align*}
\]

Subtract the second equation from the first equation:
\[
\begin{align*}
\frac{1}{x} + \frac{1}{y} + \frac{1}{z} &= \frac{1}{10} \\
\frac{1}{y} + \frac{1}{z} &= \frac{1}{15} \\
\frac{12}{x} + \frac{12}{y} + \frac{4}{z} &= 1
\end{align*}
\]

\[
1 + \frac{1}{y} + \frac{1}{z} = \frac{1}{10}
\]
\[
\frac{1}{y} + \frac{1}{z} = \frac{1}{15}
\]

\[
\frac{1}{x} = \frac{1}{30}
\]
\[
x = 30
\]

Substitute \( x = 30 \) into the third equation:
\[
\frac{12}{30} + \frac{12}{y} + \frac{4}{z} = 1
\]
\[
\frac{12}{y} + \frac{12}{z} = \frac{3}{30}
\]
\[
\frac{12}{y} + \frac{12}{z} = \frac{1}{5}
\]

Now consider the system consisting of the last result and the second original equation. Multiply the second original equation by \(-12\) and add it to the last result to eliminate \( y \):
\[
-12 + \frac{-12}{y} = -12
\]
\[
y + \frac{12}{z} = \frac{15}{4}
\]
\[
\frac{12}{y} + \frac{4}{z} = \frac{3}{5}
\]
\[
\frac{12}{y} + \frac{4}{z} = \frac{3}{5}
\]
\[
-8 = -\frac{3}{15}
\]
\[
z = 40
\]

Plugging \( z = 40 \) to find \( y \):
\[
\frac{12}{y} + \frac{4}{z} = \frac{3}{5}
\]
\[
\frac{12}{y} + \frac{4}{z} = \frac{3}{5}
\]
\[
\frac{12}{y} = \frac{1}{2}
\]
\[
y = 24
\]

Working alone, it would take Beth 30 hours, Bill 24 hours, and Edie 40 hours to complete the job.

82. – 84. Answers will vary.
Section 11.2: Systems of Linear Equations: Matrices

1. matrix
2. augmented
3. third; fifth
4. True
5. Writing the augmented matrix for the system of equations:
   \[
   \begin{align*}
   x - 5y &= 5 \\
   4x + 3y &= 6
   \end{align*}
   \rightarrow \begin{bmatrix} 1 & -5 & 5 \\ 4 & 3 & 6 \end{bmatrix}
   \]
6. Writing the augmented matrix for the system of equations:
   \[
   \begin{align*}
   3x + 4y &= 7 \\
   4x - 2y &= 5
   \end{align*}
   \rightarrow \begin{bmatrix} 3 & 4 & 7 \\ 4 & -2 & 5 \end{bmatrix}
   \]
7. \[
   \begin{align*}
   2x + 3y &= 6 \\
   4x - 6y &= 2
   \end{align*}
   \]
   Write the system in standard form and then write the augmented matrix for the system of equations:
   \[
   \begin{align*}
   2x + 3y &= 6 \\
   4x - 6y &= -2
   \end{align*}
   \rightarrow \begin{bmatrix} 2 & 3 & 6 \\ 4 & -6 & -2 \end{bmatrix}
   \]
8. \[
   \begin{align*}
   9x - y &= 0 \\
   3x - y &= 0
   \end{align*}
   \]
   Write the system in standard form and then write the augmented matrix for the system of equations:
   \[
   \begin{align*}
   9x - y &= 0 \\
   3x - y &= 4
   \end{align*}
   \rightarrow \begin{bmatrix} 9 & -1 & 0 \\ 3 & -1 & 4 \end{bmatrix}
   \]
9. Writing the augmented matrix for the system of equations:
   \[
   \begin{align*}
   0.01x - 0.03y &= 0.06 \\
   0.13x + 0.10y &= 0.20
   \end{align*}
   \rightarrow \begin{bmatrix} 0.01 & -0.03 & 0.06 \\ 0.13 & 0.10 & 0.20 \end{bmatrix}
   \]
10. Writing the augmented matrix for the system of equations:
    \[
    \begin{align*}
    \frac{4}{3}x - \frac{3}{2}y &= \frac{3}{4} \\
    \frac{1}{4}x + \frac{1}{3}y &= 2
    \end{align*}
    \rightarrow \begin{bmatrix} \frac{4}{3} & -\frac{3}{2} & \frac{3}{4} \\ \frac{1}{4} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}
    \]
11. Writing the augmented matrix for the system of equations:
    \[
    \begin{align*}
    x - y + z &= 10 \\
    3x + 3y &= 5 \\
    x + y + 2z &= 2
    \end{align*}
    \rightarrow \begin{bmatrix} 1 & -1 & 1 & 10 \\ 3 & 3 & 0 & 5 \\ 1 & 1 & 2 & 2 \end{bmatrix}
    \]
12. Writing the augmented matrix for the system of equations:
    \[
    \begin{align*}
    5x - y - z &= 0 \\
    x + y &= 5 \\
    2x - 3z &= 2
    \end{align*}
    \rightarrow \begin{bmatrix} 5 & -1 & -1 & 0 \\ 1 & 1 & 0 & 5 \\ 2 & 0 & -3 & 2 \end{bmatrix}
    \]
13. Writing the augmented matrix for the system of equations:
    \[
    \begin{align*}
    x + y - z &= 2 \\
    3x - 2y &= 2 \\
    5x + 3y - z &= 1
    \end{align*}
    \rightarrow \begin{bmatrix} 1 & 1 & -1 & 2 \\ 3 & -2 & 0 & 2 \\ 5 & 3 & -1 & 1 \end{bmatrix}
    \]
14. Writing the augmented matrix for the system of equations:
    \[
    \begin{align*}
    x - 5z &= 2 \\
    x + 2y - 3z &= -2
    \end{align*}
    \]
    Write the system in standard form and then write the augmented matrix for the system of equations:
    \[
    \begin{align*}
    2x + 3y - 4z &= 0 \\
    x - 5z &= -2 \\
    x + 2y - 3z &= -2
    \end{align*}
    \rightarrow \begin{bmatrix} 2 & 3 & -4 & 0 \\ 1 & 0 & -5 & -2 \\ 1 & 2 & -3 & -2 \end{bmatrix}
    \]
15. Writing the augmented matrix for the system of equations:
    \[
    \begin{align*}
    x - y - z &= 10 \\
    2x + y + 2z &= -1 \\
    -3x + 4y &= 5
    \end{align*}
    \rightarrow \begin{bmatrix} 1 & -1 & -1 & 10 \\ 2 & 1 & 2 & -1 \\ -3 & 4 & 0 & 5 \end{bmatrix}
    \]
16. Writing the augmented matrix for the system of equations:
    \[
    \begin{align*}
    x - y + 2z &= 5 \\
    x + 3y - 4z + 2w &= 2 \\
    3x - y - 5z - w &= -1
    \end{align*}
    \rightarrow \begin{bmatrix} 1 & -1 & 2 & -1 & 5 \\ 1 & 3 & -4 & 2 & 2 \\ 3 & -1 & -5 & -1 & 1 \end{bmatrix}
    \]
17. \[ \begin{bmatrix} 1 & -3 & -2 \\ 2 & -5 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} x - 3y = -2 \\ 2x - 5y = 5 \end{bmatrix} \]

\[ R_2 = -2 \eta_1 + r_2 \]

\[ \begin{bmatrix} 1 & -3 & -2 \\ 2 & -5 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & -2 \\ -2(1) + 2 & 2(-3) - 5 & -2(-2) + 5 \end{bmatrix} \]

\[ \begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & 9 \end{bmatrix} \]

18. \[ \begin{bmatrix} 1 & -3 & -3 \\ 2 & -5 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} x - 3y = -3 \\ 2x - 5y = -4 \end{bmatrix} \]

\[ R_2 = -2 \eta_1 + r_2 \]

\[ \begin{bmatrix} 1 & -3 & -3 \\ 2 & -5 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & -3 \\ -2(1) + 2 & -2(-3) - 5 & -2(-3) - 4 \end{bmatrix} \]

\[ \begin{bmatrix} 1 & -3 & -3 \\ 0 & 1 & 2 \end{bmatrix} \]

19. \[ \begin{bmatrix} 1 & -3 & 4 & 3 \\ 3 & -5 & 6 & 6 \\ -5 & 3 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} x - 3y + 4z = 3 \\ 3x - 5y + 6z = 6 \\ -5x + 3y + 4z = 6 \end{bmatrix} \]

\[ R_2 = -3 \eta_1 + r_2 \]

\[ \begin{bmatrix} 1 & -3 & 4 & 3 \\ 3 & -5 & 6 & 6 \\ -5 & 3 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 4 \\ -3(1) + 3 & -3(-3) - 5 & -3(4) + 6 & -3(3) + 6 \\ -5 & 3 & 4 & 6 \\ 0 & 4 & -6 & -3 \\ -5 & 3 & 4 & 6 \end{bmatrix} \]

\[ \begin{bmatrix} 1 & -3 & 4 \\ -5 & 3 & 4 \\ 0 & 4 & -6 \\ -5 & 3 & 4 \end{bmatrix} \]

20. \[ \begin{bmatrix} 1 & -3 & 3 & -5 \\ -4 & -5 & -3 & -5 \\ -3 & -2 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 3 \\ -3 & -2 & 4 \\ 0 & 17 & 9 \\ -3 & 2 & 4 \\ 0 & 11 & 13 \\ 1 & -3 & 3 \\ 0 & 17 & 9 \\ 0 & 11 & 13 \end{bmatrix} \]

\[ R_2 = 4 \eta_1 + r_2 \]

\[ \begin{bmatrix} 1 & -3 & 3 & -5 \\ -4 & -5 & -3 & -5 \\ -3 & -2 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 3 \\ -3 & -2 & 4 \\ 0 & 17 & 9 \\ -3 & 2 & 4 \\ 0 & 11 & 13 \\ 1 & -3 & 3 \\ 0 & 17 & 9 \\ 0 & 11 & 13 \end{bmatrix} \]

\[ R_3 = 3 \eta_1 + r_3 \]

\[ \begin{bmatrix} 1 & -3 & 3 & -5 \\ 0 & -17 & 9 & -25 \\ 3(1) - 3 & 3(-3) - 2 & 3(3) + 4 & 3(-5) + 6 \\ 1 & -3 & 3 & -5 \\ 0 & 17 & 9 & -25 \\ 0 & 11 & 13 & -9 \end{bmatrix} \]

21. \[ \begin{bmatrix} 1 & -3 & 2 & -6 \\ 2 & -5 & 3 & -4 \\ -3 & -6 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} x - 3y + 2z = -6 \\ 2x - 5y + 3z = -4 \\ -3x - 6y + 4z = 6 \end{bmatrix} \]

\[ R_2 = -2 \eta_1 + r_2 \]

\[ \begin{bmatrix} 1 & -3 & 2 & -6 \\ 2 & -5 & 3 & -4 \\ -3 & -6 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 2 \\ -2(1) + 2 & -2(-3) - 5 & -2(2) + 3 & -2(-6) - 4 \\ -3 & -6 & 4 & 6 \\ 1 & -3 & 2 \\ 0 & 1 & -1 \\ -3 & 6 & 4 \end{bmatrix} \]

\[ R_3 = 5 \eta_1 + r_3 \]

\[ \begin{bmatrix} 1 & -3 & 4 & 3 \\ 0 & 4 & -6 & -3 \\ -5 & 3 & 4 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -3 & 4 \\ 0 & 4 & -6 \\ -5 & 3 & 4 \end{bmatrix} \]

\[ \begin{bmatrix} 0 & 4 & -6 \\ -5 & 3 & 4 \\ 0 & -12 & 24 & 21 \end{bmatrix} \]
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\[ R_3 = 3r_1 + r_3 \]
\[
\begin{bmatrix}
1 & -3 & 2 & -6 \\
0 & 1 & -1 & 8 \\
-3 & -6 & 4 & 6 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & 2 & -6 \\
0 & 1 & -1 & 8 \\
3(-3) & 3(-1) & -6 & 3(-6) + 6 \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & -3 & 2 & -6 \\
0 & 1 & -1 & 8 \\
1 & -3 & 2 & -6 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & 2 & -6 \\
0 & 1 & -1 & 8 \\
0 & -1 & -1 & 8 \\
\end{bmatrix}
\]
\[
x - 3y - 4z = -6
\]
\[
6x - 5y + 6z = -6
\]
\[
-x + y + 4z = 6
\]

22. \[ R_2 = -6r_1 + r_2 \]
\[
\begin{bmatrix}
1 & -3 & -4 & -6 \\
6 & -5 & 6 & -6 \\
-1 & 1 & 4 & 6 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & -4 & -6 \\
6(-3) & 6(-4) + 6 & -6(-6) + 6 \\
1 & -3 & -4 & -6 \\
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & -3 & -4 \\
0 & 1 & -1 \\
-1 & 1 & 4 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -3 & -4 \\
0 & 1 & -1 \\
0 & 13 & 30 \\
\end{bmatrix}
\]

b. \[ R_3 = r_1 + r_3 \]
\[
\begin{bmatrix}
1 & -3 & 2 & -2 \\
5 & -3 & 1 & -2 \\
-4 & 1 & 4 & 6 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
5x - 3y + z = -2 \\
-4x + y + 4z = 6 \\
\end{bmatrix}
\]

23. \[ R_3 = 2r_2 + r_3 \]
\[
\begin{bmatrix}
1 & 7 & -11 & 2 \\
2 & -5 & 6 & -2 \\
-4 & 1 & 4 & 6 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 7 & -11 \\
2(2) & -5 & 6 \\
-4 & 1 & 4 \\
\end{bmatrix}
\]
\[
2(2) + (-5) + 1 \\
2(6) + 4 \\
2(-2) + 6 \\
\]
\[
\begin{bmatrix}
1 & 7 & -11 \\
2 & -5 & 6 \\
-4 & 1 & 4 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 7 & -11 \\
2 & -5 & 6 \\
0 & -9 & 16 \\
\end{bmatrix}
\]

24. \[ R_4 = -r_2 + r_1 \]
\[
\begin{bmatrix}
4 & -3 & 1 & 2 \\
3 & -5 & 2 & 6 \\
-3 & -6 & 4 & 6 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
4 & -3 & 1 & 2 \\
3 & -5 & 2 & 6 \\
-3 & -6 & 4 \\
\end{bmatrix}
\]
\[
2(-3) + 0(2) + 6(-6) + 2 \\
\]
\[
\begin{bmatrix}
1 & 2 & -3 & -4 \\
3 & -5 & 2 & 6 \\
-3 & -6 & 4 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & -3 & -4 \\
3 & -5 & 2 & 6 \\
0 & -9 & 16 \\
\end{bmatrix}
\]

25. \[
\begin{cases}
x = 5 \\
y = -1
\end{cases}
\]
Consistent; \( x = 5, y = -1 \), or using ordered pairs \((5, -1)\).

26. \[
\begin{cases}
x = -4 \\
y = 0
\end{cases}
\]
Consistent; \( x = -4, y = 0 \), or using ordered pairs \((-4, 0)\).

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27. \[ \begin{cases} x = 1 \\ y = 2 \\ 0 = 3 \end{cases} \]
Inconsistent

28. \[ \begin{cases} x = 0 \\ y = 0 \\ 0 = 2 \end{cases} \]
Inconsistent

29. \[ \begin{cases} x + 2z = -1 \\ y - 4z = -2 \\ 0 = 0 \end{cases} \]
Consistent; \( z \) is any real number
or \( \{(x, y, z) \mid x = -1 - 2z, y = -2 + 4z, z \text{ is any real number}\} \)

30. \[ \begin{cases} x + 4z = 4 \\ y + 3z = 2 \\ 0 = 0 \end{cases} \]
Consistent; \( x = 4 - 4z \\ y = 2 - 3z \\ z \) is any real number
or \( \{(x, y, z) \mid x = 4 - 4z, y = 2 - 3z, z \text{ is any real number}\} \)

31. \[ \begin{cases} x = 1 \\ x_2 + x_4 = 2 \\ x_3 + 2x_4 = 3 \end{cases} \]
Consistent; \( x_1 = 1 \\ x_2 = 2 - x_4 \\ x_3 = 3 - 2x_4 \\ x_4 \) is any real number
or \( \{(x_1, x_2, x_3, x_4) \mid x_1 = 1, x_2 = 2 - x_4, x_3 = 3 - 2x_4, x_4 \text{ is any real number}\} \)

32. \[ \begin{cases} x_1 = 1 \\ x_2 + 2x_4 = 2 \\ x_3 + 3x_4 = 0 \end{cases} \]
Consistent; \( x_1 = 1 \\ x_2 = 2 - 2x_4 \\ x_3 = -3x_4 \\ x_4 \) is any real number
or \( \{(x_1, x_2, x_3, x_4) \mid x_1 = 1, x_2 = 2 - 2x_4, x_3 = -3x_4, x_4 \text{ is any real number}\} \)

33. \[ \begin{cases} x_2 + x_3 + 3x_4 = 3 \\ 0 = 0 \end{cases} \]
Consistent; \( x_1 = 2 - 4x_4 \\ x_2 = 3 - x_3 - 3x_4 \\ x_3, x_4 \) are any real numbers
or \( \{(x_1, x_2, x_3, x_4) \mid x_1 = 2 - 4x_4, x_2 = 3 - x_3 - 3x_4, x_3 \text{ and } x_4 \text{ are any real numbers}\} \)

34. \[ \begin{cases} x_2 = 2 \\ x_3 + 2x_4 = 3 \end{cases} \]
Consistent; \( x_1 = 1 \\ x_2 = 2 \\ x_3 = 3 - 2x_4 \\ x_4 \) is any real number
or \( \{(x_1, x_2, x_3, x_4) \mid x_1 = 1, x_2 = 2, x_3 = 3 - 2x_4, x_4 \text{ is any real number}\} \)

35. \[ \begin{cases} x_1 + x_4 = -2 \\ x_2 + 2x_4 = 2 \\ x_3 - x_4 = 0 \\ 0 = 0 \end{cases} \]
Consistent; \( x_1 = -2 - x_4 \\ x_2 = 2 - 2x_4 \\ x_3 = x_4 \\ x_4 \) is any real number
or \( \{(x_1, x_2, x_3, x_4) \mid x_1 = -2 - x_4, x_2 = 2 - 2x_4, x_3 = x_4, x_4 \text{ is any real number}\} \)
Section 11.2: Systems of Linear Equations: Matrices

36. \[
\begin{align*}
\begin{cases}
  x_1 = 1 \\
  x_2 = 2 \\
  x_3 = 3 \\
  x_4 = 0
\end{cases}
\end{align*}
\]
Consistent;
\[
\begin{align*}
\begin{cases}
  x_1 = 1 \\
  x_2 = 2 \\
  x_3 = 3 \\
  x_4 = 0
\end{cases}
\end{align*}
\]
or (1, 2, 3, 0)

37. \[
\begin{align*}
\begin{cases}
  x + y = 8 \\
  x - y = 4
\end{cases}
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
1 & 1 & 8 \\
1 & -1 & 4
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 1 & 8 \\
0 & -2 & -4
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 1 & 8 \\
0 & 1 & 2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 6 \\
0 & 1 & 2
\end{bmatrix}
\]
\(R_2 = -\frac{1}{2}r_2\)
\(R_3 = -r_2 + r_1\)
The solution is \(x = 6, y = 2\) or using ordered pairs (6, 2).

38. \[
\begin{align*}
\begin{cases}
  x + 2y = 5 \\
  x - y = 3
\end{cases}
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
1 & 2 & 5 \\
1 & 1 & 3
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & 5 \\
0 & -1 & -2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & 5 \\
0 & 1 & 2
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 2
\end{bmatrix}
\]
\(R_2 = -r_2\)
\(R_3 = -2r_2 + r_1\)
The solution is \(x = 1, y = 2\) or using ordered pairs (1, 2).

39. \[
\begin{align*}
\begin{cases}
  2x - 4y = -2 \\
  3x + 2y = 3
\end{cases}
\end{align*}
\]
Write the augmented matrix:

40. \[
\begin{align*}
\begin{cases}
  3x + 3y = 3 \\
  4x + 2y = \frac{8}{3}
\end{cases}
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
3 & 3 & 3 \\
4 & 2 & \frac{8}{3}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 1 & 1 \\
0 & 2 & \frac{8}{3}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & \frac{1}{3} \\
0 & 1 & \frac{2}{3}
\end{bmatrix}
\]
\(R_2 = -\frac{4}{3}r_2\)
\(R_3 = -\frac{1}{2}r_2\)
\(R_1 = r_2 + r_1\)
The solution is \(x = \frac{1}{3}, y = \frac{2}{3}\) or using ordered pairs \(\left(\frac{1}{3}, \frac{2}{3}\right)\).

41. \[
\begin{align*}
\begin{cases}
  x + 2y = 4 \\
  2x + 4y = 8
\end{cases}
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
1 & 2 & 4 \\
2 & 4 & 8
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & 4 \\
0 & 0 & 0
\end{bmatrix}
\]
This is a dependent system.
\(x + 2y = 4\)
\(x = 4 - 2y\)
The solution is \(x = 4 - 2y, y \text{ is any real number}\) or \{(x, y) | x = 4 – 2y, y is any real number\}

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42. \[
\begin{align*}
3x - y &= 7 \\
9x - 3y &= 21
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
3 & -1 & 7 \\
9 & -3 & 21
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -\frac{1}{3} & \frac{7}{3} \\
0 & \frac{2}{3} & 1
\end{bmatrix}
\quad (R_1 = \frac{1}{3} \eta_1)
\]
\[
\rightarrow
\begin{bmatrix}
1 & \frac{1}{2} & \frac{7}{3} \\
0 & 0 & 0
\end{bmatrix}
\quad (R_2 = -9 \eta_1 + r_2)
\]
This is a dependent system.

3x - y = 7
3x - 7y = y
The solution is \( y = 3x - 7 \), \( x \) is any real number
or \( \{ (x, y) \mid y = 3x - 7, x \text{ is any real number} \} \)

43. \[
\begin{align*}
x + 3y &= 6 \\
x - y &= \frac{1}{2}
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
2 & 3 & 6 \\
1 & -1 & \frac{1}{2}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & \frac{3}{2} & 3 \\
0 & -\frac{5}{2} & -\frac{5}{2}
\end{bmatrix}
\quad (R_1 = \frac{1}{2} \eta_1)
\]
\[
\rightarrow
\begin{bmatrix}
1 & \frac{1}{2} & \frac{3}{2} \\
0 & 1 & 1
\end{bmatrix}
\quad (R_2 = -r_1 + r_2)
\]
\[
\rightarrow
\begin{bmatrix}
1 & 0 & \frac{3}{2} \\
0 & 1 & 1
\end{bmatrix}
\quad (R_2 = -\frac{1}{2} r_2)
\]
\[
\rightarrow
\begin{bmatrix}
1 & 0 & \frac{3}{2} \\
0 & 1 & 1
\end{bmatrix}
\quad (R_1 = -\frac{1}{2} r_2 + \eta_1)
\]
The solution is \( x = \frac{3}{2} \), \( y = 1 \) or \( \left( \frac{3}{2}, 1 \right) \).

44. \[
\begin{align*}
\frac{1}{2}x + y &= -2 \\
-x - 2y &= 8
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
\frac{1}{2} & 1 & -2 \\
1 & -2 & 8
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & -4 \\
0 & -4 & 8
\end{bmatrix}
\quad (R_1 = 2 \eta_1)
\]
\[
\rightarrow
\begin{bmatrix}
1 & 2 & -4 \\
0 & -4 & 12
\end{bmatrix}
\quad (R_2 = -r_1 + r_2)
\]
\[
\rightarrow
\begin{bmatrix}
1 & 2 & -4 \\
0 & 1 & -3
\end{bmatrix}
\quad (R_2 = -\frac{1}{2} r_2)
\]
\[
\rightarrow
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & -3
\end{bmatrix}
\quad (R_1 = -2 r_2 + \eta_1)
\]
The solution is \( x = 2, y = -3 \) or \( (2, -3) \).

45. \[
\begin{align*}
3x - 5y &= 3 \\
15x + 5y &= 21
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
3 & -5 & 3 \\
15 & 5 & 21
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -\frac{5}{3} & 1 \\
0 & 30 & 6
\end{bmatrix}
\quad (R_1 = \frac{1}{3} \eta_1)
\]
\[
\rightarrow
\begin{bmatrix}
1 & -\frac{5}{3} & 1 \\
0 & 1 & \frac{1}{5}
\end{bmatrix}
\quad (R_2 = -15 \eta_1 + r_2)
\]
\[
\rightarrow
\begin{bmatrix}
1 & 0 & \frac{3}{5} \\
0 & 1 & \frac{1}{5}
\end{bmatrix}
\quad (R_1 = \frac{5}{3} r_2 + \eta_1)
\]
The solution is \( x = \frac{4}{5}, y = \frac{1}{5} \) or \( \left( \frac{4}{5}, \frac{1}{5} \right) \).

46. \[
\begin{align*}
x + \frac{1}{2}y &= \frac{3}{2} \\
x + \frac{1}{2}y &= \frac{3}{2}
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
2 & -1 & -1 \\
1 & \frac{1}{2} & \frac{3}{2}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & 1 & \frac{1}{2}
\end{bmatrix}
\quad (R_1 = \frac{1}{2} \eta_1)
\]
\[
\rightarrow
\begin{bmatrix}
1 & 0 & -\frac{1}{2} \\
0 & 1 & \frac{1}{2}
\end{bmatrix}
\quad (R_1 = \frac{1}{2} r_2 + \eta_1)
\]
The solution is \( x = \frac{1}{2}, y = 2 \) or \( \left( \frac{1}{2}, 2 \right) \).

47. \[
\begin{align*}
x - y &= 6 \\
2x - 3z &= 16 \\
2y + z &= 4
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
1 & -1 & 0 & 6 \\
2 & 0 & -3 & 16 \\
0 & 2 & 1 & 4
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & 0 & 6 \\
0 & 2 & -3 & 4 \\
0 & 2 & 1 & 4
\end{bmatrix}
\quad (R_2 = -2 \eta_1 + r_2)
\]
\[
\rightarrow
\begin{bmatrix}
1 & -1 & 0 & 6 \\
0 & 1 & -\frac{3}{2} & 2 \\
0 & 2 & 1 & 4
\end{bmatrix}
\quad (R_2 = \frac{1}{2} r_2)
\]
The solution is \( x = 2, y = -3 \) or \( (2, -3) \).
Section 11.2: Systems of Linear Equations: Matrices

### 49. Systems of Linear Equations

The solution is \( x = 8, y = 2, z = 0 \) or \((8, 2, 0)\).

### 50. Systems of Linear Equations

The solution is \( x = -3, y = 2, z = 1 \) or \((-3, 2, 1)\).
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11.00

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\[
\begin{bmatrix}
1 & \frac{1}{2} & -\frac{3}{2} & 0 \\
0 & 1 & -\frac{2}{3} & -\frac{7}{3} \\
0 & -\frac{11}{2} & \frac{3}{2} & 7 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -\frac{7}{6} & \frac{7}{6} \\
0 & 1 & -\frac{2}{3} & -\frac{7}{3} \\
0 & 0 & -\frac{13}{6} & -\frac{35}{6} \\
\end{bmatrix}
\begin{bmatrix}
R_2 = \frac{1}{3} r_2 \\
R_3 = -\frac{1}{3} r_2 + r_1 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & -\frac{7}{6} & \frac{7}{6} \\
0 & 1 & -\frac{2}{3} & -\frac{7}{3} \\
0 & 0 & 1 & \frac{7}{3} \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & \frac{56}{13} \\
0 & 1 & 0 & \frac{7}{13} \\
0 & 0 & 1 & \frac{35}{13} \\
\end{bmatrix}
\begin{bmatrix}
R_1 = \frac{7}{6} r_3 + r_1 \\
R_2 = \frac{7}{3} r_3 + r_2 \\
\end{bmatrix}
\]

The solution is \( x = \frac{56}{13}, y = -\frac{7}{13}, z = \frac{35}{13} \) or \( \left( \frac{56}{13}, -\frac{7}{13}, \frac{35}{13} \right) \).

51. \[
\begin{align*}
2x - 3y - z &= 2 \\
2x + 3y + z &= 2 \\
3x + 2y &= 0
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
2 & -2 & -2 \\
2 & 3 & 1 \\
3 & 2 & 0 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & -1 & 1 \\
2 & 3 & 1 & 2 \\
3 & 2 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
R_1 = \frac{1}{2} r_1 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & -1 & -1 & 1 \\
0 & 5 & 3 & 0 \\
1 & -1 & -1 & 1 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 5 & 3 \\
0 & 5 & 3 & -3 \\
0 & 0 & 0 & -3 \\
\end{bmatrix}
\begin{bmatrix}
R_2 = -2 r_1 + r_2 \\
R_3 = -3 r_1 + r_3 \\
\end{bmatrix}
\]

There is no solution. The system is inconsistent.

52. \[
\begin{align*}
2x - 3y - z &= 0 \\
-x + 2y + z &= 5 \\
3x - 4y - z &= 1
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
2 & -3 & -1 & 0 \\
-1 & 2 & 1 & 5 \\
3 & -4 & -1 & 1 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & -1 & -5 \\
2 & -3 & -1 & 0 \\
3 & -4 & -1 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\text{Interchange} r_1 \text{ and } r_2 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & -2 & -1 & -5 \\
0 & 1 & 1 & 10 \\
0 & 2 & 2 & 16 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & -1 & -5 \\
0 & 1 & 1 & 10 \\
0 & 0 & 0 & -4 \\
\end{bmatrix}
\begin{bmatrix}
R_2 = -2 r_1 + r_2 \\
R_3 = -3 r_1 + r_3 \\
\end{bmatrix}
\]

There is no solution. The system is inconsistent.

53. \[
\begin{align*}
-x + y + z &= -1 \\
-x + 2y - 3z &= -4 \\
3x - 2y - 7z &= 0
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
-1 & 1 & 1 & -1 \\
-1 & 2 & -3 & -4 \\
3 & -2 & -7 & 0 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -1 & -1 & 1 \\
-1 & 2 & -3 & -4 \\
3 & -2 & -7 & 0 \\
\end{bmatrix}
\begin{bmatrix}
R_1 = -r_1 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & -1 & -1 & 1 \\
0 & 1 & -4 & -3 \\
0 & 1 & -4 & -3 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & -5 & -2 \\
0 & 1 & -4 & -3 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
R_2 = r_2 + r_1 \\
R_3 = -r_2 + r_3 \\
\end{bmatrix}
\]

The matrix in the last step represents the system
\[
\begin{align*}
x - 5z &= -2 \\
y - 4z &= -3 \\
0 &= 0
\end{align*}
\]
and, equivalently,
\[
\begin{align*}
x &= 5z - 2 \\
y &= 4z - 3 \\
0 &= 0
\end{align*}
\]
The solution is \( x = 5z - 2, \ y = 4z - 3, \) and \( z \) is any real number or \( \{(x, y, z) \mid x = 5z - 2, y = 4z - 3, z \text{ is any real number}\} \).
Section 11.2: Systems of Linear Equations: Matrices

54. \[
\begin{align*}
2x - 3y - z &= 0 \\
3x + 2y + 2z &= 2 \\
x + 5y + 3z &= 2
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
2 & -3 & -1 & 0 \\
3 & 2 & 2 & 2 \\
1 & 5 & 3 & 2
\end{bmatrix}
\]
\[
\rightarrow \begin{bmatrix}
1 & 5 & 3 & 2 \\
3 & 2 & 2 & 2 \\
2 & -3 & -1 & 0
\end{bmatrix}
\]
(Interchange \(r_1\) and \(r_3\))
\[
\rightarrow \begin{bmatrix}
1 & 5 & 3 & 2 \\
0 & -13 & -7 & -4 \\
0 & -13 & -7 & -4
\end{bmatrix}
\]
\((R_2 = -3r_1 + r_2)\)
\[
\rightarrow \begin{bmatrix}
1 & 5 & 3 & 2 \\
0 & 1 & \frac{7}{13} & \frac{4}{13} \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
\((R_3 = -r_2 + r_3)\)
\[
\rightarrow \begin{bmatrix}
1 & 5 & 3 & 2 \\
0 & 1 & \frac{7}{13} & \frac{4}{13} \\
0 & 0 & \frac{4}{13} & \frac{6}{13}
\end{bmatrix}
\]
\((R_1 = -5r_2 + r_1)\)

The matrix in the last step represents the system:
\[
\begin{align*}
x + \frac{4}{13}z &= \frac{6}{13} \\
y + \frac{7}{13}z &= \frac{4}{13} \\
0 &= 0
\end{align*}
\]
or, equivalently,
\[
\begin{align*}
x &= -\frac{4}{13}z + \frac{6}{13} \\
y &= -\frac{7}{13}z + \frac{4}{13} \\
0 &= 0
\end{align*}
\]
The solution is \(x = -\frac{4}{13}z + \frac{6}{13}, y = -\frac{7}{13}z + \frac{4}{13}\),

\(z\) is any real number or \(\{(x, y, z) | x = -\frac{4}{13}z + \frac{6}{13}, y = -\frac{7}{13}z + \frac{4}{13}\}\).

55. \[
\begin{align*}
2x - 2y + 3z &= 6 \\
4x - 3y + 2z &= 0 \\
-2x + 3y - 7z &= 1
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
2 & -2 & 3 & 6 \\
4 & -3 & 2 & 0 \\
-2 & 3 & -7 & 1
\end{bmatrix}
\]
\[
\rightarrow \begin{bmatrix}
1 & -1 & \frac{3}{2} & 3 \\
4 & -3 & 2 & 0 \\
-2 & 3 & -7 & 1
\end{bmatrix}
\]
\((R_1 = \frac{1}{2}r_1)\)
\[
\rightarrow \begin{bmatrix}
1 & -1 & \frac{3}{2} & 3 \\
0 & 1 & -4 & 12 \\
0 & 1 & -4 & 7
\end{bmatrix}
\]
\((R_2 = -4r_1 + r_2)\)
\[
\rightarrow \begin{bmatrix}
1 & 0 & -\frac{5}{2} & -9 \\
0 & 1 & -4 & -12 \\
0 & 0 & 0 & 19
\end{bmatrix}
\]
\((R_3 = r_2 + r_1)\)

There is no solution. The system is inconsistent.

56. \[
\begin{align*}
3x - 2y + 2z &= 6 \\
7x - 3y + 2z &= 1 \\
2x - 3y + 4z &= 0
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
3 & -2 & 2 & 6 \\
7 & -3 & 2 & 1 \\
2 & -3 & 4 & 0
\end{bmatrix}
\]
\[
\rightarrow \begin{bmatrix}
1 & -\frac{2}{3} & \frac{2}{3} & 2 \\
7 & -3 & 2 & -1 \\
2 & -3 & 4 & 0
\end{bmatrix}
\]
\((R_1 = \frac{1}{3}r_1)\)
\[
\rightarrow \begin{bmatrix}
1 & -\frac{2}{3} & \frac{2}{3} & 2 \\
0 & \frac{2}{3} & -\frac{5}{3} & -17 \\
0 & -\frac{5}{3} & \frac{8}{3} & -4
\end{bmatrix}
\]
\((R_2 = -7r_1 + r_2)\)
\[
\rightarrow \begin{bmatrix}
1 & -\frac{2}{3} & \frac{2}{3} & 2 \\
0 & \frac{2}{3} & -\frac{5}{3} & -15 \\
0 & 0 & 0 & -19
\end{bmatrix}
\]
\((R_3 = r_2 + r_3)\)

There is no solution. The system is inconsistent.
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57. \[
\begin{align*}
3x - 2y + z &= -5 \\
x + 3y - 2z &= 14
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
1 & 1 & -1 & 6 \\
3 & -2 & 1 & -5 \\
1 & 3 & -2 & 14
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & 1 & -1 & 6 \\
0 & -5 & 4 & -23 \\
0 & 2 & -1 & 8
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & 1 & -1 & 6 \\
0 & 1 & -\frac{4}{5} & \frac{23}{5} \\
0 & 0 & \frac{3}{5} & -\frac{6}{5}
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 3 \\
0 & 0 & 1 & -2
\end{bmatrix}
\]
The solution is \(x = 1, y = 3, z = -2\) or \((1, 3, -2)\).

58. \[
\begin{align*}
x + y - z &= -4 \\
2x - 3y + 4z &= -15 \\
5x + y - 2z &= 12
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
1 & -1 & 1 & -4 \\
2 & -3 & 4 & -15 \\
5 & 1 & -2 & 12
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & -1 & 1 & -4 \\
0 & -1 & 2 & -7 \\
0 & 6 & -7 & 32
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & -1 & 1 & -4 \\
0 & 1 & -2 & 7 \\
0 & 6 & -7 & 32
\end{bmatrix}
\]
The solution is \(x = -3, y = \frac{1}{2}, z = 1\) or \((-3, \frac{1}{2}, 1)\).

59. \[
\begin{align*}
x - 2y - z &= -3 \\
2x - 4y + z &= -7 \\
2x + 2y - 3z &= 4
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
1 & 2 & -1 & -3 \\
2 & -4 & 1 & -7 \\
-2 & 2 & -3 & 4
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & 2 & -1 & -3 \\
0 & -8 & 3 & -1 \\
0 & 6 & -5 & -2
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 3 \\
0 & 0 & 1 & -2
\end{bmatrix}
\]
The solution is \(x = 1, y = 3, z = -2\) or \((1, 3, -2)\).
Section 11.2: Systems of Linear Equations: Matrices

60. \[
\begin{align*}
 \begin{cases}
 x + 4y - 3z &= -8 \\
 3x - y + 3z &= 12 \\
 2x + y + 6z &= 1
\end{cases}
\end{align*}
\]

Write the augmented matrix:
\[
\begin{bmatrix}
 1 & 4 & -3 & | & -8 \\
 3 & -1 & 3 & | & 12 \\
 1 & 1 & 6 & | & 1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
 1 & 4 & -3 & | & -8 \\
 0 & -13 & 36 & | & 36 \\
 0 & -3 & 9 & | & 9
\end{bmatrix}
\]

\[
\begin{align*}
 R_2 &= -3r_1 + r_2 \\
 R_3 &= -r_1 + r_3
\end{align*}
\]

\[
\begin{bmatrix}
 0 & -1 & 12 & | & 36 \\
 0 & -3 & 9 & | & 9
\end{bmatrix}
\rightarrow
\begin{bmatrix}
 0 & 1 & -\frac{12}{3} & | & -\frac{36}{9} \\
 0 & -3 & 9 & | & 9
\end{bmatrix}
\]

\[
R_2 = -\frac{1}{3}r_2
\]

\[
\begin{align*}
 R_3 &= -r_1 + r_3 \\
 R_3 &= 3r_2 + r_3
\end{align*}
\]

The solution is \( x = \frac{1}{3}, y = \frac{2}{3}, z = 1 \) or \( \left( \frac{1}{3}, \frac{2}{3}, 1 \right) \).

61. \[
\begin{align*}
 \begin{cases}
 3x + y - z &= \frac{2}{3} \\
 2x - y + z &= 1 \\
 4x + 2y + z &= \frac{8}{3}
\end{cases}
\end{align*}
\]

Write the augmented matrix:
\[
\begin{bmatrix}
 3 & 1 & -1 & | & \frac{2}{3} \\
 2 & -1 & 1 & | & 1 \\
 4 & 2 & 0 & | & \frac{8}{3}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
 1 & \frac{1}{3} & -\frac{1}{3} & | & \frac{2}{3} \\
 2 & -1 & 1 & | & 1 \\
 4 & 2 & 0 & | & \frac{8}{3}
\end{bmatrix}
\]

\[
R_1 = \frac{1}{3}r_1
\]

62. \[
\begin{align*}
 \begin{cases}
 x + y &= 1 \\
 2x - y + z &= 1 \\
x + 2y + z &= \frac{8}{3}
\end{cases}
\end{align*}
\]

Write the augmented matrix:
\[
\begin{bmatrix}
 1 & 1 & 0 & | & 1 \\
 2 & -1 & 1 & | & 1 \\
 1 & 2 & 1 & | & \frac{8}{3}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
 1 & 1 & 0 & | & 1 \\
 0 & -3 & 1 & | & -1 \\
 0 & 1 & 1 & | & \frac{5}{3}
\end{bmatrix}
\]

\[
\begin{align*}
 R_2 &= -2r_1 + r_2 \\
 R_3 &= -r_1 + r_3
\end{align*}
\]

Interchange \( r_2 \) and \( r_3 \)

\[
\begin{bmatrix}
 1 & 1 & 0 & | & 1 \\
 0 & 1 & 1 & | & \frac{5}{3} \\
 0 & -3 & 1 & | & -1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
 1 & 1 & 0 & | & 1 \\
 0 & 1 & 1 & | & \frac{5}{3} \\
 0 & 0 & 1 & | & 4
\end{bmatrix}
\]

\[
R_3 = 3r_2 + r_3
\]
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\[
\begin{bmatrix}
1 & 1 & 0 & 1 \\
0 & 1 & 1 & \frac{5}{3} \\
0 & 0 & 1 & 1
\end{bmatrix}
\quad (R_3 = \frac{1}{3} r_3)
\]

\[
\begin{bmatrix}
1 & 1 & 0 & 1 \\
0 & 1 & 0 & \frac{2}{3} \\
0 & 0 & 1 & 1
\end{bmatrix}
\quad (R_2 = r_2 - r_3)
\]

\[
\begin{bmatrix}
1 & 0 & 0 & \frac{1}{3} \\
0 & 1 & 0 & \frac{2}{3} \\
0 & 0 & 1 & 1
\end{bmatrix}
\quad (R_1 = r_1 - r_2)
\]

The solution is \( x = \frac{2}{3}, y = \frac{2}{3}, z = 1 \) or \( \left( \frac{1}{3}, \frac{2}{3}, 1 \right) \).

63.

\[
\begin{align*}
&x + y + z + w = 4 \\
&2x - y + z = 0 \\
&3x + 2y + z - w = 6 \\
&x - 2y - 2z + 2w = -1
\end{align*}
\]

Write the augmented matrix:

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 4 \\
2 & -1 & 1 & 0 & 0 \\
3 & 2 & 1 & -1 & 6 \\
1 & -2 & 2 & -1 & -1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 4 \\
0 & -3 & -1 & -2 & -8 \\
0 & -1 & -2 & -4 & -6 \\
0 & -3 & -3 & 1 & -5
\end{bmatrix}
\quad (R_2 = -2r_1 + r_2, \quad R_3 = -3r_1 + r_3, \quad R_4 = -r_1 + r_4)
\]

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 4 \\
0 & -1 & -2 & -4 & -6 \\
0 & -3 & -3 & 1 & -5
\end{bmatrix}
\quad \text{(Interchange \( r_2 \) and \( r_3 \))}
\]

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 4 \\
0 & 1 & 2 & 4 & 6 \\
0 & -3 & -1 & -2 & -8 \\
0 & -3 & -3 & 1 & -5
\end{bmatrix}
\quad (R_2 = -r_2)
\]

\[
\begin{bmatrix}
1 & 0 & -1 & -3 & -2 \\
0 & 1 & 2 & 4 & 6 \\
0 & 0 & 5 & 10 & 10 \\
0 & 0 & 3 & 13 & 13
\end{bmatrix}
\quad (R_1 = -r_2 + r_1, \quad R_3 = 3r_2 + r_3, \quad R_4 = 3r_2 + r_4)
\]

64.

\[
\begin{align*}
&x + y + z + w = 4 \\
&-x + 2y + z = 0 \\
&2x + 3y + z - w = 6 \\
&-2x + y - 2z + 2w = -1
\end{align*}
\]

Write the augmented matrix:

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 4 \\
-1 & 2 & 1 & 0 & 0 \\
2 & 3 & 1 & -1 & 6 \\
-2 & 1 & -2 & 2 & -1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 4 \\
0 & 3 & 2 & 1 & 4 \\
0 & 1 & -1 & -3 & -2 \\
0 & 3 & 0 & 4 & 7
\end{bmatrix}
\quad (R_2 = r_1 + r_2, \quad R_3 = -2r_1 + r_3, \quad R_4 = 2r_1 + r_4)
\]

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 4 \\
0 & 1 & -1 & -3 & -2 \\
0 & 3 & 2 & 1 & 4 \\
0 & 3 & 0 & 4 & 7
\end{bmatrix}
\quad \text{(Interchange \( r_2 \) and \( r_3 \))}
\]

\[
\begin{bmatrix}
1 & 0 & -1 & -3 & -2 \\
0 & 1 & 2 & 4 & 6 \\
0 & 0 & 5 & 10 & 10 \\
0 & 0 & 3 & 13 & 13
\end{bmatrix}
\quad (R_1 = -r_2 + r_1, \quad R_3 = -3r_2 + r_3, \quad R_4 = -3r_2 + r_4)
\]
Section 11.2: Systems of Linear Equations: Matrices

Write the matrix as the corresponding system:
\[ \begin{align*}
2x + 6y + 2z &= 10 \\
3y + 2z &= -2 \\
4 + 2w &= 2 \\
w &= 1
\end{align*} \]

Substitute and solve:
\[ \begin{align*}
x &= 2 \\
y &= 1 \\
z &= 0 \\
w &= 1
\end{align*} \]

The solution is \( (x, y, z, w) = (2, 1, 0, 1) \).
68. \[
\begin{aligned}
2x + y - z &= 4 \\
-x + y + 3z &= 1
\end{aligned}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
2 & 1 & -1 \\
-1 & 1 & 3
\end{bmatrix} \quad \rightarrow \quad \begin{bmatrix}
1 & 1 & -2 \\
2 & 3 & -1
\end{bmatrix}
\]
\( \text{interchange } r_1 \text{ and } r_2 \)

The matrix in the last step represents the system:
\[
\begin{aligned}
x & = 1 + \frac{4}{3}z \\
y &= 2 - \frac{5}{3}z
\end{aligned}
\]
Thus, the solution is:
\[
\begin{aligned}
x &= 1 + \frac{4}{3}z \\
y &= 2 - \frac{5}{3}z \\
z &= \text{any real number}
\end{aligned}
\]

69. \[
\begin{aligned}
2x + 3y - z &= 3 \\
x - y - z &= 0 \\
-x + y + z &= 0 \\
x + y + 3z &= 5
\end{aligned}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
2 & 3 & -1 \\
1 & -1 & -1 \\
-1 & 1 & 1 \\
1 & 1 & 3
\end{bmatrix} \quad \rightarrow \quad \begin{bmatrix}
1 & 1 & -10 \\
2 & 3 & -13 \\
-1 & 1 & 10 \\
1 & 1 & 3
\end{bmatrix}
\]
\( \text{interchange } r_1 \text{ and } r_2 \)

The matrix in the last step represents the system:
\[
\begin{aligned}
x - 8z &= -7 \\
y - 7z &= -7 \\
z &= \frac{19}{18}
\end{aligned}
\]
Substitute and solve:
\[
\begin{aligned}
y - 7\left(\frac{19}{18}\right) &= 7 \\
x - 8\left(\frac{19}{18}\right) &= -7
\end{aligned}
\]
Thus, the solution is:
\[
\begin{aligned}
x &= \frac{13}{9} \\
y &= \frac{7}{18} \\
z &= \frac{19}{18}
\end{aligned}
\]

70. \[
\begin{aligned}
x - 3y + z &= 1 \\
2x - 4y &= 0 \\
x - 3y + 2z &= 1 \\
x - 2y &= 5
\end{aligned}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
1 & -3 & 1 & 1 \\
2 & -1 & -4 & 0 \\
1 & -3 & 2 & 1 \\
1 & -2 & 0 & 5
\end{bmatrix} \quad \rightarrow \quad \begin{bmatrix}
1 & -1 & -10 \\
0 & 5 & 1 & 3 \\
0 & 0 & 0 & 0 \\
0 & 2 & 4 & 5
\end{bmatrix}
\]
\( \text{interchange } r_3 \text{ and } r_4 \)

The matrix in the last step represents the system:
\[
\begin{aligned}
x - 3y + z &= 1 \\
x &= \frac{7}{9} \\
z &= \frac{19}{18}
\end{aligned}
\]
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21 2
31 2
41 2

\( \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \)

\( R_2 = 2r_1 + r_2 \)

\( R_3 = -r_1 + r_2 \)

\( R_4 = -r_1 + r_2 \)

interchange \( \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \) and \( \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \)

\( \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \)

\( \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \)

\( R_1 = 3r_1 + r_2 \)

\( R_3 = -5r_2 + r_2 \)

\( \begin{bmatrix} 1 & 0 & 0 & 13 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & -22 \end{bmatrix} \)

\( \begin{bmatrix} 1 & 0 & 0 & 13 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & -22 \end{bmatrix} \)

\( R_2 = 2r_1 + r_2 \)

\( R_3 = r_3 + r_2 \)

\( R_4 = r_3 + r_4 \)

There is no solution. The system is inconsistent.

71. \( \begin{align*}
x + y + z - w &= 4 \\
x - y + 2z + 3w &= 3
\end{align*} \)

Write the augmented matrix:

\( \begin{bmatrix} 4 & 1 & 1 & -1 & 4 \\ 1 & -1 & 2 & 3 & 3 \end{bmatrix} \)

\( \begin{bmatrix} 1 & -1 & 2 & 3 & 3 \\ 4 & 1 & 1 & -1 & 14 \end{bmatrix} \)

interchange \( r_1 \) and \( r_2 \)

\( \begin{bmatrix} 1 & -1 & 2 & 3 & 3 \\ 0 & 5 & -7 & -13 & -8 \end{bmatrix} \)

\( \begin{bmatrix} 1 & -1 & 2 & 3 & 3 \\ 0 & 5 & -7 & -13 & -8 \end{bmatrix} \)

\( R_2 = -4r_1 + r_2 \)

The matrix in the last step represents the system

\( \begin{align*}
x &= -y + 2z + 3w = 3 \\
5y - 7z - 13w &= -8
\end{align*} \)

The second equation yields

\( 5y - 7z - 13w = -8 \)

\( 5y = 7z + 13w - 8 \)

\( y = \frac{7}{5}z + \frac{13}{5}w - \frac{8}{5} \)

The first equation yields

\( x - y + 2z + 3w = 3 \)

\( x = 3 + y - 2z - 3w \)

Substituting for \( y \):

\( x = 3 + \left( -\frac{8}{5} + \frac{7}{5}z + \frac{13}{5}w \right) - 2z - 3w \)

\( x = -\frac{3}{5}z - \frac{2}{5}w + \frac{7}{5} \)

Thus, the solution is

\( x = -\frac{3}{5}z - \frac{2}{5}w + \frac{7}{5}, \)

\( y = \frac{7}{5}z + \frac{13}{5}w - \frac{8}{5}, \) and \( w \) are any real numbers or

\( \{(x, y, z, w) \mid x = -\frac{3}{5}z - \frac{2}{5}w + \frac{7}{5}, y = \frac{7}{5}z + \frac{13}{5}w - \frac{8}{5}, \) \( z \) and \( w \) are any real numbers \}.

72. \( \begin{align*}
-4x + y &= 5 \\
2x - y + z - w &= 5 \\
z + w &= 4
\end{align*} \)

Write the augmented matrix:

\( \begin{bmatrix} -4 & 1 & 0 & 0 & 5 \\ 2 & -1 & 1 & -1 & 5 \\ 0 & 0 & 1 & 1 & 4 \end{bmatrix} \)

\( \begin{bmatrix} 1 & -1 & 0 & 0 & -\frac{5}{4} \\ 0 & 1 & 1 & 1 & 4 \\ 1 & -1 & 0 & 0 & -\frac{5}{4} \end{bmatrix} \)

\( \begin{bmatrix} 0 & -1 & 2 & -2 & 15 \\ 0 & 0 & 1 & 1 & 4 \end{bmatrix} \)

\( \begin{bmatrix} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & -5 \\ 0 & 0 & 1 & 1 & 4 \end{bmatrix} \)

\( \begin{bmatrix} 1 & 0 & 0 & -3 & 13 \\ 0 & 1 & 0 & 4 & -7 \end{bmatrix} \)

\( \begin{bmatrix} 1 & 0 & 0 & -3 & 13 \\ 0 & 1 & 0 & 4 & -7 \end{bmatrix} \)

\( R_1 = -\frac{1}{4}r_1 + r_2 \)

\( R_2 = -2r_2 \)

\( R_1 = \frac{1}{4}r_2 + r_1 \)

\( R_2 = 2r_3 + r_2 \)

The matrix in the last step represents the system

\( \begin{align*}
x + w &= -3 \\
y + 4w &= -7 \quad \text{or, equivalently,} \\
z + w &= 4
\end{align*} \)

The solution is \( x = -3 - w, \ y = -7 - 4w, \ z = 4 - w, \) \( w \) is any real number or \( \{(x, y, z, w) \mid x = -3 - w, \) \( y = -7 - 4w, \) \( z = 4 - w, w \) is any real number \}

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73. Each of the points must satisfy the equation

\[ y = ax^2 + bx + c \]

(1, 2): \[ 2 = a + b + c \]

(−2, −7): \[ -7 = 4a - 2b + c \]

(2, −3): \[ -3 = 4a + 2b + c \]

Set up a matrix and solve:

\[ \begin{bmatrix} 1 & 1 & 1 & 2 \\ 4 & 2 & 1 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & -6 & -3 & -15 \\ 0 & -2 & -3 & -11 \end{bmatrix} \]

\[ \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & \frac{1}{3} & \frac{5}{2} \\ 0 & -2 & -3 & -11 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & \frac{1}{3} & \frac{5}{2} \\ 0 & 0 & 1 & 3 \end{bmatrix} \]

\[ \begin{bmatrix} 1 & 0 & \frac{1}{3} & \frac{7}{2} \\ 0 & 1 & \frac{1}{3} & \frac{7}{2} \\ 0 & 0 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 1 & \frac{1}{3} & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix} \]

\[ \begin{bmatrix} R_1 = -r_2 + r_3 \\ R_2 = -4r_1 + r_2 \\ R_3 = -4r_1 + r_3 \end{bmatrix} \]

The solution is \( a = 1, b = -4, c = 2 \); so the equation is \( y = x^2 - 4x + 2 \).

74. Each of the points must satisfy the equation

\[ y = ax^2 + bx + c \]

(1, −1): \[ -1 = a + b + c \]

(3, −1): \[ -1 = 9a + 3b + c \]

(−2, 14): \[ 14 = 4a - 2b + c \]

Set up a matrix and solve:

\[ \begin{bmatrix} 1 & 1 & 1 & -1 \\ 9 & 3 & 1 & -1 \\ 4 & -2 & 1 & 14 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & -6 & -8 & 8 \\ 0 & -6 & -3 & 18 \end{bmatrix} \]

\[ \begin{bmatrix} R_2 = -9r_1 + r_2 \\ R_3 = -4r_1 + r_3 \end{bmatrix} \]

The solution is \( a = -2, b = 1, c = 3 \); so the equation is \( y = -2x^2 + x + 3 \).

75. Each of the points must satisfy the equation

\[ f(x) = ax^3 + bx^2 + cx + d \]

\( f(-3) = -112 \): \[ -27a + 9b - 3c + d = -112 \]

\( f(-1) = -2 \): \[ -a + b - c + d = -2 \]

\( f(1) = 4 \): \[ a + b + c + d = 4 \]

\( f(2) = 13 \): \[ 8a + 4b + 2c + d = 13 \]

Set up a matrix and solve:

\[ \begin{bmatrix} -27 & 9 & -3 & 1 & -112 \\ -1 & 1 & -1 & 1 & -2 \\ 1 & 1 & 1 & 1 & 4 \\ 8 & 4 & 2 & 1 & 13 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ -1 & 1 & -1 & 1 & -2 \\ -27 & 9 & -3 & 1 & -112 \\ 8 & 4 & 2 & 1 & 13 \end{bmatrix} \]

\( \begin{bmatrix} R_2 = r_1 + r_2 \\ R_3 = 27r_1 + r_3 \\ R_4 = -8r_1 + r_4 \end{bmatrix} \)

Interchange \( r_3 \) and \( r_1 \)

\[ \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & 2 & 0 & 2 & -4 \\ 0 & 36 & 24 & 28 & -4 \\ 0 & -4 & -6 & -7 & -19 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 36 & 24 & 28 & -4 \\ 0 & -4 & -6 & -7 & -19 \end{bmatrix} \]

\( \begin{bmatrix} R_2 = \frac{1}{2}r_2 \\ R_3 = r_1 + r_2 \\ R_4 = -8r_1 + r_4 \end{bmatrix} \)

\[ \begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 24 & -8 & -40 \\ 0 & 0 & -6 & -3 & -15 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 24 & -8 & -40 \\ 0 & 0 & -6 & -3 & -15 \end{bmatrix} \]

The solution is \( a = 1, b = -4, c = 2 \); so the equation is \( y = x^2 - 4x + 2 \).
Section 11.2: Systems of Linear Equations: Matrices

The solution is \( a = 3, b = -4, c = 0, d = 5 \); so the equation is \( f(x) = 3x^3 - 4x^2 + 5 \).

76. Each of the points must satisfy the equation \( f(x) = ax^3 + bx^2 + cx + d \).

\[ f(-2) = -10: \quad -8a + 4b - 2c + d = -10 \]
\[ f(-1) = 3: \quad -a + b - c + d = 3 \]
\[ f(1) = 5: \quad a + b + c + d = 5 \]
\[ f(3) = 15: \quad 27a + 9b + 3c + d = 15 \]

Set up a matrix and solve:

\[
\begin{bmatrix}
-8 & 4 & -2 & 1 & -10 \\
-1 & 1 & -1 & 1 & 3 \\
1 & 1 & 1 & 1 & 5 \\
27 & 9 & 3 & 1 & 15
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 1 & 1 & 1 & 5 \\
-1 & 1 & -1 & 1 & 3 \\
-8 & 4 & -2 & 1 & -10 \\
27 & 9 & 3 & 1 & 15
\end{bmatrix}
\]

\( \text{Interchange } r_3 \text{ and } r_4 \)

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 5 \\
0 & 2 & 0 & 2 & 8 \\
0 & 12 & 6 & 9 & 30 \\
0 & -18 & -24 & -26 & -120
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 1 & 1 & 1 & 5 \\
0 & 2 & 0 & 2 & 8 \\
0 & 12 & 6 & 9 & 30 \\
0 & -18 & -24 & -26 & -120
\end{bmatrix}
\]

\( R_2 = r_4 + r_2 \)
\( R_3 = 8r_4 + r_3 \)
\( R_4 = -27r_4 + r_4 \)

\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 5 \\
0 & 2 & 0 & 2 & 8 \\
0 & 12 & 6 & 9 & 30 \\
0 & -18 & -24 & -26 & -120
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 1 & 1 & 1 & 5 \\
0 & 2 & 0 & 2 & 8 \\
0 & 12 & 6 & 9 & 30 \\
0 & -18 & -24 & -26 & -120
\end{bmatrix}
\]

\( R_2 = \frac{1}{2} r_2 \)

The solution is \( a = 1, b = -2, c = 0, d = 6 \); so the equation is \( f(x) = x^3 - 2x^2 + 6 \).

77. Let \( x \) = the number of servings of salmon steak. Let \( y \) = the number of servings of baked eggs. Let \( z \) = the number of servings of acorn squash. Protein equation: \( 30x + 15y + 3z = 78 \)

Carbohydrate equation: \( 20x + 2y + 25z = 59 \)

Vitamin A equation: \( 2x + 20y + 32z = 75 \)

Set up a matrix and solve:

\[
\begin{bmatrix}
30 & 15 & 3 & 78 \\
20 & 2 & 25 & 59 \\
2 & 20 & 32 & 75
\end{bmatrix}
\rightarrow
\begin{bmatrix}
2 & 20 & 32 & 75 \\
20 & 2 & 25 & 59 \\
30 & 15 & 3 & 78
\end{bmatrix}
\]

\( \text{Interchange } r_3 \text{ and } r_1 \)

\[
\begin{bmatrix}
2 & 20 & 32 & 75 \\
20 & 2 & 25 & 59 \\
30 & 15 & 3 & 78
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 10 & 16 & 37.5 \\
20 & 2 & 25 & 59 \\
30 & 15 & 3 & 78
\end{bmatrix}
\]

\( R_2 = -20r_2 + r_2 \)
\( R_3 = -30r_2 + r_3 \)

The solution is \( a = 3, b = -4, c = 0, d = 5 \); so the equation is \( f(x) = 3x^3 - 4x^2 + 5 \).
Chapter 11: Systems of Equations and Inequalities

78. Let \( x \) be the number of servings of pork chops.
Let \( y \) be the number of servings of corn on the cob.
Let \( z \) be the number of servings of 2% milk.
Protein equation: \( 23x + 3y + 9z = 47 \)
Carbohydrate equation: \( 16y + 13z = 58 \)
Calcium equation: \( 10x + 10y + 300z = 630 \)

Set up a matrix and solve:
\[
\begin{bmatrix}
1 & 10 & 16 & 37.5 \\
0 & -198 & -295 & -691 \\
0 & 0 & -\frac{3457}{66} & -\frac{3457}{66}
\end{bmatrix}
\]
\( R_3 = -\frac{66}{3457} r_3 \)
\[
\begin{bmatrix}
1 & 10 & 16 & 37.5 \\
0 & -198 & -295 & -691 \\
0 & 0 & 1 & 1
\end{bmatrix}
\]
\( R_3 = -\frac{66}{3457} r_3 \)

Substitute \( z = 1 \) and solve:
\(-198y - 295(1) = -691 \)
\(-198y = -396 \)
\( y = 2 \)
\( x + 10(2) + 16(1) = 37.5 \)
\( x + 36 = 37.5 \)
\( x = 1.5 \)
The dietitian should serve 1.5 servings of salmon steak, 2 servings of baked eggs, and 1 serving of acorn squash.

79. Let \( x \) be the amount invested in Treasury bills.
Let \( y \) be the amount invested in Treasury bonds.
Let \( z \) be the amount invested in corporate bonds.
Total investment equation: \( x + y + z = 10,000 \)
Annual income equation: \( 0.06x + 0.07y + 0.08z = 680 \)
Condition on investment equation: \( z = 0.5x \)
\( x - 2z = 0 \)

Set up a matrix and solve:
\[
\begin{bmatrix}
1 & 1 & 1 & 10,000 \\
0 & 0.06 & 0.07 & 0.08 & 680 \\
1 & 0 & -2 & 0
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & 1 & 1 & 10,000 \\
0 & 0.01 & 0.02 & 80 \\
0 & -1 & -3 & -10,000
\end{bmatrix}
\]
\( R_3 = -0.06r_3 + r_2 \)
\( R_3 = -r_3 + r_5 \)
\( R_2 = -0.06r_1 + r_2 \)
\( R_3 = r_2 + r_3 \)
\( R_2 = 100r_2 \)
\( R_2 = -r_3 \)
\( R_1 = -r_2 + r_3 \)
\( R_2 = -2r_3 \)
\( R_2 = 2r_3 \)
\( R_1 = r_3 \)
\( R_2 = r_3 + r_1 \)
\( R_2 = -2r_3 + r_2 \)
\( R_2 = 2r_3 + r_2 \)

Carletta should invest $4000 in Treasury bills, $4000 in Treasury bonds, and $2000 in corporate bonds.

80. Let \( x \) be the fixed delivery charge; let \( y \) be the cost of each tree, and let \( z \) be the hourly labor charge.
1st subdivision: \( x + 250y + 166z = 7520 \)
2nd subdivision: \( x + 200y + 124z = 5945 \)
3rd subdivision: \( x + 300y + 200z = 8985 \)

Set up a matrix and solve:
\[
\begin{bmatrix}
1 & 250 & 166 & 7520 \\
1 & 200 & 124 & 5945 \\
1 & 300 & 200 & 8985
\end{bmatrix}
\]
Section 11.2: Systems of Linear Equations: Matrices

81. Let \( x \) = the number of Deltas produced.
Let \( y \) = the number of Betas produced.
Let \( z \) = the number of Sigmas produced.

Painting equation: \( 10x + 16y + 8z = 240 \)
Drying equation: \( 3x + 5y + 2z = 69 \)
Polishing equation: \( 2x + 3y + z = 41 \)

Set up a matrix and solve:

\[
\begin{bmatrix}
1 & 16 & 8 & 240 \\
3 & 5 & 2 & 69 \\
1 & 1 & 2 & 33
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 4 & 48 \\
0 & 1 & -2 & -15 \\
0 & 0 & 1 & 10
\end{bmatrix}
\]

The company should produce 8 Deltas, 5 Betas, and 10 Sigmas.

82. Let \( x \) = the number of cases of orange juice produced; let \( y \) = the number of cases of grapefruit juice produced; and let \( z \) = the number of cases of tomato juice produced.

Sterilizing equation: \( 9x + 10y + 12z = 398 \)
Filling equation: \( 6x + 4y + 4z = 164 \)
Labeling equation: \( x + 2y + z = 58 \)

Set up a matrix and solve:

\[
\begin{bmatrix}
9 & 10 & 12 & 398 \\
6 & 4 & 4 & 164 \\
1 & 2 & 1 & 58
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 1 & 58 \\
0 & 1 & 4 & 164 \\
0 & 0 & 1 & 58
\end{bmatrix}
\]

The company should prepare 6 cases of orange juice, 20 cases of grapefruit juice, and 12 cases of tomato juice.
Chapter 11: Systems of Equations and Inequalities

83. Rewrite the system to set up the matrix and solve:

\[
\begin{align*}
-4 + 8 - 2I_2 &= 0 \\
8 &= 5I_4 + I_1 \\
4 &= 3I_3 + I_1 \\
I_3 + I_4 &= I_1 \\
I_1 - I_2 - I_4 &= 0
\end{align*}
\]

The solution is \( I_1 = 1, I_2 = 2, I_3 = 3, I_4 = 1 \).

84. Rewrite the system to set up the matrix and solve:

\[
\begin{align*}
I_1 &= I_3 + I_2 \\
24 - 6I_1 - 3I_2 &= 0 \\
12 + 24 - 6I_1 - 6I_2 &= 0
\end{align*}
\]

The solution is \( I_1 = 3, I_2 = 1, I_3 = 2 \).

85. Let \( x \) = the amount invested in Treasury bills.
Let \( y \) = the amount invested in corporate bonds.
Let \( z \) = the amount invested in junk bonds.

a. Total investment equation:
\[ x + y + z = 20,000 \]

Annual income equation:
\[ 0.07x + 0.09y + 0.11z = 2000 \]

Set up a matrix and solve:
\[
\begin{bmatrix}
1 & 1 & 1 & 20,000 \\
0.07 & 0.09 & 0.11 & 2000
\end{bmatrix}
\]

The solution is \( I_1 = \frac{44}{23}, I_2 = 2, I_3 = \frac{16}{23} \).

\( I_4 = \frac{28}{23} \).
Section 11.2: Systems of Linear Equations: Matrices

The matrix in the last step represents the system\[\begin{align*}
\begin{bmatrix}
-1 & 1 & -2 \\
0 & 1 & 2
\end{bmatrix}
\end{align*}
\]
\[(R_2 = r_2 - 7r_1)
\]
Therefore the solution is \[\begin{align*}
x &= -10,000 + z, \\
y &= 30,000 - 2z,
\end{align*}\]
\(z\) is any real number.

Possible investment strategies:

<table>
<thead>
<tr>
<th>Amount Invested At</th>
<th>7%</th>
<th>9%</th>
<th>11%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10,000</td>
<td>10,000</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>8000</td>
<td>11,000</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>6000</td>
<td>12,000</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>4000</td>
<td>13,000</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>2000</td>
<td>14,000</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>0</td>
<td>15,000</td>
<td></td>
</tr>
</tbody>
</table>

b. Total investment equation: \[x + y + z = 25,000\]
Annual income equation: \[0.07x + 0.09y + 0.11z = 2000\]
Set up a matrix and solve:
\[
\begin{bmatrix}
1 & 1 & 1 & 30,000 \\
0.07 & 0.09 & 0.11 & 2000
\end{bmatrix}
\]
\[
\rightarrow
\begin{bmatrix}
1 & 1 & 1 & 30,000 \\
7 & 9 & 11 & 200,000
\end{bmatrix}
\] \[(R_2 = 100r_2)\]
\[
\rightarrow
\begin{bmatrix}
1 & 1 & 1 & 30,000 \\
0 & 2 & 4 & 25,000
\end{bmatrix}
\] \[R_2 = r_2 - 7r_1\]
\[
\rightarrow
\begin{bmatrix}
1 & 1 & 1 & 30,000 \\
0 & 1 & 2 & 12,500
\end{bmatrix}
\] \[R_2 = \frac{1}{2} r_2\]
\[
\rightarrow
\begin{bmatrix}
1 & 0 & -1 & 12,500 \\
0 & 1 & 2 & 12,500
\end{bmatrix}
\] \[R_1 = r_1 - r_2\]

Thus, the solution is \[\begin{align*}
x &= z + 35,000, \\
y &= -2z - 5000, \quad z\text{ is any real number.}
\end{align*}\]

However, \(y\) and \(z\) cannot be negative. From \[y = -2z - 5000,\] we must have \(y = z = 0.\)

One possible investment strategy

<table>
<thead>
<tr>
<th>Amount Invested At</th>
<th>7%</th>
<th>9%</th>
<th>11%</th>
</tr>
</thead>
<tbody>
<tr>
<td>30,000</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

This will yield \((30,000)(0.07) = 2100\), which is more than the required income.

d. Answers will vary.
86. Let \( x \) = the amount invested in Treasury bills.
Let \( y \) = the amount invested in corporate bonds.
Let \( z \) = the amount invested in junk bonds.
Let \( I \) = income
Total investment equation: \( x + y + z = 25,000 \)
Annual income equation:
\[
0.07x + 0.09y + 0.11z = I
\]
Set up a matrix and solve:
\[
\begin{bmatrix}
1 & 1 & 1 & 25,000 \\
0.07 & 0.09 & 0.11 & I
\end{bmatrix}
\]
\[
\rightarrow
\begin{bmatrix}
1 & 1 & 1 & 25,000 \\
7 & 9 & 11 & 100I
\end{bmatrix}
\]
\[
\rightarrow
\begin{bmatrix}
1 & 1 & 1 & 25,000 \\
0 & 2 & 4 & 100I - 175,000
\end{bmatrix}
\]
\[
\rightarrow
\begin{bmatrix}
1 & 1 & 1 & 25,000 \\
0 & 1 & 2 & 50I - 87,500
\end{bmatrix}
\]
\[
\rightarrow
\begin{bmatrix}
1 & 0 & -1 & 112,500 - 50I \\
0 & 1 & 2 & 50I - 87,500
\end{bmatrix}
\]
The matrix in the last step represents the system:
\[
\begin{align*}
x - z &= 112,500 - 50I \\
y + 2z &= 50I - 87,500
\end{align*}
\]
Thus, the solution is \( x = 112,500 - 50I + z \), \( y = 50I - 87,500 - 2z \), \( z \) is any real number.

a. \( I = 1500 \)
\[
x = 112,500 - 50I + z = 112,500 - 50(1500) + z = 37,500 + z
\]
y = 50I - 87,500 - 2z = 50(1500) - 87,500 - 2z = -12,500 - 2z
\( z \) is any real number.
Since \( y \) and \( z \) cannot be negative, we must have \( y = z = 0 \). Investing all of the money at 7\% yields $1750, which is more than the $1500 needed.

b. \( I = 2000 \)
\[
x = 112,500 - 50I + z = 112,500 - 50(2000) + z = 12,500 + z
\]
y = 50I - 87,500 - 2z = 50(2000) - 87,500 - 2z = 12,500 - 2z
\( z \) is any real number.

Possible investment strategies:

<table>
<thead>
<tr>
<th>Amount Invested At</th>
<th>7%</th>
<th>9%</th>
<th>11%</th>
</tr>
</thead>
<tbody>
<tr>
<td>12,500</td>
<td>12,500</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>15,500</td>
<td>6500</td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>18,750</td>
<td>0</td>
<td>6250</td>
<td></td>
</tr>
</tbody>
</table>

c. \( I = 2500 \)
\[
x = 112,500 - 50I + z = 112,500 - 50(2500) + z = -12,500 + z
\]
y = 50I - 87,500 - 2z = 50(2500) - 87,500 - 2z = 37,500 - 2z
\( z \) is any real number.
Possible investment strategies:

<table>
<thead>
<tr>
<th>Amount invested at</th>
<th>7%</th>
<th>9%</th>
<th>11%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12,500</td>
<td>12,500</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>10,500</td>
<td>13,500</td>
<td></td>
</tr>
<tr>
<td>6250</td>
<td>0</td>
<td>18,750</td>
<td></td>
</tr>
</tbody>
</table>

d. Answers will vary.

87. Let \( x \) = the amount of supplement 1.
Let \( y \) = the amount of supplement 2.
Let \( z \) = the amount of supplement 3.
\[
\begin{align*}
0.20x + 0.40y + 0.30z &= 40 & \text{Vitamin C} \\
0.30x + 0.20y + 0.50z &= 30 & \text{Vitamin D}
\end{align*}
\]
Multiplying each equation by 10 yields:
\[
\begin{align*}
2x + 4y + 3z &= 400 \\
3x + 2y + 5z &= 300
\end{align*}
\]
Set up a matrix and solve:
\[
\begin{bmatrix}
2 & 4 & 3 & 400 \\
3 & 2 & 5 & 300
\end{bmatrix}
\]
\[
\rightarrow
\begin{bmatrix}
1 & 2 & \frac{3}{2} & 200 \\
3 & 2 & 5 & 300
\end{bmatrix}
\]
\[
\rightarrow
\begin{bmatrix}
1 & 2 & \frac{3}{2} & 200 \\
0 & -4 & \frac{1}{2} & -300
\end{bmatrix}
\]
\[
\rightarrow
\begin{bmatrix}
1 & 2 & \frac{3}{2} & 200 \\
0 & 1 & -\frac{1}{8} & 75
\end{bmatrix}
\]
Possible investment strategies:

<table>
<thead>
<tr>
<th>Amount Invested</th>
<th>7%</th>
<th>9%</th>
<th>11%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12,500</td>
<td>12,500</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>10,500</td>
<td>13,500</td>
<td></td>
</tr>
<tr>
<td>6250</td>
<td>0</td>
<td>18,750</td>
<td></td>
</tr>
</tbody>
</table>

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Section 11.3: Systems of Linear Equations: Determinants

\[
\begin{bmatrix}
1 & 0 & \frac{7}{4} & 50 \\
0 & 1 & -\frac{3}{8} & 75
\end{bmatrix}
\quad (R_1 = r_1 - 2r_2)
\]

The matrix in the last step represents the system
\[
\begin{aligned}
x + \frac{7}{4}z &= 50 \\
y - \frac{3}{8}z &= 75
\end{aligned}
\]

Therefore the solution is \( x = 50 - \frac{7}{4}z \),
\( y = 75 + \frac{1}{8}z \), \( z \) is any real number.

Possible combinations:

<table>
<thead>
<tr>
<th>Supplement 1</th>
<th>Supplement 2</th>
<th>Supplement 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>50mg</td>
<td>75mg</td>
<td>0mg</td>
</tr>
<tr>
<td>36mg</td>
<td>76mg</td>
<td>8mg</td>
</tr>
<tr>
<td>22mg</td>
<td>77mg</td>
<td>16mg</td>
</tr>
<tr>
<td>8mg</td>
<td>78mg</td>
<td>24mg</td>
</tr>
</tbody>
</table>

88. Let \( x \) = the amount of powder 1.
Let \( y \) = the amount of powder 2.
Let \( z \) = the amount of powder 3.
\[
\begin{align*}
0.20x + 0.40y + 0.30z &= 12 & \text{Vitamin B}_{12} \\
0.30x + 0.20y + 0.40z &= 12 & \text{Vitamin E}
\end{align*}
\]
Multiplying each equation by 10 yields
\[
\begin{align*}
2x + 4y + 3z &= 120 \\
3x + 2y + 4z &= 120
\end{align*}
\]
Set up a matrix and solve:
\[
\begin{bmatrix}
2 & 4 & 3 & 120 \\
3 & 2 & 4 & 120
\end{bmatrix}
\quad \rightarrow \quad
\begin{bmatrix}
2 & 4 & 3 & 120 \\
0 & -4 & -0.5 & -60
\end{bmatrix}
\quad (R_2 = r_2 - \frac{3}{2}r_1)
\]
\[
\begin{bmatrix}
2 & 4 & 3 & 120 \\
0 & -4 & -0.5 & -60
\end{bmatrix}
\quad \rightarrow \quad
\begin{bmatrix}
2 & 0 & 2.5 & 60 \\
0 & -4 & -0.5 & -60
\end{bmatrix}
\quad (R_1 = r_1 + r_2)
\]
The matrix in the last step represents the system
\[
\begin{align*}
2x + 2.5z &= 60 \\
-4y - 0.5z &= -60
\end{align*}
\]
Thus, the solution is \( x = 30 - 1.25z \),
\( y = 15 - 0.125z \), \( z \) is any real number.

Possible combinations:

Section 11.3

1. \( ad - bc \)
2. \[
\begin{bmatrix}
5 & 3 \\
-3 & -4
\end{bmatrix}
\]
3. False; If \( ad=bc \), the the det = 0.
4. False; The solution cannot be determined.
5. False; See Thm (15)
6. False; See Thm (14)
7. \[
\begin{bmatrix}
6 & 4 \\
-1 & 3
\end{bmatrix}
\]
\( = 6(3) - (-1)(4) = 18 + 4 = 22 \)
8. \[
\begin{bmatrix}
8 & -3 \\
4 & 2
\end{bmatrix}
\]
\( = 8(2) - 4(-3) = 16 + 12 = 28 \)
9. \[
\begin{bmatrix}
-3 & -1 \\
4 & 2
\end{bmatrix}
\]
\( = -3(2) - 4(-1) = -6 + 4 = -2 \)
10. \[
\begin{bmatrix}
-4 & 2 \\
-5 & 3
\end{bmatrix}
\]
\( = -4(3) - (-5)(2) = -12 + 10 = -2 \)
11. \[
\begin{bmatrix}
3 & 4 & 2 \\
1 & -1 & 5 \\
1 & 2 & -2
\end{bmatrix}
\]
\[
= 3 \left[ \left( (-1)(-2) - 2(5) \right) - 4 \left[ 1(-2) - 5 \right] \right] 
+ 2 \left[ 1(2) - 1(-1) \right]
\]
\( = 3(-8) - 4(-7) + 2(3) \)
\( = -24 + 28 + 6 \)
\( = 10 \)

Answers will vary.
12. \[
\begin{vmatrix} 1 & 3 & -2 \\ 6 & 1 & -5 \\ 8 & 2 & 3 \end{vmatrix} = 1[1(3) - 2(-5)] - 3[6(3) - 8(-5)] + (-2)[6(2) - 8(1)] \\
= 1(13) - 3(58) - 2(4) \\
= 13 - 174 - 8 \\
= -169
\]

13. \[
\begin{vmatrix} 4 & -1 & 2 \\ 6 & -1 & 0 \\ 1 & -3 & 4 \end{vmatrix} = 4[-1(4) - 0(-3)] + [1(64) - 1(0)] + [2(63) - 1(-1)] \\
= 4(-4) + 1(24) + 2(-17) \\
= -16 + 24 - 34 \\
= -26
\]

14. \[
\begin{vmatrix} 3 & -9 & 4 \\ 1 & 4 & 0 \\ 8 & -3 & 1 \end{vmatrix} = 3\begin{vmatrix} 4 & 0 \\ 8 & 1 \end{vmatrix} - 9\begin{vmatrix} 1 & 0 \\ 8 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & 4 \\ 8 & -3 \end{vmatrix} \\
= 3[4(1) - 0(1)] - 9[1(1) - 0(1)] + 4[1(-3) - 4(1)] \\
= 3(4) + 9(1) + 4(-35) \\
= 12 + 9 - 140 \\
= -119
\]

15. \[
\begin{align*}
\begin{cases} x + y &= 8 \\
x - y &= 4 
\end{cases}
\end{align*}
\]
Find the solutions by Cramer's Rule:
\[
\begin{align*}
D &= \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2 \\
D_x &= \begin{vmatrix} 8 & 1 \\ 4 & -1 \end{vmatrix} = -12 \\
D_y &= \begin{vmatrix} 1 & 8 \\ 1 & 4 \end{vmatrix} = -4 \\
\end{align*}
\]
\[
\frac{D_x}{D} = -6, \quad \frac{D_y}{D} = 2
\]
The solution is (6, 2).

16. \[
\begin{align*}
\begin{cases} x + 2y &= 5 \\
x - y &= 3 
\end{cases}
\end{align*}
\]
\[
\begin{align*}
D &= \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = -2 \\
D_x &= \begin{vmatrix} 5 & 2 \\ 3 & -1 \end{vmatrix} = -8 \\
D_y &= \begin{vmatrix} 1 & 5 \\ 1 & 3 \end{vmatrix} = -2
\end{align*}
\]
Find the solutions by Cramer's Rule:
\[
\begin{align*}
x &= \frac{D_x}{D} = \frac{-8}{-2} = 4, \\
y &= \frac{D_y}{D} = \frac{-2}{-2} = 1
\end{align*}
\]
The solution is (4, 1).

17. \[
\begin{align*}
\begin{cases} 5x - y &= 13 \\
2x + 3y &= 12 
\end{cases}
\end{align*}
\]
\[
\begin{align*}
D &= \begin{vmatrix} 5 & -1 \\ 2 & 3 \end{vmatrix} = 15 + 2 = 17 \\
D_x &= \begin{vmatrix} 13 & -1 \\ 12 & 3 \end{vmatrix} = 39 + 12 = 51 \\
D_y &= \begin{vmatrix} 5 & 13 \\ 2 & 12 \end{vmatrix} = 60 - 26 = 34
\end{align*}
\]
Find the solutions by Cramer's Rule:
\[
\begin{align*}
x &= \frac{D_x}{D} = \frac{51}{17} = 3, \\
y &= \frac{D_y}{D} = \frac{34}{17} = 2
\end{align*}
\]
The solution is (3, 2).

18. \[
\begin{align*}
\begin{cases} x + 3y &= 5 \\
2x - 3y &= -8 
\end{cases}
\end{align*}
\]
\[
\begin{align*}
D &= \begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} = -3 - 6 = -9 \\
D_x &= \begin{vmatrix} 5 & 3 \\ -8 & -3 \end{vmatrix} = -15 + 24 = 9 \\
D_y &= \begin{vmatrix} 1 & 5 \\ 2 & -8 \end{vmatrix} = -8 - 10 = -18
\end{align*}
\]
Find the solutions by Cramer's Rule:
\[
\begin{align*}
x &= \frac{D_x}{D} = \frac{9}{-9} = -1, \\
y &= \frac{D_y}{D} = \frac{-18}{-9} = 2
\end{align*}
\]
The solution is (-1, 2).
19. \[ \begin{aligned} 3x + 2y &= 24 \\ x + 2y &= 0 \end{aligned} \]

\[ D = \begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} = 6 - 0 = 6 \]

\[ D_x = \begin{vmatrix} 24 & 0 \\ 0 & 2 \end{vmatrix} = 48 - 0 = 48 \]

\[ D_y = \begin{vmatrix} 3 & 24 \\ 1 & 0 \end{vmatrix} = 0 - 24 = -24 \]

Find the solutions by Cramer's Rule:

\[ x = \frac{D_x}{D} = \frac{48}{6} = 8 \quad y = \frac{D_y}{D} = \frac{-24}{6} = -4 \]

The solution is \((8, -4)\).

20. \[ \begin{aligned} 4x + 5y &= -3 \\ -2y &= -4 \end{aligned} \]

\[ D = \begin{vmatrix} 4 & 5 \\ 0 & -2 \end{vmatrix} = -8 - 0 = -8 \]

\[ D_x = \begin{vmatrix} -3 & 5 \\ -4 & -2 \end{vmatrix} = 6 - (-20) = 26 \]

\[ D_y = \begin{vmatrix} 4 & -3 \\ 0 & -4 \end{vmatrix} = -16 - 0 = -16 \]

Find the solutions by Cramer's Rule:

\[ x = \frac{D_x}{D} = \frac{26}{-8} = -\frac{13}{4} \quad y = \frac{D_y}{D} = \frac{-16}{-8} = 2 \]

The solution is \(\left(-\frac{13}{4}, 2\right)\).

21. \[ \begin{aligned} 3x - 6y &= 24 \\ 5x + 4y &= 12 \end{aligned} \]

\[ D = \begin{vmatrix} 3 & -6 \\ 5 & 4 \end{vmatrix} = 12 - (-30) = 42 \]

\[ D_x = \begin{vmatrix} 24 & -6 \\ 12 & 4 \end{vmatrix} = 96 - (-72) = 168 \]

\[ D_y = \begin{vmatrix} 3 & 24 \\ 5 & 12 \end{vmatrix} = 36 - 120 = -84 \]

Find the solutions by Cramer's Rule:

\[ x = \frac{D_x}{D} = \frac{168}{42} = 4 \quad y = \frac{D_y}{D} = \frac{-84}{42} = -2 \]

The solution is \((4, -2)\).

22. \[ \begin{aligned} 2x + 4y &= 16 \\ 3x - 5y &= -9 \end{aligned} \]

\[ D = \begin{vmatrix} 2 & 4 \\ 3 & -5 \end{vmatrix} = -10 - 12 = -22 \]

\[ D_x = \begin{vmatrix} 16 & 4 \\ -9 & -5 \end{vmatrix} = -80 + 36 = -44 \]

\[ D_y = \begin{vmatrix} 2 & 16 \\ 3 & -9 \end{vmatrix} = -18 - 48 = -66 \]

Find the solutions by Cramer's Rule:

\[ x = \frac{D_x}{D} = \frac{-44}{-22} = 2 \quad y = \frac{D_y}{D} = \frac{-66}{-22} = 3 \]

The solution is \((2, 3)\).
26. \[
\begin{align*}
3x + 3y &= 3 \\
4x + 2y &= \frac{8}{3}
\end{align*}
\]
Find the solutions by Cramer's Rule:
\[
\begin{align*}
D &= \begin{vmatrix} 3 & 3 \\ 4 & 2 \end{vmatrix} = 6 - 12 = -6 \\
D_x &= \begin{vmatrix} 3 & 3 \\ \frac{8}{3} & 2 \end{vmatrix} = 6 - 8 = -2 \\
D_y &= \begin{vmatrix} 3 & 3 \\ 4 & \frac{8}{3} \end{vmatrix} = 8 - 12 = -4 \\
x &= \frac{D_x}{D} = \frac{-2}{-6} = \frac{1}{3} \\
y &= \frac{D_y}{D} = \frac{-4}{-6} = \frac{2}{3}
\end{align*}
\]
The solution is \(\left(\frac{1}{3}, \frac{2}{3}\right)\).

27. \[
\begin{align*}
2x - 3y &= -1 \\
10x + 10y &= 5
\end{align*}
\]
Find the solutions by Cramer's Rule:
\[
\begin{align*}
D &= \begin{vmatrix} 2 & -3 \\ 10 & 10 \end{vmatrix} = 20 - (-30) = 50 \\
D_x &= \begin{vmatrix} -1 & -3 \\ 5 & 10 \end{vmatrix} = -10 - (-15) = 5 \\
D_y &= \begin{vmatrix} 2 & -1 \\ 10 & 5 \end{vmatrix} = 10 - (-10) = 20 \\
x &= \frac{D_x}{D} = \frac{5}{50} = \frac{1}{10} \\
y &= \frac{D_y}{D} = \frac{20}{50} = \frac{2}{5}
\end{align*}
\]
The solution is \(\left(\frac{1}{10}, \frac{2}{5}\right)\).

28. \[
\begin{align*}
3x - 2y &= 0 \\
5x + 10y &= 4
\end{align*}
\]
Find the solutions by Cramer's Rule:
\[
\begin{align*}
D &= \begin{vmatrix} 3 & -2 \\ 5 & 10 \end{vmatrix} = 30 - (-10) = 40 \\
D_x &= \begin{vmatrix} 0 & -2 \\ 4 & 10 \end{vmatrix} = 0 - (-8) = 8 \\
D_y &= \begin{vmatrix} 3 & 0 \\ 5 & 4 \end{vmatrix} = 12 - 0 = 12 \\
x &= \frac{D_x}{D} = \frac{8}{40} = \frac{1}{5} \\
y &= \frac{D_y}{D} = \frac{12}{40} = \frac{3}{10}
\end{align*}
\]
The solution is \(\left(\frac{1}{5}, \frac{3}{10}\right)\).

29. \[
\begin{align*}
2x + 3y &= 6 \\
x - y &= \frac{1}{2}
\end{align*}
\]
Find the solutions by Cramer's Rule:
\[
\begin{align*}
D &= \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -2 - 3 = -5 \\
D_x &= \begin{vmatrix} 6 & 3 \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = 6 - \frac{3}{2} = -\frac{15}{2} \\
D_y &= \begin{vmatrix} 2 & 6 \\ 1 & \frac{1}{2} \end{vmatrix} = 1 - 6 = -5 \\
x &= \frac{D_x}{D} = \frac{-15}{-5} = \frac{3}{2} \\
y &= \frac{D_y}{D} = \frac{-5}{-5} = 1
\end{align*}
\]
The solution is \(\left(\frac{3}{2}, 1\right)\).

30. \[
\begin{align*}
\frac{1}{2}x + y &= -1 \\
x - 2y &= 8
\end{align*}
\]
Find the solutions by Cramer's Rule:
\[
\begin{align*}
D &= \begin{vmatrix} \frac{1}{2} & 1 \\ 1 & -2 \end{vmatrix} = -1 - 1 = -2 \\
D_x &= \begin{vmatrix} -2 & 1 \\ 8 & -2 \end{vmatrix} = 4 - 8 = -4 \\
D_y &= \begin{vmatrix} \frac{1}{2} & -2 \\ 1 & 8 \end{vmatrix} = 4 - (-2) = 6 \\
x &= \frac{D_x}{D} = \frac{-2}{-2} = 2 \\
y &= \frac{D_y}{D} = \frac{6}{-2} = -3
\end{align*}
\]
The solution is \((2, -3)\).

31. \[
\begin{align*}
3x - 5y &= 3 \\
15x + 5y &= 21
\end{align*}
\]
Find the solutions by Cramer's Rule:
\[
\begin{align*}
D &= \begin{vmatrix} 3 & -5 \\ 15 & 5 \end{vmatrix} = 15 - (-75) = 90 \\
D_x &= \begin{vmatrix} 3 & -5 \\ 21 & 5 \end{vmatrix} = 15 - (-105) = 120 \\
D_y &= \begin{vmatrix} 3 & 3 \\ 15 & 21 \end{vmatrix} = 63 - 45 = 18 \\
x &= \frac{D_x}{D} = \frac{120}{90} = \frac{4}{3} \\
y &= \frac{D_y}{D} = \frac{18}{90} = \frac{1}{5}
\end{align*}
\]
The solution is \(\left(\frac{4}{3}, \frac{1}{5}\right)\).
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32. \[
\begin{aligned}
2x - y &= -1 \\
\frac{1}{2}x + \frac{1}{2}y &= \frac{3}{2}
\end{aligned}
\]

\[D = \begin{vmatrix}
2 & -1 \\
\frac{1}{2} & \frac{1}{2}
\end{vmatrix} = 1 + 1 = 2\]

\[D_x = \begin{vmatrix}
-1 & 1 \\
\frac{1}{2} & \frac{1}{2}
\end{vmatrix} = -\frac{1}{2} + \frac{3}{2} = 1\]

\[D_y = \begin{vmatrix}
2 & -1 \\
\frac{3}{2} & \frac{3}{2}
\end{vmatrix} = 3 + 1 = 4\]

Find the solutions by Cramer's Rule:

\[x = \frac{D_x}{D} = \frac{2}{2} = 1, \quad y = \frac{D_y}{D} = \frac{4}{2} = 2\]

The solution is \((1, 2, 2)\).

33. \[
\begin{aligned}
x + y - z &= 6 \\
3x - 2y + z &= -5 \\
x + 3y - 2z &= 14
\end{aligned}
\]

\[D = \begin{vmatrix}
1 & 1 & -1 \\
3 & -2 & 1 \\
1 & 3 & -2
\end{vmatrix} = 1(-2 - (-1) - 1(-9 + 2)) - 1(3 - (-1) - 1) + 1(3 - (-2) - 1) = 1 + 7 - 11 = -3
\]

\[D_x = \begin{vmatrix}
1 & 1 & 1 \\
3 & -2 & 1 \\
14 & 3 & -2
\end{vmatrix} = 6(4 - 3) - 1(10 - 14) - 1(-15 + 28) = 6 + 4 - 13 = -3
\]

\[D_y = \begin{vmatrix}
1 & 6 & -1 \\
3 & -5 & 1 \\
1 & 14 & -2
\end{vmatrix} = 1(-5 - (-1)) - 1(10 - 14) + 1(-15 + 2) = 1 + 4 = 4
\]

Find the solutions by Cramer's Rule:

\[x = \frac{D_x}{D} = -\frac{3}{3} = 1, \quad y = \frac{D_y}{D} = -\frac{4}{3} = -\frac{4}{3}, \quad z = \frac{D_z}{D} = -\frac{6}{3} = -2\]

The solution is \((1, -\frac{4}{3}, -2)\).

34. \[
\begin{aligned}
x - y + z &= -4 \\
2x - 3y + 4z &= -15 \\
5x + y - 2z &= 12
\end{aligned}
\]

\[D = \begin{vmatrix}
1 & -1 & 1 \\
2 & -3 & 4 \\
5 & 1 & -2
\end{vmatrix} = 1(-3 - (-1) + 2) - 1(4 - (-1) + 1) + 1(2 - (-3)) = 1(-2 - 5 + 6) + 1(3 - 2) = 1(-2) + 1(1) = -2 + 1 = -1
\]

\[D_x = \begin{vmatrix}
-4 & -1 & 1 \\
-15 & -3 & 4 \\
12 & 1 & -2
\end{vmatrix} = -4(1 - (-1)) + 1(-15 - (-1)) - 1(12 - 1) = -4(2) + 1(15) - 1(12 - 1) = -8 + 15 - 11 = -4
\]

\[D_y = \begin{vmatrix}
1 & -4 & 1 \\
2 & -15 & 4 \\
5 & 12 & -2
\end{vmatrix} = 1(-5 - (-1)) + 1(2 - 1) - 1(10 - 14) = 1(-4) + 1(4) + 1(4) = -4 + 4 + 4 = 4
\]

\[D_z = \begin{vmatrix}
1 & 1 & 4 \\
12 & 15 & -4 \\
12 & 15 & -4
\end{vmatrix} = 1(15 - (-1)) + 1(15 - (-1)) - 1(12 - 15) = 1(16) + 1(16) - 1(3) = 32 + 32 - 3 = 61
\]
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Find the solutions by Cramer's Rule:
\[
\begin{align*}
D_x &= \begin{vmatrix} 1 & 2 & -3 \\ 2 & -4 & 7 \\ -2 & 2 & 4 \end{vmatrix} = 1(2) - 2(-4) + 7(-2) \\
D_y &= \begin{vmatrix} 1 & 2 & -3 \\ 2 & -4 & 7 \\ -2 & 2 & 4 \end{vmatrix} = 1(2) - 2(-4) + 7(-2) \\
D_z &= \begin{vmatrix} 1 & 2 & -3 \\ 2 & -4 & 7 \\ -2 & 2 & 4 \end{vmatrix} = 1(2) - 2(-4) + 7(-2)
\end{align*}
\]

Find the solutions by Cramer's Rule:
\[
\begin{align*}
D_x &= \begin{vmatrix} -4 & -7 \\ -2 & 4 \\ -2 & 4 \end{vmatrix} = -4(4) - -7(-2) + -7(-2) \\
D_y &= \begin{vmatrix} -4 & -7 \\ -2 & 4 \\ -2 & 4 \end{vmatrix} = -4(4) - -7(-2) + -7(-2) \\
D_z &= \begin{vmatrix} -4 & -7 \\ -2 & 4 \\ -2 & 4 \end{vmatrix} = -4(4) - -7(-2) + -7(-2)
\end{align*}
\]

The solution is \((1, 3, -2)\).

Find the solutions by Cramer's Rule:
\[
\begin{align*}
D_x &= \begin{vmatrix} 1 & 2 \\ 2 & -4 \\ -2 & 2 \end{vmatrix} = 1(-4) - 2(2) + 2(-4) \\
D_y &= \begin{vmatrix} 1 & 2 \\ 2 & -4 \\ -2 & 2 \end{vmatrix} = 1(-4) - 2(2) + 2(-4) \\
D_z &= \begin{vmatrix} 1 & 2 \\ 2 & -4 \\ -2 & 2 \end{vmatrix} = 1(-4) - 2(2) + 2(-4)
\end{align*}
\]

The solution is \((-3, \frac{1}{2}, 1)\).

Find the solutions by Cramer's Rule:
\[
\begin{align*}
D_x &= \begin{vmatrix} 1 & 4 \\ 3 & -1 \\ 1 & 6 \end{vmatrix} = 1(6) - 3(-1) + 4(1) \\
D_y &= \begin{vmatrix} 1 & 4 \\ 3 & -1 \\ 1 & 6 \end{vmatrix} = 1(6) - 3(-1) + 4(1) \\
D_z &= \begin{vmatrix} 1 & 4 \\ 3 & -1 \\ 1 & 6 \end{vmatrix} = 1(6) - 3(-1) + 4(1)
\end{align*}
\]

The solution is \((3, 12, 3)\).

Find the solutions by Cramer's Rule:
\[
\begin{align*}
D_x &= \begin{vmatrix} 1 & -8 \\ 3 & 12 \\ 1 & 6 \end{vmatrix} = 1(1) - 8(3) + 1(-12) \\
D_y &= \begin{vmatrix} 1 & -8 \\ 3 & 12 \\ 1 & 6 \end{vmatrix} = 1(1) - 8(3) + 1(-12) \\
D_z &= \begin{vmatrix} 1 & -8 \\ 3 & 12 \\ 1 & 6 \end{vmatrix} = 1(1) - 8(3) + 1(-12)
\end{align*}
\]

The solution is \((1, 2, -3)\).

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37. Find the solutions by Cramer's Rule:
\[
\begin{align*}
\begin{vmatrix}
1 & -2 & 3 \\
3 & 1 & -2 \\
2 & -4 & 6
\end{vmatrix} &= 1 \begin{vmatrix}
3 & -2 \\
2 & -4
\end{vmatrix} - (-2) \begin{vmatrix}
1 & 3 \\
2 & 2
\end{vmatrix} + 3 \begin{vmatrix}
1 & 2 \\
2 & 6
\end{vmatrix} \\
&= 1((6-8) + (18+4) + 3(-12-2)) \\
&= -2 + 44 - 42 \\
&= 0
\end{align*}
\]
Since \( D = 0 \), Cramer's Rule does not apply.

38. Find the solutions by Cramer's Rule:
\[
\begin{align*}
\begin{vmatrix}
1 & -1 & 2 \\
3 & 2 & 0 \\
-2 & 2 & -4
\end{vmatrix} &= 1 \begin{vmatrix}
2 & 0 \\
2 & -4
\end{vmatrix} - (-1) \begin{vmatrix}
3 & 0 \\
-2 & -4
\end{vmatrix} + 2 \begin{vmatrix}
3 & 2 \\
-2 & 2
\end{vmatrix} \\
&= 1((-8-0) + (-12-0) + 2(6+4)) \\
&= -8 - 12 + 20 \\
&= 0
\end{align*}
\]
Since \( D = 0 \), Cramer's Rule does not apply.

39. Find the solutions by Cramer's Rule:
\[
\begin{align*}
\begin{vmatrix}
1 & 2 & -1 \\
2 & -4 & 1 \\
-2 & 2 & -3
\end{vmatrix} &= 1 \begin{vmatrix}
2 & -4 \\
2 & -3
\end{vmatrix} - (-1) \begin{vmatrix}
1 & 1 \\
-2 & -3
\end{vmatrix} + 1 \begin{vmatrix}
1 & -1 \\
-2 & 2
\end{vmatrix} \\
&= 1((-4-2) + 2(-6+2) - 1(4-8)) \\
&= 10 + 8 + 4 \\
&= 22
\end{align*}
\]
\[
\begin{align*}
x &= \frac{D_x}{D} = \frac{0 - 4 \cdot 1}{22} = 0 \\
y &= \frac{D_y}{D} = \frac{0 - 2 \cdot 3}{22} = 0 \\
z &= \frac{D_z}{D} = \frac{0 - 4 \cdot 0}{22} = 0
\end{align*}
\]
The solution is \((0, 0, 0)\).

40. Find the solutions by Cramer's Rule:
\[
\begin{align*}
\begin{vmatrix}
1 & 4 & -3 \\
3 & 1 & 6 \\
1 & 4 & 0
\end{vmatrix} &= 1 \begin{vmatrix}
4 & -3 \\
1 & 6
\end{vmatrix} - 3 \begin{vmatrix}
1 & 6 \\
1 & 4
\end{vmatrix} + (-3) \begin{vmatrix}
1 & 1 \\
1 & 4
\end{vmatrix} \\
&= 1((-6-3) - 4(18-3) - 3(3+1)) \\
&= -9 - 60 - 12 \\
&= -81
\end{align*}
\]
\[
\begin{align*}
D_x &= \frac{D}{D_x} = \frac{-1 - 3 \cdot 0}{-81} \\
D_y &= \frac{D}{D_y} = \frac{0 - 4 \cdot 0}{-81} = 0 \\
D_z &= \frac{D}{D_z} = \frac{0 - 3 \cdot 0}{-81} = 0
\end{align*}
\]
The solution is \((0, 0, 0)\).
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Find the solutions by Cramer's Rule:

\[ \begin{vmatrix} 1 & 0 & -3 \\ 3 & 0 & 3 \\ 1 & 0 & 6 \end{vmatrix} = 0 \quad \text{[By Theorem (12)]} \]

\[ \begin{vmatrix} 1 & 4 & 0 \\ 3 & -1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0 \quad \text{[By Theorem (12)]} \]

The solution is \((0, 0, 0)\).

41. \[
\begin{cases}
x - 2y + 3z = 0 \\
3x + y - 2z = 0 \\
2x - 4y + 6z = 0
\end{cases}
\]

\[ D = \begin{vmatrix} 1 & -2 & 3 \\ 3 & 1 & -2 \\ 2 & -4 & 6 \end{vmatrix} = 0 \]

Thus, \(x = \frac{D_x}{D} = 0\), \(y = \frac{D_y}{D} = 0\), \(z = \frac{D_z}{D} = 0\).

44. \[
\begin{vmatrix} x & y & z \\ u & v & w \end{vmatrix} = 4
\]

By Theorem (11), the value of a determinant changes sign if any two rows are interchanged.

Thus, \(u v w = -4\).

45. \[
\begin{vmatrix} x & y & z \\ u & v & w \end{vmatrix} = 4
\]

By Theorem (14), if any row of a determinant is multiplied by a nonzero number \(k\), the value of the determinant is also changed by a factor of \(k\).

Thus, \(u v w = 2(4) = 8\).

46. \[
\begin{vmatrix} x & y & z \\ u & v & w \end{vmatrix} = 4
\]

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### Section 11.3: Systems of Linear Equations: Determinants

47. Let \[
\begin{vmatrix}
1 & 2 & 3 \\
x-3 & y-6 & z-9 \\
2u & 2v & 2w
\end{vmatrix}
= 2(-1)(-1)(4)
\]

48. Let \[
\begin{vmatrix}
1 & 2 & 3 \\
x & y & z
\end{vmatrix}
= 2(-1)(-1)(4)
\]

49. Let \[
\begin{vmatrix}
1 & 2 & 3 \\
x & y & z
\end{vmatrix}
= 2(-1)(-1)(4)
\]

50. Let \[
\begin{vmatrix}
1 & 2 & 3 \\
x & y & z
\end{vmatrix}
= 2(-1)(-1)(4)
\]

51. Solve for \(x\):
\[
\begin{vmatrix}
1 & 2 & 3 \\
4 & 3 & 5
\end{vmatrix}
= -2
\]

52. Solve for \(x\):
\[
\begin{vmatrix}
1 & 2 & 3 \\
4 & 3 & 5
\end{vmatrix}
= -2
\]
53. Solve for $x$:

$$x \begin{bmatrix} 3 & 2 & 4 \\ 1 & x & 5 \\ 0 & 1 & -2 \end{bmatrix} = 0$$

$$3x - 2(5) = 2(-2) + 4(1) = 0$$
$$-6x + 15 + 4 + 4 = 0$$
$$-6x + 22 + 11 = 2$$
$$11x = 13$$
$$x = \frac{13}{11}$$

54. Solve for $x$:

$$x \begin{bmatrix} 1 & 1 \\ 4 & 3 & 2 \\ -1 & 2 & 5 \end{bmatrix} = 2$$

$$x(15 - 4) - (20 + 2) + (8 + 3) = 2$$
$$11x - 22 + 11 = 2$$
$$11x = 13$$
$$x = \frac{13}{11}$$

55. Solve for $x$:

$$x \begin{bmatrix} x & 2 & 3 \\ 1 & x & 0 \\ 6 & 1 & -2 \end{bmatrix} = 7$$

$$x(-2x) - 2(1) + 3(1 - 6x) = 7$$
$$-2x^2 + 4 + 3 - 18x = 7$$
$$-2x^2 - 18x = 0$$
$$-2x(x + 9) = 0$$
$$x = 0 \text{ or } x = -9$$

56. Solve for $x$:

$$x \begin{bmatrix} x & 3 \\ 1 & 2 \end{bmatrix} = -4x$$

$$x(2x - 3) - 1(2) + 2(1) = -4x$$
$$2x^2 - 3x - 2 + 2 = -4x$$
$$2x^2 + x = 0$$
$$x(2x + 1) = 0$$
$$x = 0 \text{ or } x = -\frac{1}{2}$$

57. Expanding the determinant:

$$x \begin{bmatrix} x & y \\ x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = 0$$

$$x(y_1 - y_2) - y(x_1 - x_2) = 0$$

$$x(y_1 - y_2) + y(x_2 - x_1) = x_2y_1 - x_1y_2$$

$$y(x_2 - x_1) = x_2y_1 - x_1y_2 + x(y_2 - y_1)$$

This is the 2-point form of the equation for a line.
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58. Any point \((x, y)\) on the line containing \((x_2, y_2)\) and \((x_3, y_3)\) satisfies:
\[
\begin{vmatrix}
  x & y & 1 \\
  x_2 & y_2 & 1 \\
  x_3 & y_3 & 1 \\
\end{vmatrix} = 0.
\]
If the point \((x_1, y_1)\) is on the line containing \((x_2, y_2)\) and \((x_3, y_3)\) [the points are collinear],
then
\[
\begin{vmatrix}
  x_1 & y_1 & 1 \\
  x_2 & y_2 & 1 \\
  x_3 & y_3 & 1 \\
\end{vmatrix} = 0.
\]
Conversely, if \(\begin{vmatrix}
  x_1 & y_1 & 1 \\
  x_2 & y_2 & 1 \\
  x_3 & y_3 & 1 \\
\end{vmatrix} = 0\), then \((x_1, y_1)\) is on the line containing \((x_2, y_2)\) and \((x_3, y_3)\), and the points are collinear.

59. If the vertices of a triangle are \((2, 3)\), \((5, 2)\), and \((6, 5)\), then:

\[
D = \frac{1}{2} \begin{vmatrix} 2 & 5 & 6 \\ 3 & 2 & 5 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2} \left[ 2(5) - 5(3) + 6(3 - 2) \right] = \frac{1}{2} [2(-3) - 5(-2) + 6(1)] = \frac{1}{2} [-6 + 10 + 6] = 5.
\]
The area of the triangle is \(|S| = 5\) square units.

60. Expanding the determinant:
\[
\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix} = x^2 \begin{vmatrix} y & 1 \\ z & 1 \end{vmatrix} - x \begin{vmatrix} y^2 & 1 \\ z^2 & 1 \end{vmatrix} + 1 \begin{vmatrix} y^2 & y \\ z^2 & z \end{vmatrix} = x^2(y - z) - x(y^2 - z^2) + 1(y^2z - z^2y).
\]

61. If \(a = 0\), then \(b \neq 0\) and \(c \neq 0\) since \(ad - bc \neq 0\), and the system is \(\begin{cases} by = s \\ cx + dy = t \end{cases}\).

The solution of the system is \(y = \frac{s}{b}\),
\[
x = \frac{t - dy}{c} = \frac{t - d\left(\frac{s}{b}\right)}{c} = \frac{tb - sd}{bc}.
\]
Using Cramer’s Rule, we get \(D = \begin{vmatrix} 0 & b \\ c & d \end{vmatrix} = -bc\),
\[
D_x = \begin{vmatrix} s & b \\ t & d \end{vmatrix} = sd - tb,
\]
\[
D_y = \begin{vmatrix} 0 & s \\ c & t \end{vmatrix} = 0 - tc = -sc,
\]
so
\[
x = \frac{D_x}{D} = \frac{ds - tb}{-bc} = \frac{td - sd}{bc} \quad \text{and} \quad y = \frac{D_y}{D} = \frac{-sc}{-bc} = \frac{s}{b},
\]
which is the solution. Note that these solutions agree if \(d = 0\).

If \(b = 0\), then \(a \neq 0\) and \(d \neq 0\) since \(ad - bc \neq 0\), and the system is \(\begin{cases} ax = s \\ cx + dy = t \end{cases}\).

The solution of the system is \(x = \frac{s}{a}\),
\[
y = \frac{t - cx}{d} = \frac{at - cs}{ad}.
\]
Using Cramer’s Rule, we get \(D = \begin{vmatrix} a & 0 \\ c & d \end{vmatrix} = ad\), \(D_x = \begin{vmatrix} s & 0 \\ t & d \end{vmatrix} = sd\), and
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\[ D_y = \begin{vmatrix} a & s \\ c & t \end{vmatrix} = at - cs \], so \( x = D_y = \frac{sd}{a} = \frac{s}{a} \)
and \( y = D_x = \frac{at - cs}{ad} \), which is the solution.

Note that these solutions agree if \( c = 0 \).

If \( c = 0 \), then \( a \neq 0 \) and \( d \neq 0 \) since \( ad - bc \neq 0 \), and the system is \( \begin{cases} ax + by = s \\ cx + dy = t \end{cases} \).

The solution of the system is \( y = \frac{t}{d} \), \( x = \frac{s - by}{a} = \frac{sd - tb}{ad} \). Using Cramer’s Rule, we get \( D = \begin{vmatrix} a & b \\ 0 & d \end{vmatrix} = ad \), \( D_x = \begin{vmatrix} s & b \\ 0 & t \end{vmatrix} = sdt \) and \( D_y = \begin{vmatrix} a & s \\ 0 & t \end{vmatrix} = at\), so \( x = D_x = \frac{sd - tb}{ad} \) and \( y = D_y = \frac{at}{d} \), which is the solution. Note that these solutions agree if \( b = 0 \).

If \( d = 0 \), then \( b \neq 0 \) and \( c \neq 0 \) since \( ad - bc \neq 0 \), and the system is \( \begin{cases} ax + by = s \\ cx = t \end{cases} \).

The solution of the system is \( x = \frac{t}{c} \), \( y = \frac{s - ax}{b} = \frac{cs - at}{bc} \). Using Cramer’s Rule, we get \( D = \begin{vmatrix} a & b \\ c & 0 \end{vmatrix} = 0 - bc = \frac{0}{bc} \), \( D_x = \begin{vmatrix} s & b \\ 0 & t \end{vmatrix} = \frac{0}{t} \) and \( D_y = \begin{vmatrix} a & s \\ c & t \end{vmatrix} = at - cs \), so \( x = \frac{D_y}{D} = \frac{-tb}{-bc} = \frac{a}{c} \) and \( y = \frac{D_x}{D} = \frac{cs - at}{bc} \), which is the solution. Note that these solutions agree if \( a = 0 \).

62. Evaluating the determinant to show the relationship:
\[
\begin{vmatrix} a_{13} & a_{12} & a_{11} \\ a_{32} & a_{33} & a_{31} \\ a_{23} & a_{22} & a_{21} \end{vmatrix}
= a_{13}(a_{22}a_{31} - a_{23}a_{32}) - a_{12}(a_{23}a_{31} - a_{21}a_{33}) + a_{11}(a_{21}a_{32} - a_{22}a_{31})
= a_{13}a_{22}a_{31} - a_{12}(a_{23}a_{31} - a_{21}a_{33}) + a_{11}(a_{21}a_{32} - a_{22}a_{31})
\]
\[
= a_{11}(a_{22}a_{31} - a_{23}a_{32}) + a_{21}(a_{23}a_{31} - a_{21}a_{33}) + a_{12}(a_{21}a_{32} - a_{22}a_{31})
\]
63. Evaluating the determinant to show the relationship:
\[
\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \\ k_a & k_a & k_a \end{vmatrix}
= a_{11}(k_{a2}a_{31} - k_{a3}a_{32}) - a_{12}(k_{a2}a_{31} - k_{a3}a_{32}) + a_{13}(k_{a2}a_{31} - k_{a3}a_{32})
= a_{11}(k_{a2}a_{31} - k_{a3}a_{32}) + a_{12}(k_{a2}a_{31} - k_{a3}a_{32}) + a_{13}(k_{a2}a_{31} - k_{a3}a_{32})
\]
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64. Set up a 3 by 3 determinant in which the first column and third column are the same and evaluate:

\[
\begin{vmatrix}
 a_{11} & a_{12} & a_{11} \\
 a_{21} & a_{22} & a_{21} \\
 a_{31} & a_{32} & a_{31}
\end{vmatrix}
\]

\[
= a_{11} \begin{vmatrix}
 a_{22} & a_{21} \\
 a_{32} & a_{31}
\end{vmatrix}
- a_{12} \begin{vmatrix}
 a_{21} & a_{22} \\
 a_{31} & a_{32}
\end{vmatrix}
+ a_{11} \begin{vmatrix}
 a_{22} & a_{21} \\
 a_{32} & a_{31}
\end{vmatrix}
\]

\[
= a_{11} (a_{22}a_{31} - a_{21}a_{32}) - a_{12} (a_{21}a_{31} - a_{23}a_{31})
+ a_{11} (a_{22}a_{31} - a_{21}a_{32})
\]

\[
= a_{11} a_{22} a_{31} - a_{11} a_{21} a_{32} - a_{12} (a_{21}a_{31} - a_{21}a_{32}) + a_{11} a_{21} a_{32} - a_{11} a_{21} a_{32}
\]

\[
= a_{11} a_{22} a_{31} - a_{11} a_{21} a_{32} - a_{12} (a_{21}a_{31} - a_{21}a_{32})
+ a_{11} a_{21} a_{32} - a_{11} a_{21} a_{32}
\]

\[
= 0
\]

65. Evaluating the determinant to show the relationship:

\[
\begin{vmatrix}
 a_{11} + ka_{21} & a_{12} + ka_{22} & a_{13} + ka_{23} \\
 a_{21} & a_{22} & a_{23} \\
 a_{31} & a_{32} & a_{33}
\end{vmatrix}
\]

\[
= (a_{11} + ka_{21}) \begin{vmatrix}
 a_{22} & a_{23} \\
 a_{32} & a_{33}
\end{vmatrix}
- (a_{12} + ka_{22}) \begin{vmatrix}
 a_{21} & a_{23} \\
 a_{31} & a_{33}
\end{vmatrix}
+ (a_{13} + ka_{23}) \begin{vmatrix}
 a_{21} & a_{22} \\
 a_{31} & a_{32}
\end{vmatrix}
\]

\[
= (a_{11} + ka_{21})(a_{22}a_{33} - a_{23}a_{32})
- (a_{12} + ka_{22})(a_{21}a_{33} - a_{23}a_{31})
+ (a_{13} + ka_{23})(a_{21}a_{32} - a_{22}a_{31})
\]

\[
= a_{11}(a_{22}a_{33} - a_{23}a_{32}) + a_{12}(a_{21}a_{33} - a_{23}a_{31})
+ a_{13}(a_{21}a_{32} - a_{22}a_{31})
\]

\[
= a_{11}(a_{22}a_{33} - a_{23}a_{32}) + a_{12}(a_{21}a_{33} - a_{23}a_{31})
+ a_{13}(a_{21}a_{32} - a_{22}a_{31})
\]

\[
= 0
\]
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13. $3A-2B = \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} - 2 \begin{bmatrix} 4 & 1 & 0 \\ -2 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 9 & -15 \\ 3 & 6 & 18 \end{bmatrix} = \begin{bmatrix} -8 & 7 & -15 \\ 7 & 0 & 22 \end{bmatrix}$

14. $2A+4B = 2 \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} + 4 \begin{bmatrix} 4 & 1 & 0 \\ -2 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 6 & -10 \\ 2 & 4 & 12 \end{bmatrix} = \begin{bmatrix} 16 & 4 & 0 \\ -16 & 10 & -10 \end{bmatrix}$

15. $AC = \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 0(4)+3(6)+(-5)(-2) & 0(1)+3(2)+(-5)(3) \\ 1(4)+2(6)+6(-2) & 1(1)+2(2)+6(3) \end{bmatrix} = \begin{bmatrix} 28 & -9 \\ 4 & 23 \end{bmatrix}$

16. $BC = \begin{bmatrix} 4 & 1 & 0 \\ -2 & 3 & -2 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 4(4)+1(6)+0(-2) & 4(1)+1(2)+0(3) \\ -2(4)+3(6)+(-2)(-2) & -2(1)+3(2)+(-2)(3) \end{bmatrix} = \begin{bmatrix} 22 & 6 \\ 14 & -2 \end{bmatrix}$

17. $CA = \begin{bmatrix} 4 & 1 \\ 6 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 4(0)+1(1) & 4(3)+1(2) & 4(-5)+1(6) \\ 6(0)+2(1) & 6(3)+2(2) & 6(-5)+2(6) \\ -2(0)+3(1) & -2(3)+3(2) & -2(-5)+3(6) \end{bmatrix} = \begin{bmatrix} 1 & 14 & -14 \\ 2 & 22 & -18 \\ 3 & 0 & 28 \end{bmatrix}$

18. $CB = \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 0 & 3 & -5 \\ -2 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 4(4)+1(-2) & 4(1)+1(3) & 4(0)+1(-2) \\ -2(4)+3(-2) & -2(1)+3(3) & -2(0)+3(-2) \end{bmatrix} = \begin{bmatrix} 14 & 7 & -2 \\ -14 & 7 & -6 \end{bmatrix}$

19. $(A+B) = \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -5 \\ -2 & 3 \end{bmatrix}$

20. $(A+B)C = \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & -5 \\ -1 & 5 & 4 \end{bmatrix}$

21. $AC-3I = \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} - 3 \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 28 & -9 \\ 4 & 23 \end{bmatrix}$
22. \( CA + 5I_3 = \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 14 & -14 \\ 2 & 22 & -18 \\ 3 & 0 & 28 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 14 & -14 \\ 2 & 27 & -18 \\ 3 & 0 & 33 \end{bmatrix} \)

23. \( CA - CB \)

\[
\begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 14 & -14 \\ 2 & 22 & -18 \\ 3 & 0 & 28 \end{bmatrix} - \begin{bmatrix} 14 & 7 & -2 \\ 20 & 12 & -4 \\ -14 & 7 & -6 \end{bmatrix} = \begin{bmatrix} -13 & 7 & -12 \\ -18 & 10 & -14 \\ 17 & -7 & 34 \end{bmatrix}
\]

24. \( AC + BC \)

\[
\begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 3 & -2 \\ -2 & 3 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 28 & -9 \\ 4 & 23 \end{bmatrix} + \begin{bmatrix} 22 & 6 \\ 14 & -2 \end{bmatrix} = \begin{bmatrix} 50 & -3 \\ 18 & 21 \end{bmatrix}
\]

25. \( a_{11} = 2(2) + (-2)(3) = -2 \\
a_{12} = 2(1) + (-2)(-1) = 4 \\
a_{13} = 2(4) + (-2)(3) = 2 \\
a_{14} = 2(6) + (-2)(2) = 8 \\
a_{21} = 1(2) + 0(3) = 2 \\
a_{22} = 1(1) + 0(-1) = 1 \\
a_{23} = 1(4) + 0(3) = 4 \\
a_{24} = 1(6) + 0(2) = 6 \\
\begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 & 6 \\ 3 & -1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 2 & 8 \\ 2 & 1 & 4 & 6 \end{bmatrix}
\]

26. \( a_{11} = 4(-6) + 1(2) = -22 \\
a_{12} = 4(6) + 1(5) = 29 \\
a_{13} = 4(1) + 1(4) = 8 \\
a_{14} = 4(0) + 1(-1) = -1 \\
a_{21} = 2(-6) + 1(2) = -10 \\
a_{22} = 2(6) + 1(5) = 17 \\
a_{23} = 2(1) + 1(4) = 6 \\
a_{24} = 2(0) + 1(-1) = -1 \\
\begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -6 & 6 & 1 & 0 \\ 2 & 5 & 4 & -1 \end{bmatrix} = \begin{bmatrix} -22 & 29 & 8 & -1 \\ -10 & 17 & 6 & -1 \end{bmatrix}
\]

27. \( CA - CB \)

\[
\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} - \begin{bmatrix} 1(1) & 2(2) & 3(3) \\ 0 & 1(-1) & 4(4) \end{bmatrix} = \begin{bmatrix} 1(1) + 2(-1) + 3(2) \\ 0(1) + (-1)(-1) + 4(2) \end{bmatrix} \begin{bmatrix} 1(2) + 2(0) + 3(4) \\ 0(2) + (-1)(0) + 4(4) \end{bmatrix} = \begin{bmatrix} 1 & 14 \\ -9 & 16 \end{bmatrix}
\]

28. \( AC + BC \)

\[
\begin{bmatrix} 1 & -1 \\ -3 & 2 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} l(1) + (-1)(3) & l(8) + (-1)(6) & l(-1) + (-1)(0) \\ -3(2) + 2(3) & -3(8) + 2(6) & -3(-1) + 2(0) \\ 0(2) + 5(3) & 0(8) + 5(6) & 0(-1) + 5(0) \end{bmatrix} = \begin{bmatrix} -1 & 2 & -1 \\ -3 & 2 & -1 \\ 0 & -12 & 3 \end{bmatrix} = \begin{bmatrix} 15 & 30 & 0 \end{bmatrix}
\]

29. \( AC + BC \)

\[
\begin{bmatrix} 1 & 0 & 1 \\ 2 & 4 & 1 \\ 3 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 6 & 2 \\ -1 \end{bmatrix} = \begin{bmatrix} l(1) + 0(6) + 1(8) & l(1) + 0(2) + 1(-1) \\ 2(1) + 4(6) + 1(8) & 2(3) + 4(2) + 1(-1) \\ 3(1) + 6(6) + 1(8) & 3(3) + 6(2) + 1(-1) \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ 34 & 13 \\ 47 & 20 \end{bmatrix}
\]
30. \[ \begin{bmatrix} 4 & -2 & 3 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 1 & -1 \\ 0 & 2 \end{bmatrix} \]

\[ \begin{bmatrix} 4(2) + (-2)(1) + 3(0) & 4(6) + (-2)(-1) + 3(2) \\ 0(2) + 1(1) + 2(0) & 0(6) + 1(-1) + 2(2) \\ -1(2) + 0(1) + 1(0) & -1(6) + 0(-1) + 1(2) \end{bmatrix} \]

\[ \begin{bmatrix} 6 & 32 \\ 1 & 3 \\ -2 & -4 \end{bmatrix} \]

31. \[ A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \]

Augment the matrix with the identity and use row operations to find the inverse:

\[ \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \]

\[ \rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix} \]

(Interchange \( r_1 \) and \( r_2 \))

\[ \rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & -2 \end{bmatrix} \]

\( R_2 = -2r_1 + r_2 \)

\[ \rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -2 & 1 \end{bmatrix} \]

\( R_2 = -r_2 \)

\[ \rightarrow \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \end{bmatrix} \]

\( R_1 = -r_2 + r_1 \)

Thus, \( A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \).

32. \[ A = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} \]

Augment the matrix with the identity and use row operations to find the inverse:

\[ \begin{bmatrix} 3 & -1 & 1 & 0 \\ -2 & 1 & 0 & 1 \end{bmatrix} \]

\[ \rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ -2 & 1 & 0 & 1 \end{bmatrix} \]

\( R_1 = r_2 + r_1 \)

\[ \rightarrow \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \]

\( R_2 = 2r_1 + r_2 \)

Thus, \( A^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \).

33. \[ A = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix} \]

Augment the matrix with the identity and use row operations to find the inverse:

\[ \begin{bmatrix} 6 & 5 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{bmatrix} \]

\[ \rightarrow \begin{bmatrix} 2 & 2 & 0 & 1 \\ 6 & 5 & 1 & 0 \end{bmatrix} \]

(Interchange \( r_1 \) and \( r_2 \))

\[ \rightarrow \begin{bmatrix} 2 & 2 & 0 & 1 \\ 0 & -1 & 1 & -3 \end{bmatrix} \]

\( R_2 = -3r_1 + r_2 \)

\[ \rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \frac{1}{2} \end{bmatrix} \]

\( R_1 = \frac{1}{2}r_1 \)

\( R_2 = -r_2 \)

\[ \rightarrow \begin{bmatrix} 1 & 0 & 1 & -\frac{3}{2} \end{bmatrix} \]

\( R_1 = -r_2 + r_1 \)

Thus, \( A^{-1} = \begin{bmatrix} 1 & -\frac{3}{2} \\ -1 & 3 \end{bmatrix} \).

34. \[ A = \begin{bmatrix} -4 & 1 \\ 6 & -2 \end{bmatrix} \]

Augment the matrix with the identity and use row operations to find the inverse:

\[ \begin{bmatrix} -4 & 1 & 1 & 0 \\ 6 & -2 & 0 & 1 \end{bmatrix} \]

\[ \rightarrow \begin{bmatrix} 1 & -\frac{4}{3} & -\frac{4}{3} & 0 \\ 6 & -2 & 0 & 1 \end{bmatrix} \]

\( R_1 = -\frac{4}{3}r_1 \)

\[ \rightarrow \begin{bmatrix} 1 & -\frac{4}{3} & -\frac{4}{3} & 0 \\ 0 & -\frac{4}{3} & \frac{3}{2} & 1 \end{bmatrix} \]

\( R_2 = -6r_1 + r_2 \)

\[ \rightarrow \begin{bmatrix} 1 & -\frac{4}{3} & -\frac{4}{3} & 0 \\ 0 & 1 & -3 & -2 \end{bmatrix} \]

\( R_2 = -2r_2 \)

\[ \rightarrow \begin{bmatrix} 1 & 0 & -1 & -\frac{3}{2} \\ 0 & 1 & -3 & -2 \end{bmatrix} \]

\( R_1 = \frac{1}{2}r_2 + r \)

Thus, \( A^{-1} = \begin{bmatrix} -1 & -\frac{1}{2} \\ -3 & -2 \end{bmatrix} \).
35. \( A = \begin{bmatrix} 2 & 1 \\ a & a \end{bmatrix} \) where \( a \neq 0 \).

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix}
2 & 1 & 1 & 0 \\
\frac{1}{a} & \frac{1}{a} & 0 & 1
\end{bmatrix}
\rightarrow \begin{bmatrix}
1 & 1 & 1 & 0 \\
0 & \frac{-1}{a} & 1 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{1}{2} & \frac{1}{2} & 0 & 0 \\
\frac{1}{2a} & \frac{1}{2a} & 1 & 1
\end{bmatrix}
\rightarrow \begin{bmatrix}
\frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & \frac{-1}{a} & 1 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 1 & 1 & 0 \\
0 & \frac{-1}{a} & 1 & 1
\end{bmatrix}
\rightarrow \begin{bmatrix}
1 & 1 & 1 & 0 \\
0 & \frac{-1}{a} & 1 & 1
\end{bmatrix}
\]

Thus, \( A^{-1} = \begin{bmatrix} 1 & -\frac{1}{a} \\ -\frac{1}{a} & 1 \end{bmatrix} \).

36. \( A = \begin{bmatrix} b & 3 \\ b & 2 \end{bmatrix} \) where \( b \neq 0 \).

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix}
b & 3 & 1 & 0 \\
b & 2 & 0 & 1
\end{bmatrix}
\rightarrow \begin{bmatrix}
b & 3 & 1 & 0 \\
0 & -1 & -1 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{1}{2}b & \frac{1}{2}b & 0 & 0 \\
0 & -1 & -1 & 1
\end{bmatrix}
\rightarrow \begin{bmatrix}
\frac{1}{2}b & \frac{1}{2}b & 0 & 0 \\
0 & -1 & -1 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{1}{2}b & \frac{1}{2}b & 0 & 0 \\
0 & 1 & 1 & -1
\end{bmatrix}
\rightarrow \begin{bmatrix}
\frac{1}{2}b & \frac{1}{2}b & 0 & 0 \\
0 & 1 & 1 & -1
\end{bmatrix}
\]

Thus, \( A^{-1} = \begin{bmatrix} \frac{1}{2}b & \frac{1}{2}b \\ 1 & -1 \end{bmatrix} \).

37. \( A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{bmatrix} \)

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix}
1 & -1 & 1 & 1 & 0 & 0 \\
0 & -2 & 1 & 0 & 1 & 0 \\
-2 & -3 & 0 & 0 & 0 & 1
\end{bmatrix}
\rightarrow \begin{bmatrix}
1 & -1 & 1 & 1 & 0 & 0 \\
0 & -2 & 1 & 0 & 1 & 0 \\
-2 & -3 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & -\frac{1}{2} & 1 & 1 & 0 & 0 \\
0 & -\frac{1}{2} & 1 & 0 & 1 & 0 \\
-5 & -2 & 2 & 0 & 1 & 0
\end{bmatrix}
\rightarrow \begin{bmatrix}
1 & -\frac{1}{2} & 1 & 1 & 0 & 0 \\
0 & -\frac{1}{2} & 1 & 0 & 1 & 0 \\
-5 & -2 & 2 & 0 & 1 & 0
\end{bmatrix}
\]

Thus, \( A^{-1} = \begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2 \end{bmatrix} \).

38. \( A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix} \)

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix}
1 & 0 & 2 & 1 & 0 & 0 \\
-1 & 2 & 3 & 0 & 1 & 0 \\
1 & -1 & 0 & 0 & 0 & 1
\end{bmatrix}
\rightarrow \begin{bmatrix}
1 & 0 & 2 & 1 & 0 & 0 \\
-1 & 2 & 3 & 0 & 1 & 0 \\
1 & -1 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 2 & 1 & 0 & 0 \\
0 & 2 & 5 & 1 & 1 & 0 \\
1 & 0 & 2 & 1 & 0 & 0
\end{bmatrix}
\rightarrow \begin{bmatrix}
1 & 0 & 2 & 1 & 0 & 0 \\
0 & 2 & 5 & 1 & 1 & 0 \\
1 & 0 & 2 & 1 & 0 & 0
\end{bmatrix}
\]

Thus, \( A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix} \).
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40. \[ A = \begin{bmatrix} 3 & 3 & 1 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix} \]

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix} 3 & 3 & 1 & | & 1 & 0 & 0 \\ 1 & 2 & 1 & | & 0 & 1 & 0 \\ 2 & -1 & 1 & | & 0 & 0 & 1 \end{bmatrix}
\]

(Interchange \( r_1 \) and \( r_2 \))

\[
\begin{bmatrix} 1 & 2 & 1 & | & 0 & 1 & 0 \\ 0 & -3 & -2 & | & 1 & -3 & 0 \\ 0 & -5 & -1 & | & 0 & -2 & 1 \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & 2 & 1 & | & 0 & 1 & 0 \\ 0 & 1 & \frac{2}{3} & | & -\frac{1}{3} & 1 & 0 \\ 0 & -5 & -1 & | & 0 & -2 & 1 \end{bmatrix}
\]

\( R_2 = -\frac{1}{3} r_2 \)

\[
\begin{bmatrix} 1 & 0 & \frac{1}{3} & | & \frac{2}{3} & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{2}{3} & | & -\frac{1}{3} & 1 & 0 \\ 0 & 0 & 1 & | & -\frac{5}{7} & \frac{9}{7} & \frac{3}{7} \end{bmatrix}
\]

(Interchange \( r_1 \) and \( r_2 \))

\[
\begin{bmatrix} 1 & 0 & 0 & | & \frac{3}{7} & -\frac{4}{7} & \frac{1}{7} \\ 0 & 1 & 0 & | & \frac{1}{7} & \frac{1}{7} & -\frac{2}{7} \\ 0 & 0 & 1 & | & -\frac{6}{7} & \frac{9}{7} & \frac{1}{7} \end{bmatrix}
\]

Thus, \( A^{-1} = \begin{bmatrix} \frac{3}{7} & -\frac{4}{7} & \frac{1}{7} \\ \frac{1}{7} & \frac{1}{7} & -\frac{2}{7} \\ -\frac{6}{7} & \frac{9}{7} & \frac{1}{7} \end{bmatrix} \).

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Section 11.4: Matrix Algebra

41. \[ \begin{align*}
2x + y &= 8 \\
x + y &= 5
\end{align*} \]
Rewrite the system of equations in matrix form:
\[ A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 5 \end{bmatrix} \]
Find the inverse of \( A \) and solve \( X = A^{-1}B \):
From Problem 29, \( A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \), so
\[ X = A^{-1}B = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 8 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}. \]
The solution is \( x = 3, y = 2 \) or \((3, 2)\).

42. \[ \begin{align*}
3x - y &= 8 \\
-2x + y &= 4
\end{align*} \]
Rewrite the system of equations in matrix form:
\[ A = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 4 \end{bmatrix} \]
Find the inverse of \( A \) and solve \( X = A^{-1}B \):
From Problem 30, \( A^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \), so
\[ X = A^{-1}B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 12 \\ 28 \end{bmatrix}. \]
The solution is \( x = 12, y = 28 \) or \((12, 28)\).

43. \[ \begin{align*}
2x + y &= 0 \\
x + y &= 5
\end{align*} \]
Rewrite the system of equations in matrix form:
\[ A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 5 \end{bmatrix} \]
Find the inverse of \( A \) and solve \( X = A^{-1}B \):
From Problem 29, \( A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \), so
\[ X = A^{-1}B = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \end{bmatrix}. \]
The solution is \( x = -5, y = 10 \) or \((-5, 10)\).

44. \[ \begin{align*}
3x - y &= 4 \\
-2x + y &= 5
\end{align*} \]
Rewrite the system of equations in matrix form:
\[ A = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 5 \end{bmatrix} \]
Find the inverse of \( A \) and solve \( X = A^{-1}B \):
From Problem 30, \( A^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \), so
\[ X = A^{-1}B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 9 \\ 23 \end{bmatrix}. \]
The solution is \( x = 9, y = 23 \) or \((9, 23)\).

45. \[ \begin{align*}
6x + 5y &= 7 \\
2x + 2y &= 2
\end{align*} \]
Rewrite the system of equations in matrix form:
\[ A = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ 2 \end{bmatrix} \]
Find the inverse of \( A \) and solve \( X = A^{-1}B \):
From Problem 31, \( A^{-1} = \begin{bmatrix} 1 & -5/3 \\ -1 & 3 \end{bmatrix} \), so
\[ X = A^{-1}B = \begin{bmatrix} 1 & -5/3 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}. \]
The solution is \( x = 2, y = -1 \) or \((2, -1)\).

46. \[ \begin{align*}
-4x + y &= 0 \\
6x - 2y &= 14
\end{align*} \]
Rewrite the system of equations in matrix form:
\[ A = \begin{bmatrix} -4 & 1 \\ 6 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 14 \end{bmatrix} \]
Find the inverse of \( A \) and solve \( X = A^{-1}B \):
From Problem 32, \( A^{-1} = \begin{bmatrix} -1 & -1/2 \\ -3 & -2 \end{bmatrix} \), so
\[ X = A^{-1}B = \begin{bmatrix} -1 & -1/2 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 14 \end{bmatrix} = \begin{bmatrix} -7 \\ -28 \end{bmatrix}. \]
The solution is \( x = -7, y = -28 \) or \((-7, -28)\).
47. \[ \begin{cases} 6x + 5y = 13 \\ 2x + 2y = 5 \end{cases} \]
Rewrite the system of equations in matrix form:
\[ A = \begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 13 \\ 5 \end{bmatrix} \]
Find the inverse of \( A \) and solve \( X = A^{-1}B \):
From Problem 31, \( A^{-1} = \begin{bmatrix} 1 & \frac{-5}{3} \\ -1 & \frac{2}{3} \end{bmatrix} \), so
\[ X = A^{-1}B = \begin{bmatrix} 1 & \frac{-5}{3} \\ -1 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 13 \\ 5 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}. \]
The solution is \( x = \frac{1}{2}, y = 2 \) or \( \left( \frac{1}{2}, 2 \right) \).

48. \[ \begin{cases} -4x + y = 5 \\ 6x - 2y = -9 \end{cases} \]
Rewrite the system of equations in matrix form:
\[ A = \begin{bmatrix} -4 & 1 \\ 6 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ -9 \end{bmatrix} \]
Find the inverse of \( A \) and solve \( X = A^{-1}B \):
From Problem 32, \( A^{-1} = \begin{bmatrix} -1 & -\frac{1}{2} \\ -3 & -2 \end{bmatrix} \), so
\[ X = A^{-1}B = \begin{bmatrix} -1 & -\frac{1}{2} \\ -3 & -2 \end{bmatrix} \begin{bmatrix} 5 \\ -9 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}. \]
The solution is \( x = \frac{1}{2}, y = 3 \) or \( \left( \frac{1}{2}, 3 \right) \).

49. \[ \begin{cases} 2x + y = -3 \\ ax + ay = -a \end{cases} \quad a \neq 0 \]
Rewrite the system of equations in matrix form:
\[ A = \begin{bmatrix} 2 & 1 \\ a & a \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} -3 \\ -a \end{bmatrix} \]
Find the inverse of \( A \) and solve \( X = A^{-1}B \):
From Problem 33, \( A^{-1} = \begin{bmatrix} 1 & -\frac{1}{a} \\ -1 & \frac{2}{a} \end{bmatrix} \), so
\[ X = A^{-1}B = \begin{bmatrix} 1 & -\frac{1}{a} \\ -1 & \frac{2}{a} \end{bmatrix} \begin{bmatrix} -3 \\ -a \end{bmatrix} = \begin{bmatrix} \frac{7}{a} \\ 1 \end{bmatrix}. \]
The solution is \( x = -2, y = 1 \) or \( (-2, 1) \).

50. \[ \begin{cases} bx + 3y = 2b + 3 \\ bx + 2y = 2b + 2 \end{cases} \quad b \neq 0 \]
Rewrite the system of equations in matrix form:
\[ A = \begin{bmatrix} b & 3 \\ b & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 2b + 3 \\ 2b + 2 \end{bmatrix} \]
Find the inverse of \( A \) and solve \( X = A^{-1}B \):
From Problem 34, \( A^{-1} = \begin{bmatrix} \frac{-3}{b} & \frac{3}{b} \\ 1 & -1 \end{bmatrix} \), so
\[ X = A^{-1}B = \begin{bmatrix} \frac{-3}{b} & \frac{3}{b} \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2b + 3 \\ 2b + 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{b} \\ \frac{2}{b} \end{bmatrix}. \]
The solution is \( x = \frac{2}{b}, y = 4 \) or \( \left( \frac{2}{b}, 4 \right) \).
53. \[ \begin{aligned} x - y + z &= 0 \\ -2y + z &= -1 \\ -2x - 3y &= -5 \end{aligned} \]

Rewrite the system of equations in matrix form:

\[ A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -1 \\ -5 \end{bmatrix} \]

Find the inverse of \( A \) and solve \( X = A^{-1}B \):

From Problem 35, \( A^{-1} = \begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2 \end{bmatrix} \), so

\[ X = A^{-1}B = \begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -5 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix} \]

The solution is \( x = -2, y = 3, z = 5 \) or \( (-2, 3, 5) \).

54. \[ \begin{aligned} x + 2z &= 6 \\ -x + 2y + 3z &= -5 \\ x - y &= 6 \end{aligned} \]

Rewrite the system of equations in matrix form:

\[ A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ -5 \\ 6 \end{bmatrix} \]

Find the inverse of \( A \) and solve \( X = A^{-1}B \):

From Problem 36, \( A^{-1} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \), so

\[ X = A^{-1}B = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} \]

The solution is \( x = 4, y = -2, z = 1 \) or \( (4, -2, 1) \).

55. \[ \begin{aligned} x - y + z &= 2 \\ -2y + z &= 2 \\ -2x - 3y &= \frac{1}{2} \end{aligned} \]

Rewrite the system of equations in matrix form:

\[ A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 2 \\ \frac{1}{2} \end{bmatrix} \]

Find the inverse of \( A \) and solve \( X = A^{-1}B \):

From Problem 35, \( A^{-1} = \begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2 \end{bmatrix} \), so

\[ X = A^{-1}B = \begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{bmatrix} \]

The solution is \( x = \frac{1}{2}, y = -\frac{1}{2}, z = 1 \) or \( \left(\frac{1}{2}, -\frac{1}{2}, 1\right) \).

56. \[ \begin{aligned} x + 2z &= 2 \\ -x + 2y + 3z &= \frac{3}{2} \\ x - y &= 2 \end{aligned} \]

Rewrite the system of equations in matrix form:

\[ A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ \frac{3}{2} \\ 2 \end{bmatrix} \]

Find the inverse of \( A \) and solve \( X = A^{-1}B \):

From Problem 36, \( A^{-1} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \), so

\[ X = A^{-1}B = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ \frac{3}{2} \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ \frac{1}{2} \end{bmatrix} \]

The solution is \( x = 1, y = -1, z = \frac{1}{2} \) or \( (1, -1, \frac{1}{2}) \).
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57. \[
\begin{align*}
\begin{cases}
x + y + z &= 9 \\
3x + 2y - z &= 8 \\
3x + y + 2z &= 1
\end{cases}
\end{align*}
\]

Rewrite the system of equations in matrix form:
\[
A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & -1 \\ 3 & 1 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 9 \\ 8 \\ 1 \end{bmatrix}
\]

Find the inverse of \( A \) and solve \( AX = B \):
\[
A^{-1} = \begin{bmatrix} 0 & \frac{1}{7} & \frac{1}{7} \\ \frac{1}{7} & -\frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & \frac{1}{7} & \frac{3}{7} \end{bmatrix}
\]

\[
X = A^{-1}B = \begin{bmatrix}
\frac{9}{7} \\
\frac{1}{7} \\
\frac{9}{7}
\end{bmatrix} \begin{bmatrix} 8 \\ 5 \\ 17 \end{bmatrix} = \begin{bmatrix} \frac{8}{7} \\ \frac{5}{7} \\ \frac{17}{7} \end{bmatrix}
\]

The solution is \( x = \frac{8}{7}, y = \frac{5}{7}, z = \frac{17}{7} \) or \( \begin{bmatrix} 8 \\ 5 \\ 17 \\ 7 \\ 7 \\ 7 \end{bmatrix} \).

58. \[
\begin{align*}
\begin{cases}
3x + 3y + z &= 8 \\
x + 2y + z &= 5 \\
2x - y + z &= 4
\end{cases}
\end{align*}
\]

Rewrite the system of equations in matrix form:
\[
A = \begin{bmatrix} 3 & 3 & 1 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix}
\]

Find the inverse of \( A \) and solve \( AX = B \):
\[
A^{-1} = \begin{bmatrix} \frac{1}{7} & -\frac{4}{7} & \frac{4}{7} \\ \frac{1}{7} & \frac{3}{7} & -\frac{3}{7} \\ -\frac{5}{7} & \frac{3}{7} & \frac{3}{7} \end{bmatrix}
\]

\[
X = A^{-1}B = \begin{bmatrix}
\frac{3}{7} \\
\frac{3}{7} \\
\frac{5}{7}
\end{bmatrix} \begin{bmatrix} 8 \\ 5 \\ 17 \end{bmatrix} = \begin{bmatrix} \frac{8}{7} \\ \frac{5}{7} \\ \frac{17}{7} \end{bmatrix}
\]

The solution is \( x = \frac{8}{7}, y = \frac{5}{7}, z = \frac{17}{7} \) or \( \begin{bmatrix} 8 \\ 5 \\ 17 \\ 7 \\ 7 \\ 7 \end{bmatrix} \).

59. \[
\begin{align*}
\begin{cases}
x + y + z &= 2 \\
3x + 2y - z &= \frac{7}{3} \\
3x + y + 2z &= \frac{10}{3}
\end{cases}
\end{align*}
\]

Rewrite the system of equations in matrix form:
\[
A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & -1 \\ 3 & 1 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ \frac{7}{3} \\ \frac{10}{3} \end{bmatrix}
\]

Find the inverse of \( A \) and solve \( AX = B \):
\[
A^{-1} = \begin{bmatrix} \frac{5}{7} & \frac{1}{7} & \frac{3}{7} \\ \frac{3}{7} & \frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{3}{7} \end{bmatrix}, \quad X = A^{-1}B = \begin{bmatrix}
\frac{5}{7} \\
\frac{3}{7} \\
\frac{2}{7}
\end{bmatrix} \begin{bmatrix} 2 \\ \frac{7}{3} \\ \frac{10}{3} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{7}{3} \\ \frac{2}{3} \end{bmatrix}
\]

The solution is \( x = \frac{1}{3}, y = \frac{7}{3}, z = \frac{2}{3} \) or \( \begin{bmatrix} \frac{1}{3}, \frac{7}{3}, \frac{2}{3} \end{bmatrix} \).

60. \[
\begin{align*}
\begin{cases}
3x + 3y + z &= 1 \\
x + 2y + z &= 0 \\
2x - y + z &= 4
\end{cases}
\end{align*}
\]

Rewrite the system of equations in matrix form:
\[
A = \begin{bmatrix} 3 & 3 & 1 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}
\]

Find the inverse of \( A \) and solve \( AX = B \):
\[
A^{-1} = \begin{bmatrix} \frac{3}{7} & \frac{4}{7} & \frac{4}{7} \\ \frac{1}{7} & \frac{3}{7} & \frac{3}{7} \\ -\frac{5}{7} & \frac{3}{7} & \frac{3}{7} \end{bmatrix}, \quad X = A^{-1}B = \begin{bmatrix}
\frac{3}{7} \\
\frac{1}{7} \\
-\frac{5}{7}
\end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}
\]

The solution is \( x = 1, y = -1, z = 1 \) or \( (1, -1, 1) \).
61. \[ A = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} \]

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}
\rightarrow \begin{bmatrix} 4 & 2 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \end{bmatrix} \quad (R_2 = -\frac{1}{2}r_1 + r_2)
\rightarrow \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \end{bmatrix} \quad (R_1 = \frac{1}{3}r_1)
\]

There is no way to obtain the identity matrix on the left. Thus, this matrix has no inverse.

62. \[ A = \begin{bmatrix} -3 & \frac{1}{2} \\ 6 & -1 \end{bmatrix} \]

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix} -3 & \frac{1}{2} & 1 & 0 \\ 6 & -1 & 0 & 1 \end{bmatrix}
\rightarrow \begin{bmatrix} -3 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & -\frac{1}{3} & 2 \end{bmatrix} \quad (R_2 = 2r_1 + r_2)
\rightarrow \begin{bmatrix} 1 & \frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \quad (R_1 = -\frac{1}{3}r_1)
\]

There is no way to obtain the identity matrix on the left. Thus, this matrix has no inverse.

63. \[ A = \begin{bmatrix} 15 & 3 \\ 10 & 2 \end{bmatrix} \]

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix} 15 & 3 & 1 & 0 \\ 10 & 2 & 0 & 1 \end{bmatrix}
\rightarrow \begin{bmatrix} 15 & 3 & 1 & 0 \\ 0 & 0 & -\frac{2}{3} & 1 \end{bmatrix} \quad (R_2 = -\frac{2}{3}r_1 + r_2)
\rightarrow \begin{bmatrix} 1 & \frac{1}{5} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} & 1 \end{bmatrix} \quad (R_1 = \frac{1}{15}r_1)
\]

There is no way to obtain the identity matrix on the left; thus, there is no inverse.

64. \[ A = \begin{bmatrix} -3 & 0 \\ 4 & 0 \end{bmatrix} \]

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix} -3 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{bmatrix}
\rightarrow \begin{bmatrix} -3 & 0 & 1 & 0 \\ 0 & 0 & \frac{4}{3} & 1 \end{bmatrix} \quad (R_2 = \frac{4}{3}r_1 + r_2)
\rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{4}{3} & 1 \end{bmatrix} \quad (R_1 = -\frac{1}{3}r_1)
\]

There is no way to obtain the identity matrix on the left; thus, there is no inverse.

65. \[ A = \begin{bmatrix} -3 & 1 & -1 \\ 1 & -4 & -7 \\ 1 & 2 & 5 \end{bmatrix} \]

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & -4 & -7 & 0 & 1 & 0 \\ 1 & 2 & 5 & 0 & 0 & 1 \end{bmatrix}
\rightarrow \begin{bmatrix} 1 & 2 & 5 & 0 & 0 & 1 \\ 1 & -4 & -7 & 0 & 1 & 0 \\ -3 & 1 & -1 & 1 & 0 & 0 \end{bmatrix} \quad \text{(Interchange } r_1 \text{ and } r_3 \text{)}
\rightarrow \begin{bmatrix} 1 & 2 & 5 & 0 & 0 & 1 \\ 0 & -6 & -12 & 0 & 1 & -1 \\ 0 & 7 & 14 & 1 & 0 & 3 \end{bmatrix} \quad (R_2 = -r_1 + r_2)
\rightarrow \begin{bmatrix} 1 & 2 & 5 & 0 & 0 & 1 \\ 0 & 7 & 14 & 1 & 0 & 3 \\ 1 & 0 & 1 & 0 & -\frac{1}{6} & \frac{1}{6} \end{bmatrix} \quad (R_3 = 3r_1 + r_3)
\rightarrow \begin{bmatrix} 1 & 2 & 5 & 0 & 0 & 1 \\ 0 & 7 & 14 & 1 & 0 & 3 \\ 1 & 0 & 1 & 0 & \frac{1}{3} & \frac{7}{3} \end{bmatrix} \quad (R_2 = -\frac{1}{6}r_2)
\rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & \frac{1}{3} & \frac{7}{3} \\ 0 & 1 & 2 & 0 & -\frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 1 & \frac{2}{6} & \frac{11}{6} \end{bmatrix} \quad (R_1 = -2r_2 + r_3)
\rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & \frac{1}{3} & \frac{7}{3} \\ 0 & 1 & 2 & 0 & -\frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 1 & \frac{2}{6} & \frac{11}{6} \end{bmatrix}
\]

There is no way to obtain the identity matrix on the left; thus, there is no inverse.
66. \( A = \begin{bmatrix} 1 & 1 & -3 \\ 2 & -4 & 1 \\ -5 & 7 & 1 \end{bmatrix} \)

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix} 1 & 1 & -3 & 1 & 0 & 0 \\ 2 & -4 & 1 & 0 & 1 & 0 \\ -5 & 7 & 1 & 0 & 0 & 1 \end{bmatrix}
\]

\[
\begin{align*}
R_2 &\rightarrow -2R_1 + R_2 \\
R_3 &\rightarrow 5R_1 + R_3
\end{align*}
\]

\[
\begin{bmatrix} 1 & 1 & -3 & 1 & 0 & 0 \\ 0 & -6 & 7 & -2 & 1 & 0 \\ 0 & 12 & -14 & 5 & 0 & 1 \\
\end{bmatrix}
\]

\[
\begin{align*}
R_2 &\rightarrow 1/6R_2 \\
R_3 &\rightarrow 1/3R_3
\end{align*}
\]

\[
\begin{bmatrix} 1 & 1 & -3 & 1 & 0 & 0 \\ 0 & 1 & 7/6 & -1/3 & 0 \\ 0 & 12 & -14 & 5 & 0 & 1 \\
\end{bmatrix}
\]

\[
\begin{align*}
R_2 &\rightarrow 1/6R_2 \\
R_3 &\rightarrow 1/6R_3
\end{align*}
\]

\[
\begin{bmatrix} 1 & 1 & -3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/2 & 1 \\
\end{bmatrix}
\]

There is no way to obtain the identity matrix on the left; thus, there is no inverse.

67. \( A = \begin{bmatrix} 25 & 61 & -12 \\ 18 & -2 & 4 \\ 8 & 35 & 21 \end{bmatrix} \)

Thus, \( A^{-1} = \begin{bmatrix} 0.01 & 0.05 & -0.01 \\ 0.01 & -0.02 & 0.01 \\ -0.02 & 0.01 & 0.03 \end{bmatrix} \)

68. \( A = \begin{bmatrix} 18 & -3 & 4 \\ 6 & -20 & 14 \\ 10 & 25 & -15 \end{bmatrix} \)

Thus, \( A^{-1} = \begin{bmatrix} 0.26 & -0.29 & -0.20 \\ -1.21 & 1.63 & 1.20 \\ -1.84 & 2.53 & 1.80 \end{bmatrix} \)

69. \( A = \begin{bmatrix} 44 & 21 & 18 & 6 \\ -2 & 10 & 15 & 5 \\ 21 & 12 & -12 & 4 \\ -8 & -16 & 4 & 9 \end{bmatrix} \)

70. \( A = \begin{bmatrix} 16 & 22 & -3 & 5 \\ 21 & -17 & 4 & 8 \\ 2 & 8 & 27 & 20 \\ 5 & 15 & -3 & -10 \end{bmatrix} \)

Thus, \( A^{-1} = \begin{bmatrix} 0.01 & 0.04 & 0.00 & 0.03 \\ 0.02 & -0.02 & 0.01 & 0.01 \\ -0.04 & 0.02 & 0.04 & 0.06 \\ 0.05 & -0.02 & 0.00 & -0.09 \end{bmatrix} \)

71. \( A = \begin{bmatrix} 18 & -12 & 7 \\ 3 & 4 & -1 \end{bmatrix} \), \( B = \begin{bmatrix} 10 \\ -9 \\ 12 \end{bmatrix} \)

Enter the matrices into a graphing utility and use \( A^{-1}B \) to solve the system. The result is shown below:

Thus, the solution to the system is \( x = 4.57 \), \( y = -6.44 \), \( z = -24.07 \) or \((4.57, -6.44, -24.07)\).
Section 11.4: Matrix Algebra

72. \( A = \begin{bmatrix} 25 & 61 & -12 \\ 18 & -12 & 7 \\ 3 & 4 & -1 \end{bmatrix}; \quad B = \begin{bmatrix} 15 \\ -3 \\ 12 \end{bmatrix} \)

Enter the matrices into a graphing utility and use \( A^{-1}B \) to solve the system. The result is shown below:

Thus, the solution to the system is \( x = 4.56, \quad y = -6.06, \quad z = -22.55 \) or \( (4.56, -6.06, -22.55) \).

73. \( A = \begin{bmatrix} 25 & 61 & -12 \\ 18 & -12 & 7 \\ 3 & 4 & -1 \end{bmatrix}; \quad B = \begin{bmatrix} 21 \\ 7 \\ -2 \end{bmatrix} \)

Thus, the solution to the system is \( x = -1.19, \quad y = 2.46, \quad z = 8.27 \) or \( (-1.19, 2.46, 8.27) \).

74. \( A = \begin{bmatrix} 25 & 61 & -12 \\ 18 & -12 & 7 \\ 3 & 4 & -1 \end{bmatrix}; \quad B = \begin{bmatrix} 25 \\ 10 \\ -4 \end{bmatrix} \)

Thus, the solution to the system is \( x = -2.05, \quad y = 3.88, \quad z = 13.36 \) or \( (-2.05, 3.88, 13.36) \).

75. \[
\begin{align*}
2x + 3y &= 11 \\
5x + 7y &= 24
\end{align*}
\]

Multiply each side of the first equation by 5, and each side of the second equation by -2. Then add the equations to eliminate \( x \): \[
\begin{align*}
10x + 15y &= 55 \\
-10x - 14y &= -48
\end{align*}
\]

\( y = 7 \)

Substitute and solve for \( x \):

Thus, the solution to the system is \( x = 2.05, \quad y = 3.88, \quad z = 13.36 \) or \( (2.05, 3.88, 13.36) \).

76. \[
\begin{align*}
2x + 8y &= -8 \\
x + 7y &= -13
\end{align*}
\]

Multiply each side of the second equation by -2. Then add the equations to eliminate \( x \):

\[
\begin{align*}
2x + 8y &= -8 \\
-2x - 14y &= 26
\end{align*}
\]

\( -6y = 18 \)

\( y = -3 \)

Substitute and solve for \( x \):

Thus, the solution to the system is \( x = 8, \quad y = -3 \) or using ordered pairs \( (8, -3) \).

77. \[
\begin{align*}
x - 2y + 4z &= 2 \\
-3x + 5y - 2z &= 17 \\
4x - 3y &= -22
\end{align*}
\]

Write the augmented matrix:

\[
\begin{bmatrix}
1 & -2 & 4 & | & 2 \\
-3 & 5 & -2 & | & 17 \\
4 & -3 & 0 & | & -22
\end{bmatrix}
\]

\[
\begin{align*}
1 & -2 & 4 & | & 2 \\
0 & -1 & 10 & | & -3 \\
0 & 5 & -16 & | & -30
\end{align*}
\]

\( R_1 = 3r_1 + r_2 \)

\( R_3 = -4r_1 + r_3 \)

\[
\begin{align*}
1 & -2 & 4 & | & 2 \\
0 & 1 & -10 & | & 17 \\
0 & 5 & 16 & | & -30
\end{align*}
\]

\( R_2 = -r_2 \)

\[
\begin{align*}
1 & -2 & 4 & | & 2 \\
0 & 1 & -10 & | & -23 \\
0 & 0 & 34 & | & -85
\end{align*}
\]

\( R_3 = -5r_2 + r_3 \)

\[
\begin{align*}
1 & -2 & 4 & | & 2 \\
0 & 1 & -10 & | & -23 \\
0 & 0 & 1 & | & 5/3
\end{align*}
\]

\( R_1 = 4r_1 + r_2 \)

\( R_2 = 10r_1 + r_3 \)

\( R_3 = r_3 / 34 \)

\[
\begin{align*}
1 & -2 & 0 & | & -8 \\
0 & 1 & 0 & | & 2 \\
0 & 0 & 1 & | & 5/3
\end{align*}
\]

The solution of the system is \( x = 1.19, \quad y = 2.46, \quad z = 8.27 \) or \( (1.19, 2.46, 8.27) \).
Chapter 11: Systems of Equations and Inequalities

78. \[
\begin{align*}
2x + 3y - z &= -2 \\
4x - 2y + z &= 6 \\
6y - 2z &= 2 \\
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
2 & 3 & -1 & -2 \\
4 & 0 & 3 & 6 \\
0 & 6 & -2 & 2 \\
\end{bmatrix}
\]
\[
\begin{align*}
R_1 &= \frac{1}{2} - \frac{1}{2} \\
R_2 &= \frac{1}{6} - \frac{5}{6} \\
R_3 &= \frac{1}{2} - \frac{1}{2} \\
R_4 &= \frac{1}{6} - \frac{5}{6} \\
R_5 &= \frac{1}{2} - \frac{1}{2} \\
R_6 &= \frac{1}{6} - \frac{5}{6} \\
R_7 &= \frac{1}{2} - \frac{1}{2} \\
R_8 &= \frac{1}{6} - \frac{5}{6} \\
R_9 &= \frac{1}{2} - \frac{1}{2} \\
R_{10} &= \frac{1}{6} - \frac{5}{6} \\
R_{11} &= \frac{1}{2} - \frac{1}{2} \\
R_{12} &= \frac{1}{6} - \frac{5}{6} \\
\end{align*}
\]
The last row of our matrix is a contradiction. Therefore, the system is inconsistent. The solution set is \(\emptyset\) or \(\emptyset\).

79. \[
\begin{align*}
5x - y + 4z &= 2 \\
-x + 5y - 4z &= 3 \\
7x + 13y - 4z &= 17 \\
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
5 & -1 & 4 & 2 \\
-1 & 5 & -4 & 3 \\
7 & 13 & -4 & 17 \\
\end{bmatrix}
\]
\[
\begin{align*}
R_1 &= -r_2 \\
R_2 &= r_1 \\
R_3 &= -7r_1 + r_3 \\
R_4 &= -5r_1 + r_2 \\
R_5 &= -5r_1 + r_2 \\
R_6 &= -5r_1 + r_2 \\
R_7 &= -5r_1 + r_2 \\
R_8 &= -5r_1 + r_2 \\
R_9 &= -5r_1 + r_2 \\
R_{10} &= -5r_1 + r_2 \\
R_{11} &= -5r_1 + r_2 \\
R_{12} &= -5r_1 + r_2 \\
\end{align*}
\]
The last row of our matrix is a contradiction. Therefore, the system is inconsistent. The solution set is \(\emptyset\) or \(\emptyset\).

80. \[
\begin{align*}
3x + 2y - z &= 2 \\
2x + y + 6z &= -7 \\
2x + 2y + 14z &= 17 \\
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
3 & 2 & -1 & 2 \\
2 & 1 & 6 & -7 \\
2 & 2 & -14 & 17 \\
\end{bmatrix}
\]
\[
\begin{align*}
R_1 &= r_1 - r_2 \\
R_2 &= r_1 - r_2 \\
R_3 &= r_1 - r_2 \\
R_4 &= r_1 - r_2 \\
R_5 &= r_1 - r_2 \\
R_6 &= r_1 - r_2 \\
R_7 &= r_1 - r_2 \\
R_8 &= r_1 - r_2 \\
R_9 &= r_1 - r_2 \\
R_{10} &= r_1 - r_2 \\
R_{11} &= r_1 - r_2 \\
R_{12} &= r_1 - r_2 \\
\end{align*}
\]
The last row of our matrix is a contradiction. Therefore, the system is inconsistent. The solution set is \(\emptyset\) or \(\emptyset\).
81. \[
\begin{align*}
2x - 3y + z &= 4 \\
-3x + 2y - z &= -3 \\
-5y + z &= 6
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
2 & -3 & 1 & 4 \\
-3 & 2 & -1 & -3 \\
0 & -5 & 1 & 6
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & -\frac{3}{2} & \frac{1}{2} & 2 \\
-3 & 2 & -1 & -3 \\
0 & -5 & 1 & 6
\end{bmatrix}
R_1 = \frac{r_1}{2}
\]
\[
\begin{bmatrix}
1 & -\frac{3}{2} & \frac{1}{2} & 2 \\
0 & -\frac{5}{2} & \frac{1}{2} & 3 \\
0 & -5 & 1 & 6
\end{bmatrix}
R_2 = 3r_1 + r_2
\]
\[
\begin{bmatrix}
1 & -\frac{3}{2} & \frac{1}{2} & 2 \\
0 & -\frac{5}{2} & \frac{1}{2} & 3 \\
0 & 1 & -\frac{1}{5} & -\frac{6}{5}
\end{bmatrix}
R_2 = -\frac{2}{3}r_2
\]
\[
\begin{bmatrix}
1 & -\frac{3}{2} & \frac{1}{2} & 2 \\
0 & 1 & -\frac{1}{5} & -\frac{6}{5} \\
0 & 0 & 0 & 0
\end{bmatrix}
R_3 = 5r_2 + r_3
\]
\[
\begin{bmatrix}
1 & 0 & \frac{1}{3} & \frac{1}{3} \\
0 & 1 & -\frac{1}{3} & -\frac{6}{5}
\end{bmatrix}
R_1 = \frac{3}{2}z + r_1
\]
Since the last row yields an identity, and no contradictions exist in the other rows, there are an infinite number of solutions. The solution is
\[
x = \frac{5}{13}z - \frac{12}{13}, \quad y = \frac{2}{13}z + \frac{10}{13}, \quad \text{and} \quad z \text{ is any real number. That is,}
\]
\[
\{(x, y, z) \mid x = \frac{5}{13}z - \frac{12}{13}, y = \frac{2}{13}z + \frac{10}{13}, z \text{ is any real number}\}
\]
82. \[
\begin{align*}
-4x + 3y + 2z &= 6 \\
3x + y - z &= -2 \\
x + 9y + z &= 6
\end{align*}
\]
Write the augmented matrix:
\[
\begin{bmatrix}
-4 & 3 & 2 & 6 \\
3 & 1 & -1 & -2 \\
1 & 9 & 1 & 6
\end{bmatrix}
\]
83. a. \[
A = \begin{bmatrix} 6 & 9 \\ 3 & 12 \end{bmatrix}; \quad B = \begin{bmatrix} 80.00 \\ 277.80 \end{bmatrix}
\]
\[
AB = \begin{bmatrix} 6 & 9 \\ 3 & 12 \end{bmatrix} \begin{bmatrix} 80.00 \\ 277.80 \end{bmatrix} = \begin{bmatrix} 6(80.00) + 9(277.80) \\ 3(80.00) + 12(277.80) \end{bmatrix} = \begin{bmatrix} 2980.20 \\ 3573.60 \end{bmatrix}
\]
Nikki’s total tuition is $2980.20, and Joe’s total tuition is $3573.60.
84. a. \[
A = \begin{bmatrix} 4000 & 3000 \\ 2500 & 3800 \end{bmatrix}; \quad B = \begin{bmatrix} 0.011 \\ 0.006 \end{bmatrix}
\]
\[
AB = \begin{bmatrix} 4000 & 3000 \\ 2500 & 3800 \end{bmatrix} \begin{bmatrix} 0.011 \\ 0.006 \end{bmatrix} = \begin{bmatrix} 4000(0.011) + 3000(0.006) \\ 2500(0.011) + 3800(0.006) \end{bmatrix} = \begin{bmatrix} 62.00 \\ 50.30 \end{bmatrix}
\]
After one month, Jamal’s loans accrued
$62.00 in interest, and Stephanie’s loans accrued $50.30 in interest.

c. \[ A(C + B) = \begin{bmatrix} 400 & 3000 \\ 2500 & 3800 \end{bmatrix} \begin{bmatrix} 1 & 0.011 \\ 1 & 0.006 \end{bmatrix} \]
\[ = \begin{bmatrix} 400 & 3000 \\ 2500 & 3800 \end{bmatrix} \begin{bmatrix} 1.011 \\ 1.006 \end{bmatrix} \]
\[ = 7062.00 \]
Jamal’s loan balance after one month was $7062.00, and Stephanie’s loan balance was $6350.30.

85. a. The rows of the 2 by 3 matrix represent stainless steel and aluminum. The columns represent 10-gallon, 5-gallon, and 1-gallon.
The 2 by 3 matrix is:
\[
\begin{bmatrix} 500 & 350 & 400 \\ 700 & 500 & 850 \end{bmatrix}
\]
The 3 by 2 matrix is:
\[
\begin{bmatrix} 500 & 700 \\ 350 & 500 \\ 400 & 850 \end{bmatrix}
\]
b. The 3 by 1 matrix representing the amount of material is:
\[
\begin{bmatrix} 15 \\ 8 \\ 3 \end{bmatrix}
\]
c. The days usage of materials is:
\[
\begin{bmatrix} 500 & 350 & 400 \\ 700 & 500 & 850 \end{bmatrix} \begin{bmatrix} 15 \\ 8 \\ 3 \end{bmatrix} = \begin{bmatrix} 11,500 \\ 17,050 \end{bmatrix}
\]
Thus, 11,500 pounds of stainless steel and 17,050 pounds of aluminum were used that day.
d. The 1 by 2 matrix representing cost is:
\[
\begin{bmatrix} 0.10 & 0.05 \end{bmatrix}
\]
e. The total cost of the day’s production was:
\[
\begin{bmatrix} 0.10 & 0.05 \end{bmatrix} \begin{bmatrix} 11,500 \\ 17,050 \end{bmatrix} = \begin{bmatrix} 2002.50 \end{bmatrix}
\]
The total cost of the day’s production was $2002.50.

86. a. The rows of the 2 by 3 matrix represent the location. The columns represent the type of car sold. The 2 by 3 matrix for January is:
\[
\begin{bmatrix} 400 & 250 & 50 \\ 450 & 200 & 140 \end{bmatrix}
\]
February is:
\[
\begin{bmatrix} 350 & 100 & 30 \\ 350 & 300 & 100 \end{bmatrix}
\]
b. Adding the matrices:
\[
\begin{bmatrix} 400 & 250 & 50 \\ 450 & 200 & 140 \end{bmatrix} + \begin{bmatrix} 350 & 100 & 30 \\ 350 & 300 & 100 \end{bmatrix} = \begin{bmatrix} 750 & 350 & 80 \\ 800 & 500 & 240 \end{bmatrix}
\]
c. The 3 by 1 matrix representing profit: \[
\begin{bmatrix} 100 \\ 150 \\ 200 \end{bmatrix}
\]
d. Multiplying to find the profit at each location:
\[
\begin{bmatrix} 750 & 350 & 80 \\ 800 & 500 & 240 \end{bmatrix} \begin{bmatrix} 100 \\ 150 \\ 200 \end{bmatrix} = \begin{bmatrix} 143,500 \\ 203,000 \end{bmatrix}
\]
The city location has a two-month profit of $143,500. The suburban location has a two-month profit of $203,000.

87. a. \[
K = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}
\]
Augment the matrix with the identity and use row operations to find the inverse:
\[
\begin{bmatrix} 2 & 1 & 1 & | & 1 & 0 & 0 \\ 1 & 1 & 0 & | & 0 & 1 & 0 \\ 1 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix}
\]
\[
\rightarrow \begin{bmatrix} 1 & 1 & 0 & | & 0 & 1 & 0 \\ 2 & 1 & 1 & | & 0 & 0 & 1 \\ 1 & 1 & 1 & | & 0 & 0 & 1 \end{bmatrix}
\]
(Interchange \( r_1 \) and \( r_2 \))
\[
\rightarrow \begin{bmatrix} 1 & 1 & 0 & | & 0 & 1 & 0 \\ 0 & -1 & 1 & | & -2 & 0 \\ 0 & 0 & 1 & | & -1 & 0 \end{bmatrix}
\]
\( R_2 = -2r_1 + r_2 \)
\( R_3 = -r_1 + r_3 \)
Section 11.4: Matrix Algebra

88. \( p^2 = \begin{bmatrix} 0.4 & 0.2 & 0.1 \\ 0.5 & 0.6 & 0.5 \\ 0.1 & 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 0.4 & 0.2 & 0.1 \\ 0.5 & 0.6 & 0.5 \\ 0.1 & 0.2 & 0.4 \end{bmatrix} \)

Thus, \( p = \begin{bmatrix} 0.27 & 0.22 & 0.18 \\ 0.55 & 0.56 & 0.55 \\ 0.18 & 0.22 & 0.27 \end{bmatrix} \)

because

Each entry represents the probability that a grandchild has a certain income level given his or her grandparents' income level.

89. \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \)

If \( D = ad - bc \neq 0 \), then \( a \neq 0 \) and \( d \neq 0 \), or \( b \neq 0 \) and \( c \neq 0 \). Assuming the former, then

Thus, \( K^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \)

b. \( M = E \cdot K^{-1} = \begin{bmatrix} 47 & 34 & 33 \\ 44 & 36 & 27 \\ 47 & 41 & 20 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 13 & 1 & 20 \\ 8 & 9 & 19 \\ 6 & 21 & 14 \end{bmatrix} \)

because

\[
\begin{align*}
a_{11} &= 47(1) + 34(-1) + 33(0) = 13 \\
a_{12} &= 47(0) + 34(1) + 33(-1) = 1 \\
a_{13} &= 47(-1) + 34(1) + 33(1) = 20 \\
a_{21} &= 44(1) + 36(-1) + 27(0) = 8 \\
a_{22} &= 44(0) + 36(1) + 27(-1) = 9 \\
a_{23} &= 44(-1) + 36(1) + 27(1) = 19 \\
a_{31} &= 47(1) + 41(-1) + 20(0) = 6 \\
a_{32} &= 47(0) + 41(1) + 20(-1) = 21 \\
a_{33} &= 47(-1) + 41(1) + 20(1) = 14
\end{align*}
\]

c. \( 13 \rightarrow M; 1 \rightarrow A; 20 \rightarrow T; 8 \rightarrow H; 9 \rightarrow I; 19 \rightarrow S; 6 \rightarrow F; 21 \rightarrow U; 14 \rightarrow N \)

The message: Math is fun.

89. \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \)

If \( D = ad - bc \neq 0 \), then \( a \neq 0 \) and \( d \neq 0 \), or \( b \neq 0 \) and \( c \neq 0 \). Assuming the former, then

Thus, \( K^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \)

b. \( M = E \cdot K^{-1} = \begin{bmatrix} 47 & 34 & 33 \\ 44 & 36 & 27 \\ 47 & 41 & 20 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 13 & 1 & 20 \\ 8 & 9 & 19 \\ 6 & 21 & 14 \end{bmatrix} \)

because

\[
\begin{align*}
a_{11} &= 47(1) + 34(-1) + 33(0) = 13 \\
a_{12} &= 47(0) + 34(1) + 33(-1) = 1 \\
a_{13} &= 47(-1) + 34(1) + 33(1) = 20 \\
a_{21} &= 44(1) + 36(-1) + 27(0) = 8 \\
a_{22} &= 44(0) + 36(1) + 27(-1) = 9 \\
a_{23} &= 44(-1) + 36(1) + 27(1) = 19 \\
a_{31} &= 47(1) + 41(-1) + 20(0) = 6 \\
a_{32} &= 47(0) + 41(1) + 20(-1) = 21 \\
a_{33} &= 47(-1) + 41(1) + 20(1) = 14
\end{align*}
\]

c. \( 13 \rightarrow M; 1 \rightarrow A; 20 \rightarrow T; 8 \rightarrow H; 9 \rightarrow I; 19 \rightarrow S; 6 \rightarrow F; 21 \rightarrow U; 14 \rightarrow N \)

The message: Math is fun.
by the components in the rows of the second matrix and then adding those products, then the number of columns in the first must equal the number of rows in the second.

92. For real numbers this you multiply both sides by the multiplicative inverse of \( c \), \( \frac{1}{c} \), \( c \) not equal 0. Since \( c \) times \( \frac{1}{c} \) results in 1, the multiplicative identity, then \( a = b \). For matrices, this would hold true as long as \( C \) has an inverse. Then you would multiply both sides by \( C^{-1} \) to get:

\[
AC = BC \\
ACC^{-1} = BCC^{-1} \\
AI = BI \\
A = B
\]

93. If the inverse of \( A \) exists:

\[
AX = 0 \\
A^{-1}AX = A^{-1}0 \\
IX = 0 \\
X = 0
\]

If the inverse of \( A \) does not exist, then \( A \) is singular and you would not be able to multiply \( A \) by its inverse to give the identity inverse and \( X \) would have no solution.

Section 11.5

1. True

2. True

3. \[3x^4 + 6x^3 + 3x^2 = 3x^2 \left( x^2 + 2x + 1 \right) = 3x^2 \left( x + 1 \right)^2\]

4. True

5. The rational expression \( \frac{x}{x^2 - 1} \) is proper, since the degree of the numerator is less than the degree of the denominator.

6. The rational expression \( \frac{5x + 2}{x^3 - 1} \) is proper, since the degree of the numerator is less than the degree of the denominator.

7. The rational expression \( \frac{x^2 + 5}{x^2 - 4} \) is improper, so perform the division:

\[
x^2 - 4 \overline{x^2 + 5} \\
\frac{x^2 - 4}{9} = 1 + \frac{9}{x^2 - 4}
\]

The proper rational expression is:

\[
\frac{x^2 + 5}{x^2 - 4} = 1 + \frac{9}{x^2 - 4}
\]

8. The rational expression \( \frac{3x^2 - 2}{x^2 - 1} \) is improper, so perform the division:

\[
x^2 - 1 \overline{3x^2 - 2} \\
3x^2 - 3 \\
\frac{3x^2 - 2}{x^2 - 1} = 3 + \frac{1}{x^2 - 1}
\]

The proper rational expression is:

\[
\frac{3x^2 - 2}{x^2 - 1} = 3 + \frac{1}{x^2 - 1}
\]

9. The rational expression \( \frac{5x^3 + 2x - 1}{x^2 - 4} \) is improper, so perform the division:

\[
x^2 - 4 \overline{5x^3 + 2x - 1} \\
\frac{5x^3 - 20x}{22x - 1} = 5x + \frac{22x - 1}{x^2 - 4}
\]

The proper rational expression is:

\[
\frac{5x^3 + 2x - 1}{x^2 - 4} = 5x + \frac{22x - 1}{x^2 - 4}
\]

10. The rational expression \( \frac{3x^4 + x^2 - 2}{x^3 + 8} \) is improper, so perform the division:

\[
x^3 + 8 \overline{3x^4 + x^2 - 2} \\
\frac{3x^4 + 24x}{x^2 - 24x - 2} = 3x + \frac{x^2 - 24x - 2}{x^3 + 8}
\]

The proper rational expression is:

\[
\frac{3x^4 + x^2 - 2}{x^3 + 8} = 3x + \frac{x^2 - 24x - 2}{x^3 + 8}
\]
11. The rational expression
\[
\frac{x(x - 1)}{(x + 4)(x - 3)} = \frac{x^2 - x}{x^2 + x - 12}
\]
is improper, so perform the division:
\[
\frac{1}{x^2 - x + 0} = \frac{-2x + 12}{x^2 + x - 12}
\]
The proper rational expression is:
\[
\frac{-2(x - 6)}{(x + 4)(x - 3)} = \frac{2}{x + 2} + \frac{1}{x - 1}
\]

12. The rational expression
\[
\frac{2x(x^2 + 4)}{x^2 + 1} = \frac{2x^3 + 8x}{x^2 + 1}
\]
is improper, so perform the division:
\[
\frac{2x}{x^2 + 1} + \frac{2x^3 + 2x}{6x}
\]
The proper rational expression is:
\[
\frac{2x(x^2 + 4)}{x^2 + 1} = 2x + \frac{6x}{x^2 + 1}
\]

13. Find the partial fraction decomposition:
\[
\frac{4}{x(x - 1)} = \frac{A}{x} + \frac{B}{x - 1}
\]
\[
x(x - 1) \left( \frac{4}{x(x - 1)} \right) = x(x - 1) \left( \frac{A}{x} + \frac{B}{x - 1} \right)
\]
\[
4 = A(x - 1) + Bx
\]
Let \( x = 1 \), then \( 4 = A(0) + B \)
\[
B = 4
\]
Let \( x = 0 \), then \( 4 = A(-1) + B(0) \)
\[
A = -4
\]

14. Find the partial fraction decomposition
\[
\frac{3x}{(x + 2)(x - 1)} = \frac{A}{x + 2} + \frac{B}{x - 1}
\]
Multiplying both sides by \((x + 2)(x - 1)\), we obtain: \( 3x = A(x - 1) + B(x + 2) \)

15. Find the partial fraction decomposition:
\[
\frac{1}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}
\]
\[
x(x^2 + 1) \left( \frac{1}{x(x^2 + 1)} \right) = x(x^2 + 1) \left( \frac{A}{x} + \frac{Bx + C}{x^2 + 1} \right)
\]
\[
1 = A(x^2 + 1) + (Bx + C)x
\]
Let \( x = 0 \), then \( 1 = A(0^2 + 1) + (B(0) + C)(0) \)
\[
A = 1
\]
Let \( x = 1 \), then \( 1 = A(1^2 + 1) + (B(1) + C)(1) \)
\[
1 = 2A + B + C
\]
\[
1 = 2(1) + B + C
\]
\[
B + C = -1
\]
Let \( x = -1 \), then \( 1 = A((-1)^2 + 1) + (B(-1) + C)(-1) \)
\[
1 = A(1 + 1) + (-B + C)(-1)
\]
\[
1 = 2A - B - C
\]
\[
1 = 2(-1) + B - C
\]
\[
B - C = -1
\]
Solve the system of equations:
\[
B + C = -1
\]
\[
B - C = -1
\]
\[
2B = -2
\]
\[
B = -1
\]
\[
-1 + C = -1
\]
\[
C = 0
\]
\[
1 = 1 + \frac{-x}{x^2 + 1}
\]

16. Find the partial fraction decomposition:
\[
\frac{1}{(x + 1)(x^2 + 4)} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 4}
\]
Multiplying both sides by \((x + 1)(x^2 + 4)\), we obtain: \( 1 = A(x^2 + 4) + (Bx + C)(x + 1) \)
Let \( x = -1 \), then
\[
1 = A(5) + (B(-1) + C)(0)
\]
\[
5A = 1
\]
\[
A = \frac{1}{5}
\]

Let \( x = 1 \), then
\[
1 = A(1^2 + 4) + (B(1) + C)(1+1)
\]
\[
1 = 5A + (B + C)(2)
\]
\[
1 = 5(1/5) + 2B + 2C
\]
\[
1 = 1 + 2B + 2C
\]
\[
0 = 2B + 2C
\]
\[
0 = B + C
\]

Let \( x = 0 \), then
\[
1 = A(0^2 + 4) + (B(0) + C)(0+1)
\]
\[
1 = 4A + C
\]
\[
1 = 4(1/5) + C
\]
\[
1 = \frac{4}{5} + C
\]
\[
C = \frac{1}{5}
\]

Since \( B + C = 0 \), we have that \( B + \frac{1}{5} = 0 \)
\[
B = -\frac{1}{5}
\]
\[
\frac{1}{(x+1)(x^2 + 4)} = \frac{\frac{1}{5}}{x+1} + \frac{-\frac{1}{5}x + \frac{1}{5}}{x^2 + 4}
\]

17. Find the partial fraction decomposition:
\[
\frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}
\]

Multiplying both sides by \((x-1)(x-2)\), we obtain:
\[
x = A(x-2) + B(x-1)
\]

Let \( x = 1 \), then
\[
1 = A(1-2) + B(1-1)
\]
\[
1 = -A
\]
\[
A = -1
\]

Let \( x = 2 \), then
\[
2 = A(2-2) + B(2-1)
\]
\[
2 = B
\]
\[
\frac{x}{(x-1)(x-2)} = \frac{-1}{x-1} + \frac{2}{x-2}
\]

18. Find the partial fraction decomposition:
\[
\frac{3x}{(x+2)(x-4)} = \frac{A}{x+2} + \frac{B}{x-4}
\]

Multiplying both sides by \((x+2)(x-4)\), we obtain:
\[
3x = A(x-4) + B(x+2)
\]

Let \( x = -2 \), then
\[
3(-2) = A(-2 - 4) + B(-2 + 2)
\]
\[
-6 = -6A
\]
\[
A = 1
\]

Let \( x = 4 \), then
\[
3(4) = A(4 - 4) + B(4 + 2)
\]
\[
12 = 6B
\]
\[
B = 2
\]
\[
\frac{3x}{(x+2)(x-4)} = \frac{1}{x+2} + \frac{2}{x-4}
\]

19. Find the partial fraction decomposition:
\[
\frac{x^2}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}
\]

Multiplying both sides by \((x-1)^2(x+1)\), we obtain:
\[
x^2 = A(x-1)(x+1) + B(x-1) + C(x-1)^2
\]

Let \( x = 1 \), then
\[
1^2 = A(1-1)(1+1) + B(1+1) + C(1-1)^2
\]
\[
1 = A(0)(2) + B(2) + C(0)^2
\]
\[
1 = 2B
\]
\[
B = \frac{1}{2}
\]

Let \( x = -1 \), then
\[
(-1)^2 = A(-1-1)(-1+1) + B(-1+1) + C(-1-1)^2
\]
\[
1 = A(-2)(0) + B(0) + C(-2)^2
\]
\[
1 = 4C
\]
\[
C = \frac{1}{4}
\]

Let \( x = 0 \), then
\[
0^2 = A(0-1)(0+1) + B(0+1) + C(0-1)^2
\]
\[
0 = -A + B + C
\]
\[
A = B + C
\]
\[
A = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}
\]
\[
\frac{x^2}{(x-1)^2(x+1)} = \frac{3}{4} \cdot \frac{1}{x-1} + \frac{1}{(x-1)^2} + \frac{1}{4} \cdot \frac{1}{x+1}
\]

20. Find the partial fraction decomposition:
\[
\frac{x+1}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}
\]

Multiplying both sides by \(x^2(x-2)\), we obtain:
\[
x+1 = Ax(x-2) + B(x-2) + Cx^2
\]
Let \( x = 0 \), then
\[
0 + 1 = A(0)(0 - 2) + B(0 - 2) + C(0)^2
\]
\[
1 = -2B
\]
\[
B = -\frac{1}{2}
\]
Let \( x = 2 \), then
\[
2 + 1 = A(2)(2 - 2) + B(2 - 2) + C(2)^2
\]
\[
3 = 4C
\]
\[
C = \frac{3}{4}
\]
Let \( x = 1 \), then \( 2 = -A - B + C \)
\[
A = -B + C - 2
\]
\[
A = -\left(-\frac{1}{2}\right) + \frac{3}{4} - 2 = -\frac{3}{4}
\]
\[
\frac{x + 1}{x^2(x - 2)} = \frac{-\frac{3}{4}}{x} + \frac{-\frac{1}{2}}{x^2} + \frac{\frac{3}{4}}{x - 2}
\]

21. Find the partial fraction decomposition:
\[
\frac{1}{x^3 - 8} = \frac{1}{(x - 2)(x^2 + 2x + 4)}
\]
\[
\frac{1}{(x - 2)(x^2 + 2x + 4)} = \frac{A}{x - 2} + \frac{Bx + C}{x^2 + 2x + 4}
\]
Multiplying both sides by \((x - 2)(x^2 + 2x + 4)\), we obtain:
\[
1 = A(x^2 + 2x + 4) + (Bx + C)(x - 2)
\]
Let \( x = 1 \), then
\[
1 = A\left(1^2 + 2(1) + 4\right) + (B(1) + C)(1 - 2)
\]
\[
1 = 12A
\]
\[
A = \frac{1}{12}
\]
Let \( x = 0 \), then
\[
1 = A\left(0^2 + 2(0) + 4\right) + (B(0) + C)(0 - 2)
\]
\[
1 = 4A - 2C
\]
\[
1 = 4(1/12) - 2C
\]
\[
-2C = \frac{2}{3}
\]
\[
C = -\frac{1}{3}
\]

Let \( x = 1 \), then
\[
1 = A\left(1^2 + 2(1) + 4\right) + (B(1) + C)(1 - 2)
\]
\[
1 = 7A - B - C
\]
\[
1 = 7(1/12) - B + \frac{1}{3}
\]
\[
B = -\frac{1}{12}
\]
\[
\frac{1}{x^3 - 8} = \frac{\frac{1}{12}}{x - 2} + \frac{-\frac{1}{12}x - \frac{1}{3}}{x^2 + 2x + 4}
\]
\[
= \frac{1}{12x} + \frac{-\frac{1}{12}(x + 4)}{x^2 + 2x + 4}
\]

22. Find the partial fraction decomposition:
\[
\frac{2x + 4}{x^3 - 1} = \frac{2x + 4}{(x - 1)(x^2 + x + 1)} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + x + 1}
\]
Multiplying both sides by \((x - 1)(x^2 + x + 1)\), we obtain:
\[
2x + 4 = A(x^2 + x + 1) + (Bx + C)(x - 1)
\]
Let \( x = 1 \), then
\[
2(1) + 4 = A\left(1^2 + 1 + 1\right) + (B(1) + C)(1 - 1)
\]
\[
6 = 3A
\]
\[
A = 2
\]
Let \( x = 0 \), then
\[
2(0) + 4 = A\left(0^2 + 0 + 1\right) + (B(0) + C)(0 - 1)
\]
\[
4 = -A - C
\]
\[
4 = 2 - C
\]
\[
C = -2
\]
Let \( x = -1 \), then
\[
2(-1) + 4 = A\left((-1)^2 + (-1) + 1\right) + (B(-1) + C)(-1 - 1)
\]
\[
2 = A + 2B - 2C
\]
\[
2 = 2 + 2B - 2(-2)
\]
\[
2B = -4
\]
\[
B = -2
\]
\[
\frac{2x + 4}{x^3 - 1} = \frac{2}{x - 1} + \frac{-2x - 2}{x^2 + x + 1}
\]

23. Find the partial fraction decomposition:
\[
\frac{x^2}{(x - 1)^2(x + 1)^2} = \frac{A}{(x - 1)} + \frac{B}{(x - 1)^2} + \frac{C}{x + 1} + \frac{D}{(x + 1)^2}
\]
Multiplying both sides by \((x - 1)^2(x + 1)^2\), we obtain:
\[
x^2 = A(x - 1)(x + 1)^2 + B(x + 1)^2 + C(x - 1)^2(x + 1) + D(x - 1)^2
\]

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Let \( x = 1 \), then
\[
1^2 = A(1-1)(1+1)^2 + B(1+1)^2 + C(1-1)^2(1+1) + D(1-1)^2
\]
\[
1 = 4B
\]
\[
B = \frac{1}{4}
\]
Let \( x = -1 \), then
\[
(-1)^2 = A(-1-1)(-1+1)^2 + B(-1+1)^2 + C(-1-1)^2(-1+1) + D(-1-1)^2
\]
\[
1 = 4D
\]
\[
D = \frac{1}{4}
\]
Let \( x = 0 \), then
\[
0^2 = A(0-1)(0+1)^2 + B(0+1)^2 + C(0-1)^2(0+1) + D(0-1)^2
\]
\[
0 = -A + B + C + D
\]
\[
A - C = B + D
\]
\[
A - C = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}
\]
Let \( x = 2 \), then
\[
2^2 = A(2-1)(2+1)^2 + B(2+1)^2 + C(2-1)^2(2+1) + D(2-1)^2
\]
\[
4 = 9A + 9B + 3C + D
\]
\[
9A + 3C = 4 - 9B - D
\]
\[
9A + 3C = 4 - 9\left(\frac{1}{4}\right) - \frac{1}{4} = \frac{3}{2}
\]
\[
3A + C = \frac{1}{2}
\]
Solve the system of equations:
\[
A - C = \frac{1}{2}
\]
\[
3A + C = \frac{1}{2}
\]
\[
4A = 1
\]
\[
A = \frac{1}{4}
\]
\[
\frac{3}{4} + C = \frac{1}{2}
\]
\[
C = -\frac{1}{4}
\]
\[
\frac{x^2}{(x-1)^4(x+1)^2} = \frac{1}{4} \frac{x}{x-1} + \frac{1}{4} \frac{x}{(x-1)^2} + \frac{1}{4} \frac{1}{x+1} + \frac{1}{4} \frac{1}{(x+1)^2}
\]

24. Find the partial fraction decomposition:
\[
\frac{x+1}{x^2(x-2)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{D}{(x-2)^2}
\]
Multiplying both sides by \( x^2(x-2)^2 \), we obtain:
\[
x+1 = Ax(x-2)^2 + B(x-2)^2 + Cx^2(x-2) + Dx^2
\]
Let \( x = 0 \), then
\[
0+1 = A(0)(0-2)^2 + B(0-2)^2 + C(0)(0-2) + D(0)^2
\]
\[
1 = 4B
\]
\[
B = \frac{1}{4}
\]
Let \( x = 2 \), then
\[
2+1 = A(2)(2-2)^2 + B(2-2)^2 + C(2)(2-2) + D(2)^2
\]
\[
3 = 4D
\]
\[
D = \frac{3}{4}
\]
Let \( x = 1 \), then
\[
1+1 = A(1)(1-2)^2 + B(1-2)^2 + C(1)(1-2) + D(1)^2
\]
\[
2 = A + B - C + D
\]
\[
A - C = 2 - B - D
\]
\[
A - C = 2 - \frac{1}{4} - \frac{3}{4} = 1
\]
Let \( x = 3 \), then
\[
3+1 = A(3)(3-2)^2 + B(3-2)^2 + C(3)(3-2) + D(3)^2
\]
\[
4 = 3A + B + 9C + 9D
\]
\[
3A + 9C = 4 - B - 9D
\]
\[
3A + 9C = 4 - \frac{1}{4} - 9\left(\frac{3}{4}\right) = -3
\]
\[
A + 3C = -1
\]
Solve the system of equations:
\[
\begin{align*}
A - C &= 1 \\
A + 3C &= -1 \\
A &= C + 1
\end{align*}
\]
\[
A + 3C = -1
\]
\[
(C+1) + 3C = -1
\]
\[
4C = -2
\]
\[
C = -\frac{1}{2}
\]
25. Find the partial fraction decomposition:
\[
\frac{x - 3}{(x + 2)(x + 1)^2} = \frac{A}{x + 2} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}
\]
Multiplying both sides by \((x + 2)(x + 1)^2\), we obtain:
\[
x - 3 = A(x + 1)^2 + B(x + 2)(x + 1) + C(x + 2)
\]
Let \(x = -2\), then
\[
-2 - 3 = A(-2 + 1)^2 + B(-2 + 2)(-2 + 1) + C(-2 + 2)
\]
\[
-5 = A
\]
\[
A = -5
\]
Let \(x = 1\), then
\[
-1 - 3 = A(-1 + 1)^2 + B(-1 + 2)(-1 + 1) + C(-1 + 2)
\]
\[
-4 = C
\]
\[
C = -4
\]
Let \(x = 0\), then
\[
0 - 3 = A(0 + 1)^2 + B(0 + 2)(0 + 1) + C(0 + 2)
\]
\[
-3 = 2B + 2C
\]
\[
-3 = -5 + 2B + 2(-4)
\]
\[
2B = 10
\]
\[
B = 5
\]
\[
\frac{x - 3}{(x + 2)(x + 1)^2} = \frac{-5}{x + 2} + \frac{5}{x + 1} + \frac{-4}{(x + 1)^2}
\]
26. Find the partial fraction decomposition:
\[
\frac{x^2 + x}{(x + 2)(x - 1)^2} = \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2}
\]
Multiplying both sides by \((x + 2)(x - 1)^2\), we obtain:
\[
x^2 + x = A(x - 1)^2 + B(x + 2)(x - 1) + C(x + 2)
\]
Let \(x = -2\), then
\[
(-2)^2 + (-2) = A(-2 - 1)^2 + B(-2 + 2)(-2 - 1) + C(-2 + 2)
\]
\[
2 = 9A
\]
\[
A = \frac{2}{9}
\]
Let \(x = 1\), then
\[
1^2 + 1 = A(1 - 1)^2 + B(1 + 2)(1 - 1) + C(1 + 2)
\]
\[
2 = 3C
\]
\[
C = \frac{2}{3}
\]
Let \(x = 0\), then
\[
0^2 + 0 = A(0 - 1)^2 + B(0 + 2)(0 - 1) + C(0 + 2)
\]
\[
0 = A - 2B + 2C
\]
\[
2B = A + 2C
\]
\[
2B = \frac{2}{9} + 2 \left(\frac{2}{3}\right) = \frac{14}{9}
\]
\[
B = \frac{7}{9}
\]
\[
\frac{x^2 + x}{(x + 2)(x - 1)^2} = \frac{-\frac{2}{9}}{x + 2} + \frac{\frac{7}{9}}{x - 1} + \frac{\frac{2}{3}}{(x - 1)^2}
\]
27. Find the partial fraction decomposition:
\[
\frac{x + 4}{x^2(x^2 + 4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2 + 4}
\]
Multiplying both sides by \(x^2(x^2 + 4)\), we obtain:
\[
x + 4 = Ax(x^2 + 4) + B(x^2 + 4) + (Cx + D)x^2
\]
Let \(x = 0\), then
\[
0 + 4 = A(0)(x^2 + 4) + B(0^2 + 4) + (C(0) + D)(0)^2
\]
\[
4 = 4B
\]
\[
B = 1
\]
Let \(x = 1\), then
\[
1 + 4 = A(1)(1^2 + 4) + B(1^2 + 4) + (C(1) + D)(1)^2
\]
\[
5 = 5A + 5B + C + D
\]
\[
5 = 5A + 5 + C + D
\]
\[
5A + C + D = 0
\]
Let \(x = -1\), then
\[
-1 + 4 = A(-1)((-1)^2 + 4) + B((-1)^2 + 4) + (C(-1) + D)((-1)^2)
\]
\[
3 = -5A + 5B - C + D
\]
\[
3 = -5A + 5 - C + D
\]
\[
-5A - C + D = -2
\]
Let \(x = 2\), then
\[
2 + 4 = A(2)(2^2 + 4) + B(2^2 + 4) + (C(2) + D)(2)^2
\]
\[
6 = 16A + 8B + 8C + 4D
\]
\[
6 = 16A + 8 + 8C + 4D
\]
\[
16A + 8C + 4D = -2
\]
Chapter 11: Systems of Equations and Inequalities

Solve the system of equations:
\[
\begin{align*}
5A + C + D &= 0 \\
-5A - C + D &= -2 \\
2D &= -2
\end{align*}
\]

\[D = -1\]

- \[5A + C = 1 = -5A\]
- \[16A + 8(\bar{5}A) + 4(-1) = -2\]
- \[16A + 8 - 40A - 4 = -2\]
- \[24A = -6\]

\[A = \frac{1}{4}\]

\[C = 1 - 5\left(\frac{1}{4}\right) = 1 - \frac{5}{4} = \frac{1}{4}\]

\[\frac{x + 4}{x^2(x^2 + 4)} = \frac{\frac{1}{4} + \frac{1}{4}x - \frac{1}{4}(x + 4)}{x^2 + 4}\]

28. Find the partial fraction decomposition:
\[
\frac{10x^2 + 2x}{(x-1)^2(x^2 + 2)} = A\frac{1}{x-1} + B\frac{1}{x-1}^2 + \frac{Cx + D}{x^2 + 2}
\]

Multiply both sides by \((x-1)^2(x^2 + 2)\):
\[
10x^2 + 2x = A(x-1)(x^2 + 2) + B(x^2 + 2) + (Cx + D)(x-1)^2
\]

Let \(x = 1\), then
\[
10(1)^2 + 2(1) = A(1-1)(1^2 + 2) + B(1^2 + 2) + (C(1) + D)(1-1)^2
\]
\[12 = 3B\]
\[B = 4\]

Let \(x = 0\), then
\[
10(0)^2 + 2(0) = A(0-1)(0^2 + 2) + B(0^2 + 2) + (C(0) + D)(0-1)^2
\]
\[0 = -2A + 2B + D\]
\[0 = -2A + 8 + D\]

\[2A - D = 8\]
\[D = 2A - 8\]

Let \(x = -1\), then
\[
10(-1)^2 + 2(-1) = A(-1-1)((-1)^2 + 2) + B((-1)^2 + 2) + (C(-1) + D)((-1-1)^2
\]
\[8 = -6A + 3B - 4C + 4D\]
\[8 = -6A + 12 - 4C + 4D\]

\[-6A - 4C + 4D = -4\]

Let \(x = 2\), then
\[
10(2)^2 + 2(2) = A(2-1)(2^2 + 2) + B(2^2 + 2) + (C(2) + D)(2-1)^2
\]
\[44 = 6A + 6B + 2C + D\]
\[44 = 6A + 24 + 2C + D\]

\[6A + 2C + D = 20\]

Solve the system of equations (Substitute for \(D\)):
\[
D = 2A - 8
\]
\[6A + 2C + D = 20\]
\[6A + 2C + (2A - 8) = 20\]
\[8A + 2C = 28\]

\[A - 2C = 14\]
\[9A = 42\]
\[A = \frac{14}{3}\]

\[2C = A - 14 = \frac{14}{3} - 14 = -\frac{28}{3}\]
\[C = -\frac{14}{3}\]

\[D = 2A - 8 = 2\left(\frac{14}{3}\right) - 8 = \frac{4}{3}\]

\[
10x^2 + 2x = \frac{14}{x-1} + \frac{4}{x^2 + 2} + \frac{-14}{x} + \frac{4}{x^2 + 2}
\]
29. Find the partial fraction decomposition:

\[ \frac{x^2 + 2x + 3}{(x+1)(x^2 + 2x + 4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2 + 2x + 4} \]

Multiplying both sides by \((x+1)(x^2 + 2x + 4)\), we obtain:

\[ x^2 + 2x + 3 = A(x^2 + 2x + 4) + (Bx+C)(x+1) \]

Let \(x = -1\), then

\[ (-1)^2 + 2(-1) + 3 = A((-1)^2 + 2(-1) + 4) + (B(-1)+C)(-1+1) \]

\[ 2 = 3A \]

Let \(x = 0\), then

\[ 0^2 + 2(0) + 3 = A(0^2 + 2(0) + 4) + (B(0)+C)(0+1) \]

\[ 3 = 4A + C \]

\[ 3 = 4(2/3) + C \]

\[ C = \frac{1}{3} \]

Let \(x = 1\), then

\[ 1^2 + 2(1) + 3 = A(1^2 + 2(1) + 4) + (B(1)+C)(1+1) \]

\[ 6 = 7A + 2B + 2C \]

\[ 6 = 7\left(\frac{2}{3}\right) + 2B + 2\left(\frac{1}{3}\right) \]

\[ 2B = 6 - \frac{14}{3} - \frac{2}{3} = \frac{2}{3} \]

\[ B = \frac{1}{3} \]

\[ \frac{x^2 + 2x + 3}{(x+1)(x^2 + 2x + 4)} = \frac{\frac{2}{3}}{x+1} + \frac{\frac{1}{3}x + \frac{1}{3}}{x^2 + 2x + 4} \]

30. Find the partial fraction decomposition:

\[ \frac{x^2 - 11x - 18}{x(x^2 + 3x + 3)} = \frac{A}{x^2} + \frac{Bx+C}{x^2 + 3x + 3} \]

Multiplying both sides by \(x(x^2 + 3x + 3)\), we obtain:

\[ x^2 - 11x - 18 = A(x^2 + 3x + 3) + (Bx+C)x \]

Let \(x = 0\), then

\[ 0^2 - 11(0) - 18 = A(0^2 + 3(0) + 3) + (B(0)+C)(0) \]

\[ -18 = 3A \]

\[ A = -6 \]

Let \(x = 1\), then

\[ 1^2 - 11(1) - 18 = A\left(1^2 + 3(1) + 3\right) + (B(1)+C)(1) \]

\[ -28 = 7A + B + C \]

\[ -28 = 7(-6) + B + C \]

\[ B + C = 14 \]

Let \(x = -1\), then

\[ (-1)^2 - 11(-1) - 18 = A((-1)^2 + 3(-1) + 3) \]

\[ + (B(-1)+C)(-1) \]

\[ -6 = A + B - C \]

\[ -6 = -6 + B - C \]

\[ B - C = 0 \]

Add the last two equations and solve:

\[ B + C = 14 \]

\[ B - C = 0 \]

\[ 2B = 14 \]

\[ B = 7 \]

\[ B + C = 14 \]

\[ 7 + C = 14 \]

\[ C = 7 \]

\[ \frac{x^2 - 11x - 18}{x(x^2 + 3x + 3)} = \frac{-6}{x} + \frac{7x + 7}{x^2 + 3x + 3} \]

31. Find the partial fraction decomposition:

\[ \frac{x}{(3x - 2)(2x + 1)} = \frac{A}{3x - 2} + \frac{B}{2x + 1} \]

Multiplying both sides by \((3x - 2)(2x + 1)\), we obtain: \(x = A(2x+1) + B(3x-2)\)

Let \(x = -\frac{1}{2}\), then

\[ -\frac{1}{2} = A\left(2\left(-\frac{1}{2}\right)+1\right) + B\left(3\left(-\frac{1}{2}\right)-2\right) \]

\[ -\frac{1}{2} = -\frac{7}{2} B \]

\[ B = \frac{1}{7} \]

Let \(x = \frac{2}{3}\), then

\[ \frac{2}{3} = A\left(2\left(\frac{2}{3}\right)+1\right) + B\left(3\left(\frac{2}{3}\right)-2\right) \]

\[ \frac{2}{3} = \frac{7}{3} A \]

\[ A = \frac{2}{7} \]

\[ x \]

\[ \frac{x}{(3x - 2)(2x + 1)} = \frac{\frac{2}{7}}{3x - 2} + \frac{\frac{1}{7}}{2x + 1} \]
32. Find the partial fraction decomposition:
\[
\frac{1}{(2x+3)(4x-1)} = \frac{A}{2x+3} + \frac{B}{4x-1}
\]
Multiplying both sides by \((2x+3)(4x-1)\), we obtain:
\[1 = A(4x-1) + B(2x+3)\]
Let \(x = -\frac{3}{2}\), then
\[1 = A\left(4\left(-\frac{3}{2}\right)-1\right) + B\left(2\left(-\frac{3}{2}\right)+3\right)\]
\[1 = -7A\]
\[A = -\frac{1}{7}\]
Let \(x = \frac{1}{4}\), then
\[1 = A\left(4\left(\frac{1}{4}\right)-1\right) + B\left(2\left(\frac{1}{4}\right)+3\right)\]
\[1 = \frac{7}{2}B\]
\[B = \frac{2}{7}\]
\[
\frac{1}{(2x+3)(4x-1)} = -\frac{1}{7} \cdot \frac{2}{2x+3} + \frac{2}{7} \cdot \frac{1}{4x-1}
\]
33. Find the partial fraction decomposition:
\[
\frac{x}{x^2+2x-3} = \frac{x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}
\]
Multiplying both sides by \((x+3)(x-1)\), we obtain:
\[x = A(x+3) + B(x-1)\]
Let \(x = 1\), then
\[1 = A(1+3) + B(1+3)\]
\[1 = 4B\]
\[B = \frac{1}{4}\]
Let \(x = -3\), then
\[-3 = A(-3+3) + B(-3+3)\]
\[-3 = -4A\]
\[A = \frac{3}{4}\]
\[
\frac{x}{x^2+2x-3} = \frac{3}{4} \cdot \frac{1}{x+3} + \frac{1}{4} \cdot \frac{1}{x-1}
\]
34. Find the partial fraction decomposition:
\[
\frac{x^2-x-8}{(x+1)(x^2+5x+6)} = \frac{x^2-x-8}{(x+1)(x+2)(x+3)}
\]
Multiplying both sides by \((x+1)(x+2)(x+3)\), we obtain:
\[x^2-x-8 = A(x+2)(x+3) + B(x+1)(x+3)
\]
Let \(x = -1\), then
\[(-1)^2-(-1)-8 = A(-1+2)(-1+3)
\]
\[6 = 2A\]
\[A = 3\]
Let \(x = -2\), then
\[(-2)^2-(-2)-8 = A(-2+2)(-2+3)
\]
\[0 = -2\]
\[B = 2\]
Let \(x = -3\), then
\[(-3)^2-(-3)-8 = A(-3+2)(-3+3)
\]
\[0 = -3\]
\[C = 2\]
\[
\frac{x^2-x-8}{(x+1)(x^2+5x+6)} = \frac{-3}{x+1} + \frac{2}{x+2} + \frac{2}{x+3}
\]
35. Find the partial fraction decomposition:
\[
\frac{x^2+2x+3}{(x^2+4)^2} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2}
\]
Multiplying both sides by \((x^2+4)^2\), we obtain:
\[x^2+2x+3 = (Ax+B)(x^2+4) + Cx+D\]
\[x^2+2x+3 = Ax^3 + Bx^2 + 4Ax + 4B + Cx + D\]
\[A = 1; \quad B = 1; \quad C = 2\]
\[4A+C = 2; \quad 4B+D = 3\]
\[4(0) + C = 2; \quad 4(1) + D = 3\]
\[C = 2; \quad D = -1\]
\[
\frac{x^2+2x+3}{(x^2+4)^2} = \frac{1}{x^2+4} + \frac{2x-1}{(x^2+4)^2}
\]
36. Find the partial fraction decomposition:
\[
\frac{x^3 + 1}{(x^2 + 16)^2} = \frac{Ax + B}{x^2 + 16} + \frac{Cx + D}{(x^2 + 16)^2}
\]
Multiplying both sides by \((x^2 + 16)^2\), we obtain:
\[
x^3 + 1 = (Ax + B)(x^2 + 16) + Cx + D
\]
\[
x^3 + 1 = Ax^3 + Bx^2 + 16Ax + 16B + Cx + D
\]
\[
x^3 + 1 = Ax^3 + Bx^2 + (16A + C)x + 16B + D
\]
\[A = 1; \quad B = 0; \quad 16A + C = 0 \]
\[16(1) + C = 0 \quad C = -16 \]
\[16B + D = 1 \quad 16(0) + D = 1 \quad D = 1 \]
\[
\frac{x^3 + 1}{(x^2 + 16)^2} = \frac{x}{x^2 + 16} + \frac{-16x + 1}{(x^2 + 16)^2}
\]

37. Find the partial fraction decomposition:
\[
\frac{7x + 3}{x^3 - 2x^2 - 3x} = \frac{7x + 3}{x(x-3)(x+1)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x+1}
\]
Multiplying both sides by \(x(x-3)(x+1)\), we obtain:
\[
7x + 3 = A(x-3)(x+1) + Bx(x+1) + Cx(x-3)
\]
Let \(x = 0\), then
\[
7(0) + 3 = A(0-3)(0+1) + B(0)(0+1) + C(0)(0-3) \quad 3 = -3A \quad A = -1
\]
Let \(x = 3\), then
\[
7(3) + 3 = A(3-3)(3+1) + B(3)(3+1) + C(3)(3-3) \quad 24 = 12B \quad B = 2
\]
Let \(x = -1\), then
\[
7(-1) + 3 = A(-1-3)(-1+1) + B(-1)(-1+1) + C(-1)(-1-3) \quad -4 = 4C \quad C = -1
\]
\[
\frac{7x + 3}{x^3 - 2x^2 - 3x} = \frac{-1}{x} + \frac{2}{x-3} + \frac{-1}{x+1}
\]

38. Find the partial fraction decomposition:
\[
\frac{x^3 + 1}{x^3 - x^4} = \frac{x^3 + 1}{x^3(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x-1}
\]
Multiplying both sides by \(x^4(x-1)\), we obtain:
\[
x^3 + 1 = Ax^3(x-1) + Bx^2(x-1) + Cx(x-1) + D(x-1) + Ex^4
\]
Let \(x = 0\), then
\[
0^4 + 1 = A \cdot 0^3(0-1) + B \cdot 0^2(0-1) + C \cdot 0(0-1) + D(0-1) + E \cdot 0^4
\]
\[1 = -D \quad D = -1
\]
Let \(x = 1\), then
\[
1^4 + 1 = A \cdot 1^3(1-1) + B \cdot 1^2(1-1) + C \cdot 1(1-1)
\]
\[1 = 2D + E \cdot 1^4 \quad 2 = E
\]
Let \(x = -1\), then
\[
(-1)^4 + 1 = A(-1)^3(-1-1) + B(-1)^2(-1-1)
\]
\[+ C(-1)(-1-1) + D(-1-1) + E(-1)^4
\]
\[0 = 2A - 2B + 2C - 2D + E
\]
\[0 = 2A - 2B + 2C - 2(-1) + 2
\]
\[2A - 2B + 2C = -4
\]
\[A - B + C = -2
\]
Let \(x = 2\), then
\[
2^4 + 1 = A \cdot 2^3(2-1) + B \cdot 2^2(2-1) + C \cdot 2(2-1)
\]
\[+ D(2-1) + E \cdot 2^4
\]
\[9 = 8A + 4B + 2C + D + 16E
\]
\[9 = 8A + 4B + 2C + (-1) + 16(2)
\]
\[8A + 4B + 2C = -22
\]
\[4A + 2B + C = -11
\]
Let \(x = -2\), then
\[
(-2)^4 + 1 = A(-2)^3(-2-1) + B(-2)^2(-2-1)
\]
\[+ C(-2)(-2-1) + D(-2-1) + E(-2)^4
\]
\[-7 = 24A - 12B + 6C - 3D + 16E
\]
\[-7 = 24A - 12B + 6C - 3(-1) + 81(2)
\]
\[-42 = 24A - 12B + 6C
\]
\[-14 = 8A - 4B + 2C
\]
Chapter 11: Systems of Equations and Inequalities

Solve the system of equations by using a matrix equation:

\[
\begin{align*}
A - B + C &= -2 \\
4A + 2B + C &= -11 \\
8A - 4B + 2C &= -14
\end{align*}
\]

\[
\begin{bmatrix}
1 & -1 & 1 \\
4 & 2 & 1 \\
8 & -4 & 2
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix}
=
\begin{bmatrix}
-2 \\
-11 \\
-14
\end{bmatrix}
\]

\[
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix}
=
\begin{bmatrix}
1 & -1 & 1^{-1} \\
4 & 2 & 1 \\
8 & -4 & 2
\end{bmatrix}
\begin{bmatrix}
-2 \\
-11 \\
-14
\end{bmatrix}
\]

So, \(A = -2\), \(B = -1\), and \(C = -1\). Thus,

\[
\frac{x^3 + 1}{x^3 - x^2} = \frac{x^3 + 1}{x^3(x-1)}
\]

\[
= \frac{-2}{x^2} + \frac{-1}{x^3} + \frac{-1}{x^4} + \frac{2}{x-1}
\]

39. Perform synthetic division to find a factor:

\[
\begin{array}{c|cccc}
\text{2|} & 1 & -4 & 5 & -2 \\
\hline & & 2 & -4 & 2 \\
1 & 2 & 3 & 0
\end{array}
\]

\(x^3 - 4x^2 + 5x - 2 = (x-2)(x^2 - 2x + 1) = (x-2)(x-1)^2\)

Find the partial fraction decomposition:

\[
\frac{x^2}{x^3 - 4x^2 + 5x - 2} = \frac{x^2}{(x-2)(x-1)^2}
\]

\[
= \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}
\]

Multiplying both sides by \((x-2)(x-1)^2\), we obtain:

\(x^2 = A(x-1)^2 + B(x-2)(x-1) + C(x-2)\)

Let \(x = 2\), then

\(4 = A\)

Let \(x = 1\), then

\(1^2 = A(1-1)^2 + B(1-2)(1-1) + C(1-2)\)

\(1 = -C\)

\(C = -1\)

Let \(x = 0\), then

\(0^2 = A(0-1)^2 + B(0-2)(0-1) + C(0-2)\)

\(0 = A + 2B - 2C\)

\(0 = 4 + 2B - 2(-1)\)

\(-2B = 6\)

\(B = -3\)

\[
x^2 = \frac{4}{x-2} + \frac{-3}{x-1} + \frac{-1}{(x-1)^2}
\]

40. Perform synthetic division to find a factor:

\[
\begin{array}{c|cccc}
1 & 1 & -5 & 3 \\
\hline & & 1 & 2 & -3 \\
1 & 2 & -3 & 0
\end{array}
\]

\(x^3 + x^2 - 5x + 3 = (x-1)(x^2 + 2x - 3) = (x+3)(x-1)^2\)

Find the partial fraction decomposition:

\[
\frac{x^2 + 1}{x^3 + x^2 - 5x + 3} = \frac{x^2 + 1}{(x+3)(x-1)^2}
\]

\[
= \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}
\]

Multiplying both sides by \((x+3)(x-1)^2\), we obtain:

\(x^2 + 1 = A(x-1)^2 + B(x+3)(x-1) + C(x+3)\)

Let \(x = -3\), then

\((-3)^2 + 1 = A(-3-1)^2 + B(-3+3)(-3-1) + C(3+3)\)

\(10 = 16A\)

\(A = \frac{5}{8}\)

Let \(x = 1\), then

\(1^2 + 1 = A(1-1)^2 + B(1+3)(1-1) + C(1+3)\)

\(2 = 4C\)

\(C = \frac{1}{2}\)
Let $x = 0$, then
\[0^2 + 1 = A(0-1)^2 + B(0+3)(0-1) + C(0+3)\]
\[
1 = A - 3B + 3C
\]
\[
1 = \frac{5}{8} - 3B + \frac{3}{2}\left(\frac{1}{2}\right)
\]
\[
3B = \frac{9}{8}
\]
\[
B = \frac{3}{8}
\]
\[
\frac{x^2 + 1}{x^3 + x^2 - 5x + 3} = \frac{5}{8} + \frac{3}{8} + \frac{1}{2} + \frac{1}{2}\]

**41.** Find the partial fraction decomposition:
\[
\frac{x^3}{(x^2 + 16)^3} = \frac{Ax + B}{x^2 + 16} + \frac{Cx + D}{(x^2 + 16)^2} + \frac{Ex + F}{(x^2 + 16)^3}
\]
Multiplying both sides by $(x^2 + 16)^3$, we obtain:
\[
x^3 = (Ax + B)(x^2 + 16)^2 + (Cx + D)(x^2 + 16)
\]
\[
+ Ex + F
\]
\[
x^3 = (Ax + B)(x^4 + 32x^2 + 256) + Cx^3 + Dx^2 + 16Cx + 16D + Ex + F
\]
\[
x^3 = Ax^5 + Bx^4 + 32Ax^3 + 32Bx^2 + 256Ax
\]
\[
+ 256B + Cx^3 + Dx^2 + 16Cx + 16D + Ex + F
\]
\[
x^3 = Ax^5 + Bx^4 + (32A + C)x^3 + (32B + D)x^2
\]
\[
+ (256A + 16C + E)x
\]
\[
+ (256B + 16D + F)
\]
\[
A = 0; \quad B = 0; \quad 32A + C = 1
\]
\[
32(0) + C = 1
\]
\[
C = 1
\]
\[
32B + D = 0
\]
\[
256A + 16C + E = 0
\]
\[
32(0) + D = 0
\]
\[
256(0) + 16(1) + E = 0
\]
\[
D = 0
\]
\[
E = -16
\]
\[
256B + 16D + F = 0
\]
\[
256(0) + 16(0) + F = 0
\]
\[
F = 0
\]
\[
\frac{x^3}{(x^2 + 16)^3} = \frac{x}{(x^2 + 16)^2} + \frac{-16x}{(x^2 + 16)^3}
\]

**42.** Find the partial fraction decomposition:
\[
\frac{x^2}{(x^2 + 4)^3} = \frac{Ax + B}{x^2 + 4} + \frac{Cx + D}{(x^2 + 4)^2} + \frac{Ex + F}{(x^2 + 4)^3}
\]
Multiplying both sides by $(x^2 + 4)^3$, we obtain:
\[
x^2 = (Ax + B)(x^2 + 4)^2 + (Cx + D)(x^2 + 4)
\]
\[
+ Ex + F
\]
\[
x^2 = (Ax + B)(x^4 + 8x^2 + 16) + Cx^3 + Dx^2 + 4Cx + 4D + Ex + F
\]
\[
x^2 = Ax^5 + Bx^4 + 8Ax^3 + 8Bx^2 + 16Ax + 16B
\]
\[
+ Cx^3 + Dx^2 + 4Cx + 4D + Ex + F
\]
\[
x^2 = Ax^5 + Bx^4 + (8A + C)x^3 + (8B + D)x^2
\]
\[
+ (16A + 4C + E)x + (16B + 4D + F)
\]
\[
A = 0; \quad B = 0; \quad 8A + C = 0
\]
\[
8(0) + C = 0
\]
\[
C = 0
\]
\[
8B + D = 1
\]
\[
16A + 4C + E = 0
\]
\[
8(0) + D = 1
\]
\[
16(0) + 4(0) + E = 0
\]
\[
D = 1
\]
\[
E = 0
\]
\[
16B + 4D + F = 0
\]
\[
16(0) + 4(1) + F = 0
\]
\[
F = -4
\]
\[
\frac{x^2}{(x^2 + 4)^3} = \frac{1}{(x^2 + 4)^2} + \frac{-4}{(x^2 + 4)^3}
\]

**43.** Find the partial fraction decomposition:
\[
\frac{4}{2x^2 - 5x - 3} = \frac{A}{(x-3)(2x+1)} = \frac{A}{x-3} + \frac{B}{2x+1}
\]
Multiplying both sides by $(x - 3)(2x + 1)$, we obtain: 
\[
4 = A(2x + 1) + B(x - 3)
\]
Let $x = -\frac{1}{2}$, then
\[
4 = A\left(-1 + 1\right) + B\left(-\frac{1}{2} - 3\right)
\]
\[
4 = -\frac{7}{2}B
\]
\[
B = -\frac{8}{7}
\]
Let \( x = 3 \), then
\[
4 = A(2(3)+1) + B(3-3)
\]
\[
4 = 7A
\]
\[
A = \frac{4}{7}
\]
\[
\frac{4}{2x^2-5x-3} = \frac{4}{7} + \frac{-8}{3x+1}
\]

44. Find the partial fraction decomposition:
\[
\frac{4x}{2x^2+3x-2} = \frac{A}{x+2} + \frac{B}{2x-1}
\]
Multiplying both sides by \((x+2)(2x-1)\), we obtain:
\[
4x = A(2x-1) + B(x+2)
\]
Let \( x = \frac{1}{2} \), then
\[
4\left(\frac{1}{2}\right) = A\left(2\left(\frac{1}{2}\right)-1\right) + B\left(\frac{1}{2}+2\right)
\]
\[
2 = \frac{5}{3}B
\]
\[
B = \frac{4}{5}
\]
Let \( x = -2 \), then
\[
4(-2) = A(2(-2)-1) + B(-2+2)
\]
\[
-8 = -5A
\]
\[
A = \frac{8}{5}
\]
\[
\frac{4x}{2x^2+3x-2} = \frac{8}{5} + \frac{\frac{4}{3}}{2x-1}
\]

45. Find the partial fraction decomposition:
\[
\frac{2x+3}{x^4-9x^2} = \frac{2x+3}{x^2(x-3)(x+3)}
\]
\[
\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3} + \frac{D}{x+3}
\]
Multiplying both sides by \(x^2(x-3)(x+3)\), we obtain:
\[
2x+3 = Ax(x-3)(x+3) + B(x-3)(x+3) + Cx^2(x+3) + Dx^2(x-3)
\]
Let \( x = 0 \), then
\[
2 \cdot 0 + 3 = A \cdot 0(0-3)(0+3) + B(0-3)(0+3) + C \cdot 0^2(0+3) + D \cdot 0^2(0-3)
\]
\[
3 = -9B
\]
\[
B = -\frac{1}{3}
\]

Let \( x = 3 \), then
\[
2 \cdot 3 + 3 = A \cdot (3-3)(3+3) + B(3-3)(3+3) + C \cdot 3^2(3+3) + D \cdot 3^2(3-3)
\]
\[
9 = 54C
\]
\[
C = \frac{1}{6}
\]

Let \( x = -3 \), then
\[
2(-3) + 3 = A(-3)(-3-3)(-3+3) + B(-3-3)(-3+3) + C(-3)^2(-3+3) + D(-3)^2(-3-3)
\]
\[
-3 = -54D
\]
\[
D = \frac{1}{18}
\]

Let \( x = 1 \), then
\[
2\cdot1 + 3 = A \cdot 1(1-3)(1+3) + B(1-3)(1+3) + C \cdot 1^2(1+3) + D \cdot 1^2(1-3)
\]
\[
5 = -8A - 8B + 4C - 2D
\]
\[
5 = -8A - 8(-1/3) + 4(1/6) - 2(1/18)
\]
\[
5 = -8A + \frac{8}{3} + \frac{2}{3} - \frac{1}{9}
\]
\[
-8A = \frac{16}{9}
\]
\[
A = -\frac{2}{9}
\]
\[
\frac{2x+3}{x^4-9x^2} = \frac{-\frac{2}{9}}{x} + \frac{-\frac{1}{3}}{x^2} + \frac{\frac{1}{6}}{x-3} + \frac{\frac{1}{18}}{x+3}
\]

46. Find the partial fraction decomposition:
\[
\frac{x^2+9}{x^4-2x^2-8} = \frac{x^2+9}{(x^2+2)(x-2)(x+2)}
\]
\[
= \frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+2}
\]
Multiplying both sides by \((x^2+2)(x-2)(x+2)\), we obtain:
\[
x^2+9 = A(x^2+2)(x+2) + B(x-2)(x^2+2) + (Cx+D)(x-2)(x+2)
\]

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Let $x = 2$, then
\[ 2^2 + 9 = A(2^2 + 2)(2+2) + B(2-2)(2^2 + 2) + (C+D)(2-2)(2+2) \]
\[ 13 = 24A \]
\[ A = \frac{13}{24} \]
Let $x = -2$, then
\[ (-2)^2 + 9 = A((-2)^2 + 2)(-2+2) + B(-2-2)((-2)^2 + 2) + (C(-2)+D)(-2-2)(-2+2) \]
\[ 13 = -24B \]
\[ B = -\frac{13}{24} \]
Let $x = 0$, then
\[ 0^2 + 9 = A(0^2 + 2)(0+2) + B(0-2)(0^2 + 2) + (C(0)+D)(0-2)(0+2) \]
\[ 9 = 4A - 4B - 4D \]
\[ 9 = 4\left(\frac{13}{24}\right) - 4\left(-\frac{13}{24}\right) - 4D \]
\[ 4D = -\frac{14}{3} \]
\[ D = -\frac{7}{6} \]
Let $x = 1$, then
\[ 1^2 + 9 = A(1^2 + 2)(1+2) + B(1-2)(1^2 + 2) + (C(1)+D)(1-2)(1+2) \]
\[ 10 = 9A - 3B - 3C - 3D \]
\[ 10 = \frac{39}{8} + \frac{13}{8} - 3C + \frac{7}{2} \]
\[ 3C = 0 \]
\[ C = 0 \]
\[ \frac{x^2 + 9}{x^3 - 2x^2 - 8} = \frac{\frac{13}{24}}{x-2} + \frac{-\frac{13}{24}}{x+2} + \frac{-\frac{7}{6}}{x^2 + 2} \]

### Section 11.6

**1.** $y = 3x + 2$
The graph is a line.
$x$-intercept:  
\[ 0 = 3x + 2 \]
\[ 3x = -2 \]
\[ x = -\frac{2}{3} \]
y-intercept: $y = 3(0) + 2 = 2$

![Graph of y = 3x + 2](image)

**2.** $y = x^2 - 4$
The graph is a parabola.
$x$-intercepts:
\[ 0 = x^2 - 4 \]
\[ x^2 = 4 \]
\[ x = -2, x = 2 \]
y-intercept: $y = 0^2 - 4 = -4$
The vertex has $x$-coordinate:
\[ x = -\frac{b}{2a} = -\frac{0}{2(1)} = 0 \]
The $y$-coordinate of the vertex is $y = 0^2 - 4 = -4$.

![Graph of y = x^2 - 4](image)
3. \( y^2 = x^2 - 1 \)
\( x^2 - y^2 = 1 \)
\( \frac{x^2 - y^2}{1^2} = 1 \)

The graph is a hyperbola with center (0, 0), transverse axis along the x-axis, and vertices at \((-1, 0)\) and \((1, 0)\). The asymptotes are \( y = -x \) and \( y = x \).

![Graph of a hyperbola with center (0, 0) and vertices at (-1, 0) and (1, 0).](image)

4. \( x^2 + 4y^2 = 4 \)
\( \frac{x^2}{4} + \frac{y^2}{1} = 1 \)

The graph is an ellipse with center \((0, 0)\), major axis along the x-axis, vertices at \((-2, 0)\) and \((2, 0)\). The graph also has y-intercepts at \((0, -1)\) and \((0, 1)\).

![Graph of an ellipse with center (0, 0) and vertices at (-2, 0) and (2, 0).](image)

5. \( \begin{cases} y = x^2 + 1 \\ y = x + 1 \end{cases} \)

Solve by substitution:
\( x^2 + 1 = x + 1 \)
\( x^2 - x = 0 \)
\( x(x - 1) = 0 \)
\( x = 0 \) or \( x = 1 \)
\( y = 1 \) or \( y = 2 \)

Solutions: \((0, 1)\) and \((1, 2)\)

![Graph of two functions.](image)

6. \( \begin{cases} y = x^2 + 1 \\ y = 4x + 1 \end{cases} \)

Solve by substitution:
\( x^2 + 1 = 4x + 1 \)
\( x^2 - 4x = 0 \)
\( x(x - 4) = 0 \)
\( x = 0 \) or \( x = 4 \)
\( y = 1 \) or \( y = 17 \)

Solutions: \((0, 1)\) and \((4, 17)\)

![Graph of two functions.](image)
Section 11.6: Systems of Nonlinear Equations

7. \[ \begin{align*}
    y &= \sqrt{36-x^2} \\
    y &= 8-x 
\end{align*} \]

(2.59, 5.41) and (5.41, 2.59) are the intersection points.

Solve by substitution:
\[ \begin{align*}
    \sqrt{36-x^2} &= 8-x \\
    36-x^2 &= 64-16x+x^2 \\
    2x^2-16x+28 &= 0 \\
    x^2-8x+14 &= 0 \\
    x &= \frac{8 \pm \sqrt{64-56}}{2} \\
    &= \frac{8 \pm 2\sqrt{2}}{2} \\
    &= 4 \pm \sqrt{2} \\
\end{align*} \]

If \( x = 4+\sqrt{2} \), \( y = 8-(4+\sqrt{2}) = 4-\sqrt{2} \)

If \( x = 4-\sqrt{2} \), \( y = 8-(4-\sqrt{2}) = 4+\sqrt{2} \)

Solutions: \( (4+\sqrt{2}, 4-\sqrt{2}) \) and \( (4-\sqrt{2}, 4+\sqrt{2}) \)

8. \[ \begin{align*}
    y &= \sqrt{4-x^2} \\
    y &= 2x+4 
\end{align*} \]

\((-2, 0)\) and \((-1.2, 1.6)\) are the intersection points.

Solve by substitution:
\[ \begin{align*}
    \sqrt{4-x^2} &= 2x+4 \\
    4-x^2 &= 4x^2+16x+16 \\
    5x^2+16x+12 &= 0 \\
    (x+2)(5x+6) &= 0 \\
    x &= -2 \quad \text{or} \quad x = -\frac{6}{5} \\
    y &= 0 \quad \text{or} \quad y = \frac{8}{5} \\
\end{align*} \]

Solutions: \((-2, 0)\) and \((-\frac{6}{5}, \frac{8}{5})\)

9. \[ \begin{align*}
    y &= \sqrt{x} \\
    y &= 2-x 
\end{align*} \]

(1, 1) is the intersection point.

Solve by substitution:
\[ \begin{align*}
    \sqrt{x} &= 2-x \\
    x &= 4-4x+x^2 \\
    x^2-5x+4 &= 0 \\
    (x-4)(x-1) &= 0 \\
    x &= 4 \quad \text{or} \quad x = 1 \\
    y &= -2 \quad \text{or} \quad y = 1 \\
\end{align*} \]

Eliminate (4, -2); we must have \( y \geq 0 \).

Solution: (1, 1)

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Chapter 11: Systems of Equations and Inequalities

10. \[
\begin{align*}
  y &= \sqrt{x} \\
  y &= 6 - x
\end{align*}
\]

(4, 2) is the intersection point.

Solve by substitution:
\[ \sqrt{x} = 6 - x \]
\[ x = 36 - 12x + x^2 \]
\[ x^2 - 13x + 36 = 0 \]
\[ (x - 4)(x - 9) = 0 \]
\[ x = 4 \text{ or } x = 9 \]
\[ y = 2 \text{ or } y = -3 \]
Eliminate (9, -3); we must have \( y \geq 0 \).
Solution: (4, 2)

11. \[
\begin{align*}
  x &= 2y \\
  x &= y^2 - 2y
\end{align*}
\]

(0, 0) and (8, 4) are the intersection points.

Solve by substitution:
\[ 2y = y^2 - 2y \]
\[ y^2 - 4y = 0 \]
\[ y(y - 4) = 0 \]
\[ y = 0 \text{ or } y = 4 \]
\[ x = 0 \text{ or } x = 8 \]
Solutions: (0, 0) and (8, 4)

12. \[
\begin{align*}
  y &= x - 1 \\
  y &= x^2 - 6x + 9
\end{align*}
\]

(2, 1) and (5, 4) are the intersection points.

Solve by substitution:
\[ x^2 - 6x + 9 = x - 1 \]
\[ x^2 - 7x + 10 = 0 \]
\[ (x - 2)(x - 5) = 0 \]
\[ x = 2 \text{ or } x = 5 \]
\[ y = 1 \text{ or } y = 4 \]
Solutions: (2, 1) and (5, 4)

13. \[
\begin{align*}
  x^2 + y^2 &= 4 \\
  x^2 + 2x + y^2 &= 0
\end{align*}
\]

(–2, 0) is the intersection point.

Substitute 4 for \( x^2 + y^2 \) in the second equation.
\[ 2x + 4 = 0 \]
\[ 2x = -4 \]
\[ x = -2 \]
\[ y = \sqrt{4 - (-2)^2} = 0 \]
Solution: (–2, 0)
Section 11.6: Systems of Nonlinear Equations

14. \[ \begin{align*} x^2 + y^2 &= 8 \\
 x^2 + y^2 + 4y &= 0 \end{align*} \]

\((-2, -2)\) and \((2, -2)\) are the intersection points.

Substitute 8 for \(x^2 + y^2\) in the second equation.

\[ 8 + 4y = 0 \]

\[ 4y = -8 \]

\[ y = -2 \]

\[ x = \pm \sqrt{8 - (-2)^2} = \pm 2 \]

Solution: \((-2, -2)\) and \((2, -2)\)

15. \[ \begin{align*} y &= 3x - 5 \\
 x^2 + y^2 &= 5 \end{align*} \]

\((1, -2)\) and \((2, 1)\) are the intersection points.

Solve by substitution:

\[ x^2 + (3x - 5)^2 = 5 \]

\[ x^2 + 9x^2 - 30x + 25 = 5 \]

\[ 10x^2 - 30x + 20 = 0 \]

\[ x^2 - 3x + 2 = 0 \]

\(x - 1)(x - 2) = 0\)

\[ x = 1 \quad \text{or} \quad x = 2 \]

\[ y = -2 \quad \text{or} \quad y = 1 \]

Solutions: \((1, -2)\) and \((2, 1)\)

16. \[ \begin{align*} x^2 + y^2 &= 10 \\
 y &= x + 2 \end{align*} \]

\((1, 3)\) and \((-3, -1)\) are the intersection points.

Solve by substitution:

\[ x^2 + (x + 2)^2 = 10 \]

\[ x^2 + x^2 + 4x + 4 = 10 \]

\[ 2x^2 + 4x - 6 = 0 \]

\[ 2(x + 3)(x - 1) = 0 \]

\[ x = -3 \quad \text{or} \quad x = 1 \]

\[ y = -1 \quad \text{or} \quad y = 3 \]

Solutions: \((-3, -1)\) and \((1, 3)\)

17. \[ \begin{align*} x^2 + y^2 &= 4 \\
 y^2 - x &= 4 \end{align*} \]

\((-1, 1.73), (-1, -1.73), (0, 2), \) and \((0, -2)\) are the intersection points.

Substitute \(x + 4\) for \(y^2\) in the first equation:

\[ x^2 + x + 4 = 4 \]

\[ x^2 + x = 0 \]

\[ x(x + 1) = 0 \]

\[ x = 0 \quad \text{or} \quad x = -1 \]

\[ y^2 = 4 \quad \text{or} \quad y^2 = 3 \]

\[ y = \pm 2 \quad \text{or} \quad y = \pm \sqrt{3} \]

Solutions: \((-1, -2), (0, 2), (-1, \sqrt{3}), (-1, -\sqrt{3})\)
18. \[
\begin{align*}
x^2 + y^2 &= 16 \\
x^2 - 2y &= 8
\end{align*}
\]

\((-3.46, 2), (0, -4), \) and \((3.46, 2)\) are the intersection points.

Substitute \(2y + 8\) for \(x^2\) in the first equation.

\[
\begin{align*}
2y + 8 + y^2 &= 16 \\
y^2 + 2y - 8 &= 0
\end{align*}
\]

\((y + 4)(y - 2) = 0\)

\(y = -4\) or \(y = 2\)

\(x^2 = 0\) or \(x^2 = 12\)

\(x = 0\) or \(x = \pm 2\sqrt{3}\)

Solutions: \((0, -4), (2\sqrt{3}, 2), (-2\sqrt{3}, 2)\)

19. \[
\begin{align*}
xy &= 4 \\
x^2 + y^2 &= 8
\end{align*}
\]

\((-2, -2)\) and \((2, 2)\) are the intersection points.

Solve by substitution:

\[
\begin{align*}
x^2 + \left(\frac{4}{x}\right)^2 &= 8 \\
x^2 + 16 &= 8 \\
x^2 &= 8x^2 \\
x^4 + 16 &= 8x^2 \\
x^4 - 8x^2 + 16 &= 0 \\
(x^2 - 4)^2 &= 0 \\
x^2 &= 4 \\
x = 2\) or \(x = -2\)
\]

\(y = \frac{x}{4}\)

\(y = 2\) or \(y = -2\)

Solutions: \((-2, -2)\) and \((2, 2)\)

20. \[
\begin{align*}
x^2 &= y \\
xy &= 1
\end{align*}
\]

\((1, 1)\) is the intersection point.

Solve by substitution:

\[
\begin{align*}
x^2 &= \frac{1}{y} \\
x &= \frac{1}{x}
\end{align*}
\]

\(x^3 = 1\)

\(x = 1\)

\(y = (1)^2 = 1\)

Solution: \((1, 1)\)

21. \[
\begin{align*}
x^2 + y^2 &= 4 \\
y &= x^2 - 9
\end{align*}
\]

No solution; Inconsistent.

Solve by substitution:

\[
\begin{align*}
x^2 + (x^2 - 9)^2 &= 4 \\
x^2 + x^4 - 18x^2 + 81 &= 4 \\
x^4 - 17x^2 + 77 &= 0
\end{align*}
\]

\[
x^2 = \frac{17 \pm \sqrt{389 - 4(77)}}{2} = \frac{17 \pm \sqrt{-19}}{2}
\]

There are no real solutions to this expression.

Inconsistent.
22. \[
\begin{align*}
xy &= 1 \\
y &= 2x + 1
\end{align*}
\]

\((-1, -1)\) and \((0.5, 2)\) are the intersection points.

Solve by substitution:

\[
\begin{align*}
x(2x + 1) &= 1 \\
2x^2 + x - 1 &= 0
\end{align*}
\]

\((x + 1)(2x - 1) = 0\)

\[
x = -1 \quad \text{or} \quad x = \frac{1}{2}
\]

\[
y = -1 \quad \text{or} \quad y = 2
\]

Solutions: \((-1, -1)\) and \(\left(\frac{1}{2}, 2\right)\)

23. \[
\begin{align*}
y &= x^2 - 4 \\
y &= 6x - 13
\end{align*}
\]

\((3, 5)\) is the intersection point.

Solve by substitution:

\[
x^2 - 4 = 6x - 13
\]

\[
x^2 - 6x + 9 = 0
\]

\[
(x - 3)^2 = 0
\]

\[
x = 3
\]

\[
y = (3)^2 - 4 = 5
\]

Solution: \((3, 5)\)

24. \[
\begin{align*}
x^2 + y^2 &= 10 \\
xy &= 3
\end{align*}
\]

\((1, 3), (3, 1), (-3, -1), \text{ and } (-1, -3)\) are the intersection points.

Solve by substitution:

\[
x^2 + \left(\frac{3}{2}\right)^2 = 10
\]

\[
x^2 + \frac{9}{4} = 10
\]

\[
x^4 + 9 = 10x^2
\]

\[
x^4 - 10x^2 + 9 = 0
\]

\[
(x^2 - 9)(x^2 - 1) = 0
\]

\[
x = 3 \quad \text{or} \quad x = -3 \quad \text{or} \quad x = 1 \quad \text{or} \quad x = -1
\]

\[
y = 1 \\
y = -1 \\
y = 3 \\
y = -3
\]

Solutions: \((1, 3), (-3, -1), (3, 1), (-1, -3)\)

25. Solve the second equation for \(y\), substitute into the first equation and solve:

\[
\begin{align*}
2x^2 + y^2 &= 18 \\
xy &= 4 \\
\Rightarrow \quad y &= \frac{4}{x}
\end{align*}
\]

\[
2x^2 + \left(\frac{4}{x}\right)^2 = 18
\]

\[
2x^2 + \frac{16}{x^2} = 18
\]

\[
2x^4 + 16 = 18x^2
\]

\[
2x^4 - 18x^2 + 16 = 0
\]

\[
x^4 - 9x^2 + 8 = 0
\]

\[
(x^2 - 8)(x^2 - 1) = 0
\]

\[
x^2 = 8 \quad \text{or} \quad x^2 = 1
\]

\[
x = \pm \sqrt{8} = \pm 2\sqrt{2} \quad \text{or} \quad x = \pm 1
\]
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If \( x = 2\sqrt{2} \):
\[
y = \frac{4}{2\sqrt{2}} = \sqrt{2}
\]

If \( x = -2\sqrt{2} \):
\[
y = \frac{4}{-2\sqrt{2}} = -\sqrt{2}
\]

If \( x = 1 \):
\[
y = \frac{4}{1} = 4
\]

If \( x = -1 \):
\[
y = \frac{4}{-1} = -4
\]
Solutions:
\( (2\sqrt{2}, \sqrt{2}), (-2\sqrt{2}, -\sqrt{2}), (1, 4), (-1, -4) \)

26. Solve the second equation for \( y \), substitute into the first equation and solve:
\[
\begin{align*}
x^2 - y^2 &= 21 \\
x + y &= 7 \quad \Rightarrow \quad y = 7 - x
\end{align*}
\]
\[
\begin{align*}
x^2 - (7 - x)^2 &= 21 \\
x^2 - (49 - 14x + x^2) &= 21 \\
14x &= 70 \\
x &= 5 \\
y &= 7 - 5 = 2
\end{align*}
\]
Solution: \( (5, 2) \)

27. Substitute the first equation into the second equation and solve:
\[
\begin{align*}
y &= 2x + 1 \\
2x^2 + y^2 &= 1
\end{align*}
\]
\[
\begin{align*}
2x^2 + (2x + 1)^2 &= 1 \\
2x^2 + 4x^2 + 4x + 1 &= 1 \\
6x^2 + 4x &= 0 \\
2x(3x + 2) &= 0 \\
x &= 0 \quad \text{or} \quad 3x + 2 = 0
\end{align*}
\]
\[
\begin{align*}
x &= 0 \quad \text{or} \quad x = -\frac{2}{3}
\end{align*}
\]
If \( x = 0 \):
\[
y = 2(0) + 1 = 1
\]
If \( x = -\frac{2}{3} \):
\[
y = 2\left(-\frac{2}{3}\right) + 1 = -\frac{4}{3} + 1 = -\frac{1}{3}
\]
Solutions: \( (0, 1), \left(-\frac{2}{3}, \frac{1}{3}\right) \)

28. Solve the second equation for \( x \) and substitute into the first equation and solve:
\[
\begin{align*}
x^2 - 4y^2 &= 16 \\
2y - x &= 2 \quad \Rightarrow \quad x = 2y - 2
\end{align*}
\]
\[
\begin{align*}
(2y - 2)^2 - 4y^2 &= 16 \\
4y^2 - 8y + 4 - 4y^2 &= 16 \\
-8y &= 12 \\
y &= -\frac{3}{2} \\
x &= 2\left(-\frac{3}{2}\right) - 2 = -5
\end{align*}
\]
Solutions: \( (-5, -\frac{3}{2}) \)

29. Solve the first equation for \( y \), substitute into the second equation and solve:
\[
\begin{align*}
x + y + 1 &= 0 \quad \Rightarrow \quad y = -x - 1 \\
x^2 + y^2 + 6y - x &= -5
\end{align*}
\]
\[
\begin{align*}
x^2 + (-x - 1)^2 + 6(-x - 1) - x &= -5 \\
x^2 + x^2 + 2x + 1 - 6x - 6 - x &= -5 \\
2x^2 - 5x &= 0 \\
x(2x - 5) &= 0 \\
x &= 0 \quad \text{or} \quad x = \frac{5}{2}
\end{align*}
\]
If \( x = 0 \):
\[
y = -(0) - 1 = -1
\]
If \( x = \frac{5}{2} \):
\[
y = \frac{5}{2} - 1 = -\frac{7}{2}
\]
Solutions: \( (0, -1), \left(\frac{5}{2}, -\frac{7}{2}\right) \)

30. Solve the second equation for \( y \), substitute into the first equation and solve:
\[
\begin{align*}
2x^2 - xy + y^2 &= 8 \\
xy &= 4 \quad \Rightarrow \quad y = \frac{4}{x}
\end{align*}
\]
\[
\begin{align*}
2x^2 - x\left(\frac{4}{x}\right) + \left(\frac{4}{x}\right)^2 &= 8 \\
2x^2 - 4 + \frac{16}{x^2} &= 8 \\
2x^4 + 16 &= 12x^2 \\
(x^2 - 4)(x^2 - 2) &= 0 \\
(x - 2)(x + 2)(x - \sqrt{2})(x + \sqrt{2}) &= 0
\end{align*}
\]
\[
\begin{align*}
x &= 2 \quad \text{or} \quad x = -2 \quad \text{or} \quad x = \sqrt{2} \quad \text{or} \quad x = -\sqrt{2} \\
y &= 2 \\
y &= -2 \\
y &= 2\sqrt{2} \\
y &= -2\sqrt{2}
\end{align*}
\]
Solutions: \( (-2, -2), \left(-\sqrt{2}, -2\sqrt{2}\right), \left(\sqrt{2}, 2\sqrt{2}\right), (2, 2) \)

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Section 11.6: Systems of Nonlinear Equations

31. Solve the second equation for \( y \), substitute into the first equation and solve:
   \[
   \begin{align*}
   4x^2 - 3xy + 9y^2 &= 15 \\
   2x + 3y &= 5 \implies y = -\frac{2}{3}x + \frac{5}{3}
   
   4x^2 - 3x \left( -\frac{2}{3}x + \frac{5}{3} \right) + 9 \left( -\frac{2}{3}x + \frac{5}{3} \right)^2 &= 15 \\
   4x^2 + 2x^2 - 5x + 4x^2 - 20x + 25 &= 15 \\
   10x^2 - 25x + 10 &= 0 \\
   2x^2 - 5x + 2 &= 0 \\
   (2x-1)(x-2) &= 0
   
   x &= \frac{1}{2} \text{ or } x = 2
   
   \text{If } x = \frac{1}{2}: \quad y = -\frac{2}{3} \left( \frac{1}{2} \right) + \frac{5}{3} = \frac{4}{3} \\
   \text{If } x = 2: \quad y = -\frac{2}{3} \left( 2 \right) + \frac{5}{3} = \frac{1}{3}
   
   \text{Solutions: } \left( \frac{1}{2}, \frac{4}{3} \right), \left( 2, \frac{1}{3} \right)
   
   \end{align*}
   \]

32. \[
   \begin{align*}
   2y^2 - 3xy + 6y + 2x + 4 &= 0 \\
   2x - 3y + 4 &= 0
   
   \text{Solve the second equation for } x, \text{ substitute into the first equation and solve:}
   
   2x - 3y + 4 &= 0 \\
   2x &= 3y - 4 \\
   x &= \frac{3y - 4}{2}
   
   2y^2 - 3 \left( \frac{3y - 4}{2} \right) y + 6y + 2 \left( \frac{3y - 4}{2} \right) &= -4 \\
   2y^2 - \frac{9}{2}y^2 + 6y + 6y + 3y - 4 &= -4 \\
   \frac{-5}{2}y^2 + 15y &= 0 \\
   -5y^2 + 30y &= 0 \\
   -5y(y - 6) &= 0 \\
   y &= 0 \text{ or } y = 6
   
   \text{If } y = 0: \quad x = \frac{3(0) - 4}{2} = -2 \\
   \text{If } y = 6: \quad x = \frac{3(6) - 4}{2} = 7
   
   \text{Solutions: } (-2, 0), (7, 6)
   
   \end{align*}
   \]

33. Multiply each side of the second equation by 4 and add the equations to eliminate \( y \):
   \[
   \begin{align*}
   x^2 - 4y^2 &= -7 \quad \longrightarrow \quad x^2 - 4y^2 = -7 \\
   3x^2 + y^2 &= 31 \quad \longrightarrow \quad 12x^2 + 4y^2 = 124 \\
   13x^2 &= 117 \\
   x^2 &= 9 \\
   x &= \pm 3
   
   \text{If } x = 3: \quad (3)^2 + y^2 = 31 \implies y^2 = 4 \implies y = \pm 2
   
   \text{If } x = -3: \quad (-3)^2 + y^2 = 31 \implies y^2 = 4 \implies y = \pm 2
   
   \text{Solutions: } (3, 2), (3, -2), (-3, 2), (-3, -2)
   
   \end{align*}
   \]

34. \[
   \begin{align*}
   3x^2 - 2y^2 + 5 &= 0 \\
   2x^2 - y^2 + 2 &= 0 \\
   3x^2 - 2y^2 &= -5 \\
   2x^2 - y^2 &= -2
   
   \text{Multiply each side of the second equation by } -2 \text{ and add the equations to eliminate } y:
   
   3x^2 - 2y^2 &= -5 \\
   -4x^2 + 2y^2 &= 4 \\
   -x^2 &= 1 \\
   x &= \pm 1
   
   \text{If } x = 1: \quad 2(1)^2 - y^2 = -2 \implies y^2 = 4 \implies y = \pm 2
   
   \text{If } x = -1: \quad 2(-1)^2 - y^2 = -2 \implies y^2 = 4 \implies y = \pm 2
   
   \text{Solutions: } (1, 2), (1, -2), (-1, 2), (-1, -2)
   
   \end{align*}
   \]

35. \[
   \begin{align*}
   7x^2 - 3y^2 + 5 &= 0 \\
   3x^2 + 5y^2 &= 12 \\
   7x^2 - 3y^2 &= -5 \\
   3x^2 + 5y^2 &= 12
   
   \text{Multiply each side of the first equation by 5 and each side of the second equation by 3 and add the equations to eliminate } y:
   
   35x^2 - 15y^2 &= -25 \\
   9x^2 + 15y^2 &= 36 \\
   44x^2 &= 11 \\
   x^2 &= \frac{1}{4} \\
   x &= \pm \frac{1}{2}
   
   \end{align*}
   \]
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If \( x = \frac{1}{2} \):

\[
3 \left( \frac{1}{2} \right)^2 + 5y^2 = 12 \Rightarrow y^2 = \frac{9}{4} \Rightarrow y = \pm \frac{3}{2}
\]

If \( x = -\frac{1}{2} \):

\[
3 \left( -\frac{1}{2} \right)^2 + 5y^2 = 12 \Rightarrow y^2 = \frac{9}{4} \Rightarrow y = \pm \frac{3}{2}
\]

Solutions:

\[
\left( \frac{1}{2}, \frac{3}{2} \right), \left( \frac{1}{2}, -\frac{3}{2} \right), \left( -\frac{1}{2}, \frac{3}{2} \right), \left( -\frac{1}{2}, -\frac{3}{2} \right)
\]

36. \[
\begin{align*}
x^2 - 3y^2 + 1 &= 0 \\
2x^2 - 7y^2 + 5 &= 0 \\
x^2 - 3y^2 &= -1 \\
2x^2 - 7y^2 &= -5
\end{align*}
\]

Multiply each side of the first equation by \(-2\) and add the equations to eliminate \( x \):

\[
-2x^2 + 6y^2 = 2 \\
2x^2 - 7y^2 = -5 \\
-3y^2 = -3 \\
y^2 = 3 \\
y = \pm \sqrt{3}
\]

If \( y = \sqrt{3} \):

\[
x^2 - 3(\sqrt{3})^2 = -1 \Rightarrow x^2 = 8 \Rightarrow x = \pm 2\sqrt{2}
\]

If \( y = -\sqrt{3} \):

\[
x^2 - 3(-\sqrt{3})^2 = -1 \Rightarrow x^2 = 8 \Rightarrow x = \pm 2\sqrt{2}
\]

Solutions:

\((2\sqrt{2}, \sqrt{3}), (2\sqrt{2}, -\sqrt{3}), (-2\sqrt{2}, \sqrt{3}), (-2\sqrt{2}, -\sqrt{3}))\)

37. Multiply each side of the second equation by 2 and add the equations to eliminate \( xy \):

\[
\begin{align*}
x^2 + 2xy &= 10 \\
3x^2 - xy &= 2 \\
&\rightarrow 6x^2 - 2xy = 4 \\
7x^2 &= 14 \\
x^2 &= 2 \\
x &= \pm \sqrt{2}
\end{align*}
\]

If \( x = \sqrt{2} \):

\[
3(\sqrt{2})^2 - \sqrt{2} \cdot y = 2 \\
\Rightarrow -\sqrt{2} \cdot y = -4 \Rightarrow y = \frac{4}{\sqrt{2}} \Rightarrow y = 2\sqrt{2}
\]

If \( x = -\sqrt{2} \):

\[
3(-\sqrt{2})^2 - (-\sqrt{2}) \cdot y = 2 \\
\Rightarrow \sqrt{2} \cdot y = -4 \Rightarrow y = \frac{-4}{\sqrt{2}} \Rightarrow y = -2\sqrt{2}
\]

Solutions:

\((\sqrt{2}, 2\sqrt{2}), (-\sqrt{2}, -2\sqrt{2})\)

38. \[
\begin{align*}
5xy + 13y^2 + 36 &= 0 \\
xy + 7y^2 &= 6 \\
5xy + 13y^2 &= -36 \\
xy + 7y^2 &= 6
\end{align*}
\]

Multiply each side of the second equation by \(-5\) and add the equations to eliminate \( xy \):

\[
\begin{align*}
5xy + 13y^2 &= -36 \\
-5xy - 35y^2 &= -30 \\
-22y^2 &= -66 \\
y^2 = 3 \\
y = \pm \sqrt{3}
\end{align*}
\]

If \( y = \sqrt{3} \):

\[
x(\sqrt{3}) + 7(\sqrt{3})^2 = 6 \\
\Rightarrow \sqrt{3} \cdot x = -15 \Rightarrow x = -\frac{15}{\sqrt{3}} \Rightarrow x = -5\sqrt{3}
\]

If \( y = -\sqrt{3} \):

\[
x(-\sqrt{3}) + 7(-\sqrt{3})^2 = 6 \\
\Rightarrow -\sqrt{3} \cdot x = -15 \Rightarrow x = \frac{15}{\sqrt{3}} \Rightarrow x = 5\sqrt{3}
\]

Solutions:

\((-5\sqrt{3}, \sqrt{3}), (5\sqrt{3}, -\sqrt{3})\)

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39. \[
\begin{align*}
2x^2 + y^2 &= 2 \\
x^2 - 2y^2 + 8 &= 0
\end{align*}
\]
Multiply each side of the first equation by 2 and add the equations to eliminate \(y\):
\[
\begin{align*}
4x^2 + 2y^2 &= 4 \\
x^2 - 2y^2 &= -8
\end{align*}
\]
No real solution. The system is inconsistent.

40. \[
\begin{align*}
y^2 - x^2 + 4 &= 0 \\
2x^2 + 3y^2 &= 6
\end{align*}
\]
Multiply each side of the first equation by 2 and add the equations to eliminate \(x\):
\[
\begin{align*}
-2x^2 + 2y^2 &= -8 \\
2x^2 + 3y^2 &= 6
\end{align*}
\]
No real solution. The system is inconsistent.

41. \[
\begin{align*}
x^2 + 2y^2 &= 16 \\
4x^2 - y^2 &= 24
\end{align*}
\]
Multiply each side of the second equation by 2 and add the equations to eliminate \(y\):
\[
\begin{align*}
x^2 + 2y^2 &= 16 \\
8x^2 - 2y^2 &= 48
\end{align*}
\]
If \(x = \frac{8}{3}\):
\[
\left(\frac{8}{3}\right)^2 + 2y^2 = 16 \Rightarrow 2y^2 = \frac{80}{9}
\]
\[
y^2 = \frac{40}{9} \Rightarrow y = \pm \frac{2\sqrt{10}}{3}
\]
Solutions:
\[
\left\{ \left( \frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{3} \right), \left( \frac{-\sqrt{2}}{2}, -\frac{\sqrt{6}}{3} \right), \left( -\frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{3} \right) \right\}
\]

42. \[
\begin{align*}
4x^2 + 3y^2 &= 4 \\
2x^2 - 6y^2 &= -3
\end{align*}
\]
Multiply each side of the first equation by 2 and add the equations to eliminate \(y\):
\[
\begin{align*}
8x^2 + 6y^2 &= 8 \\
2x^2 - 6y^2 &= -3
\end{align*}
\]
If \(x = \frac{\sqrt{2}}{2}\):
\[
4 \left( \frac{\sqrt{2}}{2} \right)^2 + 3y^2 = 4 \Rightarrow 3y^2 = 2
\]
\[
y^2 = \frac{2}{3} \Rightarrow y = \pm \frac{\sqrt{6}}{3}
\]
Solutions:
\[
\left\{ \left( \frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{3} \right), \left( \frac{-\sqrt{2}}{2}, -\frac{\sqrt{6}}{3} \right), \left( -\frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{3} \right) \right\}
\]
43. \[
\begin{align*}
\begin{cases}
5x - 2y & = 3 \\
3x + 1y & = 7
\end{cases}
\quad \text{and}
\begin{cases}
5x - 2y & = -3 \\
3x + 1y & = 7
\end{cases}
\end{align*}
\]
Multiply each side of the second equation by 2 and add the equations to eliminate \(y\):
\[
\begin{align*}
\frac{5}{x^2} - \frac{2}{y^2} & = -3 \\
\frac{6}{x^2} + \frac{2}{y^2} & = 14
\end{align*}
\]
\[
\begin{align*}
\frac{11}{x^2} & = 11 \\
x^2 & = 1 \\
x & = \pm 1
\end{align*}
\]
If \(x = 1\):
\[
\frac{3}{1^2} + \frac{1}{y^2} = 7 \implies \frac{1}{y^2} = 4 \implies y^2 = \frac{1}{4}
\]
\[
\Rightarrow y = \pm \frac{1}{2}
\]
If \(x = -1\):
\[
\frac{3}{(-1)^2} + \frac{1}{y^2} = 7 \implies \frac{1}{y^2} = 4 \implies y^2 = \frac{1}{4}
\]
\[
\Rightarrow y = \pm \frac{1}{2}
\]
Solutions: \((1, \frac{1}{2}), (-1, -\frac{1}{2}), (-1, \frac{1}{2}), (1, -\frac{1}{2})\)

44. \[
\begin{align*}
\begin{cases}
\frac{2}{x^2} - \frac{3}{y^2} + 1 & = 0 \\
\frac{6}{x^2} + \frac{7}{y^2} + 2 & = 0
\end{cases}
\quad \text{and}
\begin{cases}
\frac{2}{x^2} - \frac{3}{y^2} & = -1 \\
\frac{6}{x^2} - \frac{7}{y^2} & = -2
\end{cases}
\end{align*}
\]
Multiply each side of the first equation by \(-3\) and add the equations to eliminate \(x\):
\[
\begin{align*}
\frac{-6}{x^2} + \frac{9}{y^2} & = 3 \\
\frac{6}{x^2} - \frac{7}{y^2} & = -2
\end{align*}
\]
\[
\begin{align*}
\frac{2}{y^2} & = 1 \\
y^2 & = 2 \\
y & = \pm \sqrt{2}
\end{align*}
\]
If \(y = \sqrt{2} \implies \frac{2}{x^2} - \frac{3}{\sqrt{2}}^2 = -1 \implies \frac{2}{x^2} = \frac{1}{2}
\]
\[
\Rightarrow x^2 = 4 \Rightarrow x = \pm 2
\]
If \(y = -\sqrt{2} \implies \frac{2}{x^2} - \frac{3}{(-\sqrt{2})^2} = -1 \implies \frac{2}{x^2} = \frac{1}{2}
\]
\[
\Rightarrow x^2 = 4 \Rightarrow x = \pm 2
\]
Solutions: \((2, \sqrt{2}), (2, -\sqrt{2}), (-2, \sqrt{2}), (-2, -\sqrt{2})\)

45. \[
\begin{align*}
\begin{cases}
\frac{1}{x^4} + \frac{6}{y^4} & = 6 \\
\frac{2}{x^4} - \frac{2}{y^4} & = 19
\end{cases}
\end{align*}
\]
Multiply each side of the first equation by \(-2\) and add the equations to eliminate \(x\):
\[
\begin{align*}
\frac{-2}{x^4} - \frac{12}{y^4} & = -12 \\
\frac{2}{x^4} - \frac{2}{y^4} & = 19
\end{align*}
\]
\[
\begin{align*}
\frac{14}{y^4} & = 7 \\
y^4 & = -2
\end{align*}
\]
There are no real solutions. The system is inconsistent.
46. Add the equations to eliminate $y$:
\[
\frac{1}{x^2} - \frac{1}{y^2} = 1 \\
\frac{1}{x^2} + \frac{1}{y^2} = 4 \\
\frac{2}{x^2} = 5 \\
x^4 = \frac{2}{5} \\
x = \pm \sqrt[5]{\frac{2}{5}}
\]
If $x = \sqrt[5]{\frac{2}{5}}$:
\[
\frac{1}{(\sqrt[5]{\frac{2}{5}})^2} + \frac{1}{y^2} = 4 \Rightarrow \frac{1}{y^2} = \frac{3}{2}
\]
$y^2 = \frac{2}{3} \Rightarrow y = \pm \sqrt[3]{\frac{2}{3}}$

If $x = -\sqrt[5]{\frac{2}{5}}$:
\[
\frac{1}{(\sqrt[5]{\frac{2}{5}})^2} + \frac{1}{y^2} = 4 \Rightarrow \frac{1}{y^2} = \frac{3}{2}
\]
$y^2 = \frac{2}{3} \Rightarrow y = \pm \sqrt[3]{\frac{2}{3}}$

47. \[
\begin{align*}
x^2 - 3xy + 2y^2 &= 0 \\
x^2 + xy &= 6
\end{align*}
\]
Subtract the second equation from the first to eliminate the $x^2$ term.
-4xy + 2y^2 = -6
2xy - y^2 = 3
Since $y \neq 0$, we can solve for $x$ in this equation to get
\[
x = \frac{y^2 + 3}{2y}, \quad y \neq 0
\]
Now substitute for $x$ in the second equation and solve for $y$.

48. \[
\begin{align*}
x^2 - xy - 2y^2 &= 0 \\
xy + x + 6 &= 0
\end{align*}
\]
Factor the first equation, solve for $x$, substitute into the second equation and solve:
\[
x^2 - xy - 2y^2 = 0 \\
(x - 2y)(x + y) = 0 \\
x = 2y \quad \text{or} \quad x = -y
\]
Substitute $x = 2y$ and solve:
\[
xy + x + 6 = 0 \\
(2y)y + 2y = -6 \\
2y^2 + 2y + 6 = 0 \\
2(y^2 + y + 3) = 0
\]
\[
y = \frac{-1 \pm \sqrt{1^2 - 4(1)(3)}}{2(1)} \quad \text{No real solution}
\]
Substitute $x = -y$ and solve:
\[
xy + x + 6 = 0 \\
y \cdot y + (-y) = -6 \\
-y^2 + y + 6 = 0 \\
(-y - 3)(y - 2) = 0
\]
$y = -3$ or $y = 2$
If $y = -3$: $x = 3$
If $y = 2$: $x = -2$

Solutions: (3, -3), (-2, 2)
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49. \[
\begin{align*}
\begin{cases}
y^2 + y + x^2 - x - 2 &= 0 \\
y + 1 + \frac{x - 2}{y} &= 0
\end{cases}
\end{align*}
\]
Multiply each side of the second equation by \(-y\) and add the equations to eliminate \(y\):
\[
\begin{align*}
y^2 + y + x^2 - x - 2 &= 0 \\
-y^2 - y - x + 2 &= 0
\end{align*}
\]
\[
x^2 - 2x = 0 \\
x(x - 2) = 0 \\
x = 0 \text{ or } x = 2
\]
If \(x = 0\):
\[
y^2 + y + 0^2 - 0 - 2 = 0 \Rightarrow y^2 + y = 0
\]
\[
\Rightarrow y(y + 1) = 0 \Rightarrow y = 0 \text{ or } y = -1
\]
Note: \(y \neq 0\) because of division by zero.
Solutions: \((0, -2), (0, 1), (2, -1)\)

50. \[
\begin{align*}
\begin{cases}
x^3 - 3x^2 + y^2 + 3y - 4 &= 0 \\
x - 2 + \frac{y^2 - y}{x^2} &= 0
\end{cases}
\end{align*}
\]
Multiply each side of the second equation by \(-x^2\) and add the equations to eliminate \(x\):
\[
\begin{align*}
x^3 - 3x^2 + y^2 + 3y - 4 &= 0 \\
-x^3 + 2x^2 - y^2 + y &= 0
\end{align*}
\]
\[
4y - 4 = 0 \\
4y = 4 \\
y = 1
\]
If \(y = 1\):
\[
x^3 - 3x^2 + 1^2 + 3 \cdot 1 - 4 = 0 \Rightarrow x^3 - 2x^2 = 0
\]
\[
\Rightarrow x^2(x - 2) = 0 \Rightarrow x = 0 \text{ or } x = 2
\]
Note: \(x \neq 0\) because of division by zero.
Solution: \((2, 1)\)

51. Rewrite each equation in exponential form:
\[
\begin{align*}
\log y &= 3 \Rightarrow y = e^3 \\
\log (4y) &= 5 \Rightarrow 4y = e^5
\end{align*}
\]
Substitute the first equation into the second and solve:
\[
4x^3 = x^5 \\
x^5 - 4x^3 = 0 \\
x^3(x^2 - 4) = 0
\]
\[
x^3 = 0 \text{ or } x^2 = 4 \Rightarrow x = 0 \text{ or } x = \pm 2
\]
The base of a logarithm must be positive, thus \(x \neq 0\) and \(x \neq -2\).
If \(x = 2\):
\[
y = 2^3 = 8
\]
Solution: \((2, 8)\)

52. Rewrite each equation in exponential form:
\[
\begin{align*}
\log (2y) &= 3 \Rightarrow 2y = e^3 \\
\log (4y) &= 2 \Rightarrow 4y = e^2
\end{align*}
\]
Multiply the first equation by 2 then substitute the first equation into the second and solve:
\[
2x^3 = x^2 \\
2x^3 - x^2 = 0 \\
x^2(2x - 1) = 0
\]
\[
x^2 = 0 \text{ or } x = \frac{1}{2} \Rightarrow x = \frac{1}{2} \text{ or } x = 0
\]
The base of a logarithm must be positive, thus \(x \neq 0\).
If \(x = \frac{1}{2}\):
\[
4y = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \Rightarrow y = \frac{1}{16}
\]
Solution: \(\left(\frac{1}{2}, \frac{1}{16}\right)\)

53. Rewrite each equation in exponential form:
\[
\begin{align*}
\ln x &= 4 \ln y \Rightarrow x = e^{4\ln y} = e^{\ln y^4} = y^4 \\
\log_3 x &= 2 + 2\log_3 y \\
x &= 3^{2 + 2\log_3 y} = 3^2 \cdot 3^{2\log_3 y} = 3^2 \cdot 3^{\log_3 y^2} = 9y^2
\end{align*}
\]
So we have the system
\[
\begin{cases}
x = y^4 \\
x = 9y^2
\end{cases}
\]
Therefore we have:
\[
9y^2 = y^4 \Rightarrow 9y^2 - y^4 = 0 \Rightarrow y^2(9 - y^2) = 0
\]
\[
y^2(3 + y)(3 - y) = 0 \\
y = 0 \text{ or } y = -3 \text{ or } y = 3
\]
Since \(\ln y\) is undefined when \(y \leq 0\), the only
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Solution is \( y = 3 \).

If \( y = 3 \): \( x = y^4 \Rightarrow x = 3^4 = 81 \)

Solution: \((81, 3)\)

54. Rewrite each equation in exponential form:

\[
\ln x = 5 \ln y \Rightarrow x = e^{5 \ln y} = e^{\ln y^5} = y^5
\]

\[
\log_2 x = 3 + 2 \log_2 y
\]

\[
x = 2^{3 + 2 \log_2 y} = 2^3 \cdot 2^{2 \log_2 y} = 2^3 \cdot 2^{\log_2 y^2} = 8y^2
\]

So we have the system:

\[
\begin{align*}
x &= y^5 \\
x &= 8y^2
\end{align*}
\]

Therefore we have:

\[
8y^2 = y^5
\]

\[
8y^2 - y^5 = 0
\]

\[
y^5(8 - y^3) = 0
\]

\[
y = 0 \quad \text{or} \quad 8 - y^3 = 0 \Rightarrow y = 2
\]

Since \( \ln y \) is undefined when \( y \leq 0 \), the only solution is \( y = 2 \).

If \( y = 2 \): \( x = y^5 \Rightarrow x = 2^5 = 32 \)

Solution: \((32, 2)\)

55. \[
\begin{align*}
x^2 + x + y^2 - 3y + 2 &= 0 \\
x + 1 + \frac{y^2 - y}{x} &= 0
\end{align*}
\]

\[
\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{1}{2}
\]

\[
\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}
\]

\[
x^2 + x + y^2 - 3y + 2 = 0
\]

\[
\left(-\frac{3}{2}, \frac{3}{2}\right)
\]

56. \[
\begin{align*}
y^2 + y + x^2 - x - 2 &= 0 \\
y + 1 + \frac{x - 2}{y} &= 0
\end{align*}
\]

\[
\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{5}{2}
\]

\[
x = -\left(y + \frac{1}{2}\right)^2 + \frac{9}{4}
\]

\[
y + 1 + \frac{x - 2}{y} = 0
\]

\[
\left(0, 1\right)
\]

Since \( \ln y \) is undefined when \( y \leq 0 \), the only solution is \( y = 2 \).

If \( y = 2 \): \( x = y^5 \Rightarrow x = 2^5 = 32 \)

Solution: \((32, 2)\)

57. Graph: \( y_1 = x \wedge (2/3); \ y_2 = e \wedge (-x) \)

Use INTERSECT to solve:

\[
\left(3.1, -3.1\right)
\]

Solution: \( x = 0.48, y = 0.62 \) or \((0.48, 0.62)\)

58. Graph: \( y_1 = x \wedge (3/2); \ y_2 = e \wedge (-x) \)

Use INTERSECT to solve:

\[
\left(3.1, -3.1\right)
\]

Solution: \( x = 0.65, y = 0.52 \) or \((0.65, 0.52)\)

59. Graph: \( y_1 = \sqrt[3]{2 - x^2}; \ y_2 = 4/x^3 \)

Use INTERSECT to solve:
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60. Graph: \( y_1 = \sqrt{2-x^3} \); \( y_2 = -\sqrt{2-x^3} \);

\[ y_3 = \frac{4}{x^2} \]

Use INTERSECT to solve:

Solution: \( x = -1.65, y = -0.89 \) or \((-1.65, -0.89)\)

61. Graph: \( y_1 = \sqrt[4]{12-x^4} \); \( y_2 = -\sqrt[4]{12-x^4} \);

\[ y_3 = \frac{1}{x} \]; \( y_4 = -\frac{1}{x} \)

Use INTERSECT to solve:

Solution: \( x = -1.37, y = 2.14 \) or \((-1.37, 2.14)\)

62. Graph: \( y_1 = \sqrt[6]{6-x^4} \); \( y_2 = -\sqrt[6]{6-x^4} \);

\[ y_3 = \frac{1}{x} \]

Use INTERSECT to solve:

Solutions: \( x = 0.64, y = 1.55 \); \( x = 1.55, y = 0.64 \);
\( x = -0.64, y = -1.55 \); \( x = -1.55, y = -0.64 \) or
\( (0.64, 1.55), (1.55, 0.64), (-0.64, -1.55), (-1.55, -0.64) \)

63. Graph: \( y_1 = \frac{2}{x} \); \( y_2 = \ln x \)

Use INTERSECT to solve:

Solution: \( x = 2.35, y = 0.85 \) or \((2.35, 0.85)\)
64. Graph: \( y_1 = \sqrt{4-x^2}; \quad y_2 = -\sqrt{4-x^2}; \quad y_3 = \ln x \)

Use INTERSECT to solve:

Solution: \( x = 1.90, y = 0.64; \quad x = 0.14, y = -2.00 \)
or \((1.90, 0.64), (0.14, -2.00)\)

65. Solve the first equation for \( x \), substitute into the second equation and solve:

\[
\begin{align*}
(x-1)^2 + (y-1)^2 &= 5 \\
(-2y-1)^2 + (y-1)^2 &= 5
\end{align*}
\]

\( 4y^2 + 4y + 1 + y^2 - 2y + 1 = 5 \Rightarrow 5y^2 + 2y - 3 = 0 \)

\( (5y-3)(y+1) = 0 \)

\( y = \frac{3}{5} \quad \text{or} \quad y = -1 \)

\( x = -\frac{6}{5} = -1.2 \quad \text{or} \quad x = 2 \)

The points of intersection are \( \left(-\frac{6}{5}, \frac{3}{5}\right), (2, -1) \).

66. Solve the first equation for \( x \), substitute into the second equation and solve:

\[
\begin{align*}
x + 2y + 6 &= 0 \Rightarrow x = -2y - 6 \\
(x+1)^2 + (y+1)^2 &= 5
\end{align*}
\]

\( (-2y-6+1)^2 + (y+1)^2 = 5 \)

\( 4y^2 + 20y + 25 + y^2 + 2y + 1 = 5 \)

\( 5y^2 + 22y + 21 = 0 \)

\( (y+7)(y+3) = 0 \)

\( y = -\frac{7}{5} \quad \text{or} \quad y = -3 \)

\( x = -\frac{16}{5} \quad \text{or} \quad x = 0 \)

The points of intersection are \( \left(-\frac{16}{5}, -\frac{7}{5}\right), (0, -3) \).

67. Complete the square on the second equation.

\( y^2 + 4y + 4 = x - 1 + 4 \)

\( (y+2)^2 = x + 3 \)

Substitute this result into the first equation.

\( (x-1)^2 + x + 3 = 4 \)

\( x^2 - 2x + 1 + x + 3 = 4 \)

\( x^2 - x = 0 \)

\( x(x-1) = 0 \)

\( x = 0 \quad \text{or} \quad x = 1 \)

If \( x = 0 \): \( (y+2)^2 = 0 + 3 \)

\( y + 2 = \pm\sqrt{3} \Rightarrow y = -2 \pm \sqrt{3} \)

If \( x = 1 \): \( (y+2)^2 = 1 + 3 \)

\( y + 2 = \pm 2 \Rightarrow y = -2 \pm 2 \)

The points of intersection are:

\( (0, -2 - \sqrt{3}), (0, -2 + \sqrt{3}), (1, -4), (1, 0) \).
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68. Complete the square on the second equation, substitute into the first equation and solve:
\[
\begin{align*}
(x+2)^2 + (y-1)^2 &= 4 \\
y^2 - 2y - x - 5 &= 0
\end{align*}
\]
\[
y^2 - 2y + 1 = x + 5 + 1
\]
\[
(y-1)^2 = x + 6
\]
\[
(x+2)^2 + x + 6 = 4
\]
\[
x^2 + 4x + 4 + x + 6 = 4
\]
\[
x^2 + 6x + 6 = 0
\]
\[
(x+2)(x+3) = 0
\]
\[
x = -2 \quad \text{or} \quad x = -3
\]
If \( x = -2 \): \( (y-1)^2 = -2 + 6 \Rightarrow y-1 = \pm 2 \)
\[
\Rightarrow y = -1 \text{ or } y = 3
\]
If \( x = -3 \): \( (y-1)^2 = -3 + 6 \Rightarrow y-1 = \pm \sqrt{3} \)
\[
\Rightarrow y = 1 \pm \sqrt{3}
\]
The points of intersection are:
\[
(-3, 1 - \sqrt{3}), (-3, 1 + \sqrt{3}), (-2, -1), (-2, 3).
\]

69. Solve the first equation for \( x \), substitute into the second equation and solve:
\[
\begin{align*}
y &= \frac{4}{x-3} \\
x^2 - 6x + y^2 + 1 &= 0
\end{align*}
\]
\[
y = \frac{4}{x-3}
\]
\[
x - 3 = \frac{4}{y}
\]
\[
x = \frac{4}{y} + 3
\]
\[
\left(\frac{4}{y} + 3\right)^2 - 6\left(\frac{4}{y} + 3\right) + y^2 + 1 = 0
\]
\[
\frac{16}{y^2} + \frac{24}{y} + 9 - \frac{24}{y} - 18 + y^2 + 1 = 0
\]
\[
\frac{16}{y^2} + y^2 - 8 = 0
\]
\[
16 + y^4 - 8y^2 = 0
\]
\[
y^4 - 8y^2 + 16 = 0
\]
\[
(y^2 - 4)^2 = 0
\]
\[
y^2 - 4 = 0
\]
\[
y^2 = 4
\]
\[
y = \pm 2
\]
If \( y = 2 \): \( x = \frac{4}{2} + 3 = 5 \)
If \( y = -2 \): \( x = \frac{4}{-2} + 3 = 1 \)
The points of intersection are: \((1, -2), (5, 2)\).
70. Substitute the first equation into the second equation and solve:
\[
\begin{align*}
\frac{4}{x+2} &= 4 \\
\frac{4}{x+2} + 4x + y^2 - 4 &= 0 \\
x^2 + 4x + \left(\frac{4}{x+2}\right)^2 - 4 &= 0 \\
(x+2)^2\left(x^2+4x-4\right) &= -16 \\
(x+2)^2\left(x^2+4x-4\right) &= -16 \\
x^4 + 8x^3 + 16x^2 - 16 &= -16 \\
x^2(x^2+8x+16) &= 0 \\
x^2(x+4)^2 &= 0
\end{align*}
\]
\[
x = 0 \quad \text{or} \quad x = -4
\]
y = 2 \quad \text{or} \quad y = -2
\]
The points of intersection are: (0, 2), (–4, –2).

71. Let \( x \) and \( y \) be the two numbers. The system of equations is:
\[
\begin{align*}
x - y &= 2 \quad \Rightarrow x = y + 2 \\
x^2 + y^2 &= 10
\end{align*}
\]
Solve the first equation for \( x \), substitute into the second equation and solve:
\[
\begin{align*}
(y + 2)^2 + y^2 &= 10 \\
y^2 + 4y + 4 + y^2 &= 10 \\
y^2 + 2y - 3 &= 0 \\
y^2 + 3(y - 1) &= 0 \quad \Rightarrow \quad y = -3 \quad \text{or} \quad y = 1
\end{align*}
\]
If \( y = -3 \): \( x = -3 + 2 = -1 \)
If \( y = 1 \): \( x = 1 + 2 = 3 \)
The two numbers are 1 and 3 or –1 and –3.

72. Let \( x \) and \( y \) be the two numbers. The system of equations is:
\[
\begin{align*}
x + y &= 7 \quad \Rightarrow x = 7 - y \\
x^2 - y^2 &= 21
\end{align*}
\]
Solve the first equation for \( x \), substitute into the second equation and solve:
\[
\begin{align*}
(7 - y)^2 - y^2 &= 21 \\
49 - 14y + y^2 - y^2 &= 21 \\
-14y &= -28 \\
y &= 2 \quad \Rightarrow \quad x = 7 - 2 = 5
\end{align*}
\]
The two numbers are 2 and 5.

73. Let \( x \) and \( y \) be the two numbers. The system of equations is:
\[
\begin{align*}
xy &= 4 \quad \Rightarrow x = \frac{4}{y} \\
x^2 + y^2 &= 8
\end{align*}
\]
Solve the first equation for \( x \), substitute into the second equation and solve:
\[
\begin{align*}
\left(\frac{4}{y}\right)^2 + y^2 &= 8 \\
\frac{16}{y^2} + y^2 &= 8 \\
16 + y^4 &= 8y^2 \\
y^4 - 8y^2 + 16 &= 0 \\
(y^2 - 4)^2 &= 0 \\
y^2 &= 4 \\
y &= \pm 2
\end{align*}
\]
If \( y = 2 \): \( x = \frac{4}{2} = 2 \); If \( y = -2 \): \( x = \frac{4}{-2} = -2 \)
The two numbers are 2 and 2 or –2 and –2.

74. Let \( x \) and \( y \) be the two numbers. The system of equations is:
\[
\begin{align*}
xy &= 10 \quad \Rightarrow x = \frac{10}{y} \\
x^2 - y^2 &= 21
\end{align*}
\]
Solve the first equation for \( x \), substitute into the second equation and solve:
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The two numbers are 2 and 5 or –2 and –5.

76. Let x and y be the two numbers. The system of equations is:
\[
\begin{aligned}
&x + y = xy \\
&1 - \frac{1}{x} = 3
\end{aligned}
\]
Solve the first equation for x, substitute into the second equation and solve:
\[
x(1 - y) = y \quad \Rightarrow \quad x = \frac{y}{1 - y}
\]
\[
\begin{aligned}
&\frac{1}{1 - y} + \frac{1}{y} = 5 \\
&1 - \frac{1}{y} = 5 \\
&\frac{2 - y}{y} = 5 \\
&2 - y = 5y \\
&6y = 2
\end{aligned}
\]
\[
y = \frac{1}{3} \quad \Rightarrow \quad x = \frac{1}{1 - \frac{1}{3}} = \frac{1}{2}
\]
The two numbers are \(\frac{1}{2}\) and \(\frac{1}{3}\).

77. Let x and y be the two numbers. The system of equations is:
\[
\begin{aligned}
&xy - x = y \\
x(y - 1) = y \quad \Rightarrow \quad x = \frac{y}{y - 1}
\end{aligned}
\]
\[
\begin{aligned}
&\frac{1}{y} - \frac{1}{y - 1} = 3 \\
y - 1 - \frac{1}{y} = 3 \\
y - 2 = 3y \\
y = -1
\end{aligned}
\]
The two numbers are \(-1\) and \(\frac{1}{2}\).
79. Let \( x \) = the width of the rectangle.
    Let \( y \) = the length of the rectangle.
\[
\begin{align*}
2x + 2y &= 16 \\
x &= 15
\end{align*}
\]
Solve the first equation for \( y \), substitute into the second equation and solve.
\[
\begin{align*}
2x + 2y &= 16 \\
2y &= 16 - 2x \\
y &= 8 - x \\
x(8 - x) &= 15 \\
8x - x^2 &= 15 \\
x^2 - 8x + 15 &= 0 \\
(x - 5)(x - 3) &= 0 \\
x &= 5 \quad \text{or} \quad x = 3 \\
\end{align*}
\]
The dimensions of the rectangle are 3 inches by 5 inches.

80. Let \( 2x \) = the side of the first square.
    Let \( 3x \) = the side of the second square.
\[
\begin{align*}
(2x)^2 + (3x)^2 &= 52 \\
4x^2 + 9x^2 &= 52 \\
13x^2 &= 52 \\
x^2 &= 4 \\
x &= \pm 2
\end{align*}
\]
Note that we must have \( x > 0 \).
The sides of the first square are \((2)(2) = 4\) feet and the sides of the second square are \((3)(2) = 6\) feet.

81. Let \( x \) = the radius of the first circle.
    Let \( y \) = the radius of the second circle.
\[
\begin{align*}
2\pi x + 2\pi y &= 12\pi \\
\pi x^2 + \pi y^2 &= 20\pi
\end{align*}
\]
Solve the first equation for \( y \), substitute into the second equation and solve:
\[
\begin{align*}
2\pi x + 2\pi y &= 12\pi \\
x + y &= 6 \\
y &= 6 - x
\end{align*}
\]
\[
\begin{align*}
\pi x^2 + \pi y^2 &= 20\pi \\
x^2 + y^2 &= 20 \\
x^2 + (6 - x)^2 &= 20 \\
x^2 + 36 - 12x + x^2 &= 20 \\
2x^2 - 12x + 16 &= 0 \\
x &= 4 \quad \text{or} \quad x = 2 \\
y &= 2 \quad \text{or} \quad y = 4
\end{align*}
\]
The radii of the circles are 2 centimeters and 4 centimeters.

82. Let \( x \) = the length of each of the two equal sides in the isosceles triangle.
    Let \( y \) = the length of the base.
The perimeter of the triangle: \( x + x + y = 18 \)
Since the altitude to the base \( y \) is 3, the Pythagorean theorem produces another equation.
\[
\left(\frac{y}{2}\right)^2 + 3^2 = x^2 \Rightarrow \frac{y^2}{4} + 9 = x^2
\]
Solve the system of equations:
\[
\begin{align*}
2x + y &= 18 \\
\frac{y^2}{4} + 9 &= x^2
\end{align*}
\]
Solve the first equation for \( y \), substitute into the second equation and solve.
\[
\begin{align*}
\frac{(18 - 2x)^2}{4} + 9 &= x^2 \\
324 - 72x + 4x^2 + 9 &= x^2 \\
81 - 18x + x^2 + 9 &= x^2 \\
-18x &= -90 \\
x &= 5 \\
\Rightarrow y &= 18 - 2(5) = 8
\end{align*}
\]
The base of the triangle is 8 centimeters.

83. The tortoise takes \( 9 + 3 = 12 \) minutes or 0.2 hour longer to complete the race than the hare.
Let \( r \) = the rate of the hare.
Let \( t \) = the time for the hare to complete the race. Then \( t + 0.2 \) = the time for the tortoise and \( r - 0.5 \) = the rate for the tortoise. Since the length of the race is 21 meters, the distance equations are:
\[
\begin{align*}
rt &= 21 \\
(r - 0.5)(t + 0.2) &= 21
\end{align*}
\]
Solve the first equation for \( r \), substitute into the
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second equation and solve:
\[
\left(\frac{21}{t} - 0.5\right)(t + 0.2) = 21
\]
\[
21 + \frac{4.2}{t} - 0.5t - 0.1 = 21
\]
\[
10\left(21 + \frac{4.2}{t} - 0.5t - 0.1\right) = 10t \cdot (21)
\]
\[
210t + 42 - 5t^2 - t = 210t
\]
\[
5t^2 + t - 42 = 0
\]
\[
(5t - 14)(t + 3) = 0
\]
\[
5t - 14 = 0 \quad \text{or} \quad t + 3 = 0
\]
\[
t = \frac{14}{5} = 2.8
\]
\[
t = -3 \quad \text{makes no sense, since time cannot be negative.}
\]
Solve for \(r\):
\[
r = \frac{21}{2.8} = 7.5
\]
The average speed of the hare is 7.5 meters per hour, and the average speed for the tortoise is 7 meters per hour.

84. Let \(v_1, v_2, v_3 = \) the speeds of runners 1, 2, 3.
Let \(t_1, t_2, t_3 = \) the times of runners 1, 2, 3.
Then by the conditions of the problem, we have the following system:
\[
\begin{align*}
5280 &= v_1 t_1 \\
5270 &= v_2 t_1 \\
5260 &= v_3 t_1 \\
5280 &= v_2 t_2
\end{align*}
\]
Distance between the second runner and the third runner after \(t_2\) seconds is:
\[
5280 - v_3 t_2 = 5280 - v_3 t_1 \left(\frac{t_2}{t_1}\right)
\]
\[
= 5280 - 5260 \left(\frac{5280}{5270}\right)
\]
\[
= 10.02
\]
The second place runner beats the third place runner by about 10.02 feet.

85. Let \(x = \) the width of the cardboard. Let \(y = \) the length of the cardboard. The width of the box will be \(x - 4\), the length of the box will be \(y - 4\), and the height is 2. The volume is
\[
V = (x - 4)(y - 4)(2).
\]
Solve the system of equations:
\[
\begin{align*}
xy &= 216 \quad \Rightarrow \quad y = \frac{216}{x} \\
2(x - 4)(y - 4) &= 224
\end{align*}
\]
Solve the first equation for \(y\), substitute into the second equation and solve.
\[
(2x - 8)\left(\frac{216}{x} - 4\right) = 224
\]
\[
432 - 8x - \frac{1728}{x} + 32 = 224
\]
\[
432 - 8x^2 - 1728 + 32x = 224x
\]
\[
8x^2 - 240x + 1728 = 0
\]
\[
x^2 - 30x + 216 = 0
\]
\[
(x - 12)(x - 18) = 0
\]
\[
x - 12 = 0 \quad \text{or} \quad x - 18 = 0
\]
\[
x = 12 \quad x = 18
\]
The cardboard should be 12 centimeters by 18 centimeters.

86. Let \(x = \) the width of the cardboard. Let \(y = \) the length of the cardboard.
The area of the cardboard is: \(xy = 216\)
The volume of the tube is: \(V = \pi r h = 224\)
where \(h = y\) and \(2\pi r = x\) or \(r = \frac{x}{2\pi}\).
Solve the system of equations:
\[
\begin{align*}
xy &= 216 \quad \Rightarrow \quad y = \frac{216}{x} \\
\pi \left(\frac{x}{2\pi}\right)^2 y &= 224 \quad \Rightarrow \quad \frac{x^2 y}{4\pi} = 224
\end{align*}
\]
Solve the first equation for \(y\), substitute into the second equation and solve.
\[
\frac{x^2 \left(\frac{216}{x}\right)}{4\pi} = 224
\]
\[
216x = 896\pi
\]
\[
x = \frac{896\pi}{216} = 13.03
\]
\[
y = \frac{216}{x} = \frac{216}{896\pi} = \left(\frac{216}{896\pi}\right)^2 = 16.57
\]
The cardboard should be about 13.03 centimeters by 16.57 centimeters.
Section 11.6: Systems of Nonlinear Equations

87. Find equations relating area and perimeter:
\[
\begin{align*}
    x^2 + y^2 &= 4500 \\
    3x + 3y + (x - y) &= 300
\end{align*}
\]
Solve the second equation for \( y \), substitute into the first equation and solve:
\[
\begin{align*}
    4x + 2y &= 300 \\
    2y &= 300 - 4x \\
    y &= 150 - 2x \\
    x^2 + (150 - 2x)^2 &= 4500 \\
    x^2 + 22,500 - 600x + 4x^2 &= 4500 \\
    5x^2 - 600x + 18,000 &= 0 \\
    x^2 - 120x + 3600 &= 0 \\
    (x - 60)^2 &= 0 \\
    x - 60 &= 0 \\
    x &= 60 \\
    y &= 150 - 2(60) = 30
\end{align*}
\]
The sides of the squares are 30 feet and 60 feet.

88. Let \( x \) = the length of a side of the square.
Let \( r \) = the radius of the circle.
The area of the square is \( x^2 \) and the area of the circle is \( \pi r^2 \). The perimeter of the square is \( 4x \) and the circumference of the circle is \( 2\pi r \). Find equations relating area and perimeter:
\[
\begin{align*}
    x^2 + \pi r^2 &= 100 \\
    4x + 2\pi r &= 60
\end{align*}
\]
Solve the second equation for \( x \), substitute into the first equation and solve:
\[
\begin{align*}
    4x &= 60 - 2\pi r \\
    x &= 15 - \frac{1}{2}\pi r \\
    \left(15 - \frac{1}{2}\pi r\right)^2 + \pi r^2 &= 100 \\
    225 - 15\pi r + \frac{1}{4}\pi^2 r^2 + \pi r^2 &= 100 \\
    \left(\frac{1}{4}\pi^2 + \pi\right)r^2 - 15\pi r + 125 &= 0 \\
    b^2 - 4ac &= (-15\pi)^2 - 4\left(\frac{1}{4}\pi^2 + \pi\right)(125) \\
    &= 225\pi^2 - 500\left(\frac{1}{4}\pi^2 + \pi\right) \\
    &= 100\pi^2 - 500\pi < 0
\end{align*}
\]
Since the discriminant is less than zero, it is impossible to cut the wire into two pieces whose total area equals 100 square feet.

89. Solve the system for \( l \) and \( w \):
\[
\begin{align*}
    2l + 2w &= P \\
    l &= w = A
\end{align*}
\]
Solve the first equation for \( l \), substitute into the second equation and solve:
\[
\begin{align*}
    2l &= P - 2w \\
    l &= \frac{P}{2} - w \\
    \left(\frac{P}{2} - w\right)w &= A \\
    w &= \frac{P}{2} - w \\
    \frac{P}{2}w - w^2 &= A \\
    w^2 - \frac{P}{2}w + A &= 0 \\
    w &= \frac{-\frac{P}{2} \pm \sqrt{\left(-\frac{P}{2}\right)^2 - 4A}}{2} \\
    &= \frac{-\frac{P}{2} \pm \sqrt{\frac{P^2}{4} - 4A}}{2} \\
    &= \frac{P}{2} \pm \sqrt{\frac{P^2}{4} - 4A}
\end{align*}
\]
If \( w = \frac{P + \sqrt{P^2 - 16A}}{4} \) then
\[
\begin{align*}
    l &= \frac{P}{2} - \frac{P + \sqrt{P^2 - 16A}}{4} = \frac{P - \sqrt{P^2 - 16A}}{4} \\
    &= \frac{P}{2} - \frac{\sqrt{P^2 - 16A}}{4} \\
    &= \frac{P}{2} - \frac{\sqrt{P^2 - 16A}}{4} \\
    &= \frac{P}{2} - \frac{\sqrt{P^2 - 16A}}{4}
\end{align*}
\]
If it is required that length be greater than width, then the solution is:
\[
\begin{align*}
    w &= \frac{P - \sqrt{P^2 - 16A}}{4} \\
    l &= \frac{P}{2} - \frac{P - \sqrt{P^2 - 16A}}{4} = \frac{P + \sqrt{P^2 - 16A}}{4} \\
    &= \frac{P}{2} + \frac{\sqrt{P^2 - 16A}}{4}
\end{align*}
\]
90. Solve the system for \( l \) and \( b \):
\[
\begin{align*}
    P &= b + 2l \\
    \frac{b^2}{4} &= l^2
\end{align*}
\]
Solve the first equation for \( b \), substitute into the second equation and solve.
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91. Solve the equation: $m^2 - 4(2m - 4) = 0$

$$m^2 - 8m + 16 = 0$$

$$(m - 4)^2 = 0$$

$m = 4$

Use the point-slope equation with slope 4 and the point (2, 4) to obtain the equation of the tangent line:

$$y - 4 = 4(x - 2) \Rightarrow y - 4 = 4x - 8 \Rightarrow y = 4x - 4$$

92. Solve the system:

$$\begin{align*}
  x^2 + y^2 &= 10 \\
  y &= mx + b
\end{align*}$$

Solve the system by substitution:

$$x^2 + (mx + b)^2 = 10$$

$$x^2 + m^2x^2 + 2bm - b^2 - 10 = 0$$

$$(1 + m^2)x^2 + 2bm + b^2 - 10 = 0$$

Note that the tangent line passes through (1, 3). Find the relation between $m$ and $b$:

$$3 = m(1) + b \Rightarrow b = 3 - m$$

There is one solution to the quadratic if the discriminant is zero.

$$\sqrt{(2bm)^2 - 4(m^2 + 1)(b^2 - 10)} = 0$$

$$4b^2m^2 - 4b^2m^2 + 40m^2 - 4b^2 + 40 = 0$$

$$40m^2 - 4(3 - m)^2 + 40 = 0$$

$$40m^2 - 4m^2 + 24m - 36 + 40 = 0$$

$$36m^2 + 24m + 4 = 0$$

$$9m^2 + 6m + 1 = 0$$

$$m = \frac{3}{2} - \frac{1}{3} = \frac{10}{3}$$

The equation of the tangent line is $y = \frac{1}{3}x + \frac{10}{3}$.

93. Solve the system:

$$\begin{align*}
  y &= x^2 + 2 \\
  y &= mx + b
\end{align*}$$

Solve the system by substitution:

$$x^2 + 2 = mx + b \Rightarrow x^2 - mx + 2 - b = 0$$

Note that the tangent line passes through (1, 3). Find the relation between $m$ and $b$:

$$3 = m(1) + b \Rightarrow b = 3 - m$$

Substitute into the quadratic to eliminate $b$:

$$(m - 2)^2 = 0$$

$$m - 2 = 0$$

$$m = 2$$

$$b = 3 - m = 3 - 2 = 1$$

The equation of the tangent line is $y = 2x + 1$.

94. Solve the system:

$$\begin{align*}
  x^2 + y &= 5 \\
  y &= mx + b
\end{align*}$$

Solve the system by substitution:

$$x^2 + mx + b = 5 \Rightarrow x^2 + mx + b - 5 = 0$$

Note that the tangent line passes through (–2, 1). Find the relation between $m$ and $b$:

$$1 = m(-2) + b \Rightarrow b = 2m + 1$$

Substitute into the quadratic to eliminate $b$: 

$$x^2 + m(-2) + b = 5$$

$$x^2 + 2m + b - 5 = 0$$

$$x^2 + 2m - 5 = 0$$

$$2m^2 - 4m^2 + 24m - 36 + 40 = 0$$

$$36m^2 + 24m + 4 = 0$$

$$9m^2 + 6m + 1 = 0$$

$$m = \frac{3}{2} - \frac{1}{3} = \frac{10}{3}$$

The equation of the tangent line is $y = \frac{1}{3}x + \frac{10}{3}$.
Section 11.6: Systems of Nonlinear Equations

Find when the discriminant equals 0:

\[(m - 4)^2 = 0\]
\[m = 4\]
\[b = 2m + 1 = 2(4) + 1 = 9\]
The equation of the tangent line is \(y = 4x + 9\).

95. Solve the system:

\[
\begin{align*}
2x^2 + 3y^2 &= 14 \\
y &= mx + b
\end{align*}
\]
Solve the system by substitution:

\[
\begin{align*}
2x^2 + 3(mx + b)^2 &= 14 \\
2x^2 + 3m^2x^2 + 6mbx + 3b^2 &= 14 \\
(3m^2 + 2)x^2 + 6mbx + 3b^2 - 14 &= 0
\end{align*}
\]
Note that the tangent line passes through (1, 2).
Find the relation between \(m\) and \(b\):
\[
2 = m(1) + b \Rightarrow b = 2 - m
\]
Substitute into the quadratic to eliminate \(b\):
\[
(3m^2 + 2)x^2 + (12m - 6m^2)x + (3m^2 - 12m - 2) = 0
\]
Find when the discriminant equals 0:
\[
(12m - 6m^2)^2 - 4(3m^2 + 2)(3m^2 - 12m - 2) = 0
\]
\[
144m^2 + 96m + 16 = 0
\]
\[
9m^2 + 6m + 1 = 0
\]
\[
(3m + 1)^2 = 0
\]
\[
m = -\frac{1}{3}
\]
\[
b = 2 - m = 2 - \left(-\frac{1}{3}\right) = \frac{7}{3}
\]
The equation of the tangent line is \(y = -\frac{1}{3}x + \frac{7}{3}\).

96. Solve the system:

\[
\begin{align*}
3x^2 + y^2 &= 7 \\
y &= mx + b
\end{align*}
\]
Solve the system by substitution:

\[
\begin{align*}
3x^2 + (mx + b)^2 &= 7 \\
3x^2 + m^2x^2 + 2mbx + b^2 &= 7 \\
\left(m^2 + 3\right)x^2 + 2mbx + b^2 - 7 &= 0
\end{align*}
\]
Note that the tangent line passes through (–1, 2).
Find the relation between \(m\) and \(b\):
\[
2 = m(-1) + b \Rightarrow b = m + 2
\]
There is one solution to the quadratic if the discriminant equals 0.
\[
(2mb)^2 - 4\left(m^2 + 3\right)(b^2 - 7) = 0
\]
\[
4m^2 - 48m + 84 = 0
\]
\[
28m^2 - 12b^2 + 84 = 0
\]
\[
7m^2 - 3b^2 + 21 = 0
\]
Substitute for \(b\) and solve:
\[
7m^2 - 3(m + 2)^2 + 21 = 0
\]
\[
7m^2 - 3m^2 - 12m + 12 + 21 = 0
\]
\[
4m^2 - 12m + 9 = 0
\]
\[
(2m - 3)^2 = 0
\]
\[
m = \frac{3}{2}
\]
\[
b = m + 2 = \frac{3}{2} + 2 = \frac{7}{2}
\]
The equation of the tangent line is \(y = \frac{3}{2}x + \frac{7}{2}\).

97. Solve the system:

\[
\begin{align*}
x^2 - y^2 &= 3 \\
y &= mx + b
\end{align*}
\]
Solve the system by substitution:

\[
\begin{align*}
x^2 - (mx + b)^2 &= 3 \\
x^2 - m^2x^2 - 2mbx - b^2 &= 3 \\
\left(1 - m^2\right)x^2 - 2mbx - b^2 - 3 &= 0
\end{align*}
\]
Note that the tangent line passes through (2, 1).
Find the relation between \(m\) and \(b\):
\[
1 = m(2) + b \Rightarrow b = 1 - 2m
\]
Substitute into the quadratic to eliminate \(b\):
\[
(1 - m^2)x^2 - 2m(1 - 2m)x - (1 - 2m)^2 - 3 = 0
\]
\[
(1 - m^2)x^2 + (2m^2 - 4m + 4)x - 1 + 4m - 4m^2 - 3 = 0
\]
\[
(1 - m^2)x^2 + (2m^2 - 4m + 4)x + (-4m^2 + 4m - 4) = 0
\]
Find when the discriminant equals 0:
Chapter 11: Systems of Equations and Inequalities

\[
\begin{align*}
(-2m + 4m^2)^2 - 4(1-m^2)(-4m^2 + 4m - 4) &= 0 \\
4m^2 - 16m^3 + 16m^4 - 16m^3 + 16m^4 - 16m + 16 &= 0 \\
4m^2 - 16m + 16 &= 0 \\
4m^2 - 4m + 4 &= 0 \\
(m - 2)^2 &= 0 \\
m &= 2
\end{align*}
\]

The equation of the tangent line is \( y = 2x - 3 \).

98. Solve the system:
\[
\begin{align*}
2y^2 - x^2 &= 14 \\
y &= mx + b
\end{align*}
\]

Solve the system by substitution:
\[
2(mx + b)^2 - x^2 = 14 \\
2m^2x^2 + 4mbx + 2b^2 - x^2 = 14 \\
(2m^2 - 1)x^2 + 4mbx + 2b^2 - 14 = 0
\]

Note that the tangent line passes through \((2, 3)\).

Find the relation between \(m\) and \(b\):

\[
3 = m(2) + b \Rightarrow b = 3 - 2m
\]

There is one solution to the quadratic if the discriminant equals 0.

\[
(4bm)^2 - 4(2m^2 - 1)(2b^2 - 14) = 0
\]

\[
16b^2m^2 - 16b^2m^2 + 112m^2 + 8b^2 - 56 = 0 \\
112m^2 + 8b^2 - 56 = 0 \\
14m^2 + b^2 - 7 = 0
\]

Substitute for \(b\) and solve:

\[
14m^2 + (3 - 2m)^2 - 7 = 0 \\
14m^2 + 4m^2 - 12m + 9 - 7 = 0 \\
18m^2 - 12m + 2 = 0 \\
2(3m - 1)^2 = 0 \\
m = 1/3
\]

\[
b = 3 - 2m = 3 - 2\left(\frac{1}{3}\right) = \frac{7}{3}
\]

The equation of the tangent line is \( y = \frac{1}{3}x + \frac{7}{3} \).

99. Solve for \(r_1\) and \(r_2\):
\[
\begin{align*}
\left\{ \begin{array}{c}
r_1 + r_2 = -\frac{b}{a} \\
r_1r_2 = \frac{c}{a}
\end{array} \right.
\]

Substitute and solve:

\[
r_1 = -r_2 - \frac{b}{a}
\]

\[
\begin{align*}
\left(-r_2 - \frac{b}{a}\right) r_2 &= \frac{c}{a} \\
-r_2^2 - \frac{b}{a} r_2 - \frac{c}{a} &= 0 \\
ar_2^2 + br_2 + c &= 0 \\
r_2 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
r_1 &= -r_2 - \frac{b}{a} = -\left(-\frac{b \pm \sqrt{b^2 - 4ac}}{2a}\right) - \frac{b}{a}
\end{align*}
\]

The solutions are:

\[
r_i = -\frac{b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad r_i = -\frac{b - \sqrt{b^2 - 4ac}}{2a}.
\]
100. Consider the circle with equation
\[(x-h)^2 + (y-k)^2 = r^2\]
and the third degree polynomial with equation
\[y = ax^3 + bx^2 + cx + d\].
Substituting the second equation into the first equation yields
\[(x-h)^2 + (ax^3 + bx^2 + cx + d - k)^2 = r^2\].
In order to find the roots for this equation we can expand the terms on the left hand side of the equation. Notice that \((x-h)^2\) yields a 2nd degree polynomial, and \((ax^3 + bx^2 + cx + d - k)^2\) yields a 6th degree polynomial. Therefore, we need to find the roots of a 6th degree equation, and the Fundamental Theorem of Algebra states that there will be at most six real roots. Thus, the circle and the 3rd degree polynomial will intersect at most six times.

Now consider the circle with equation
\[(x-h)^2 + (y-k)^2 = r^2\]
and the polynomial of degree \(n\) with equation
\[y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n\].
Substituting the first equation into the first equation yields
\[(x-h)^2 + (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n - k)^2 = r^2\].
In order to find the roots for this equation we can expand the terms on the left hand side of the equation. Notice that \((x-h)^2\) yields a 2nd degree polynomial, and \((a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n - k)^2\) yields a polynomial of degree 2n. Therefore, we need to find the roots of an equation of degree 2n, and the Fundamental Theorem of Algebra states that there will be at most 2n real roots. Thus, the circle and the \(n^{th}\) degree polynomial will intersect at most 2n times.

101. Since the area of the square piece of sheet metal is 100 square feet, the sheet’s dimensions are 10 feet by 10 feet. Let \(x\) = the length of the cut.

The dimensions of the box are: length = \(10 - 2x\); width = \(10 - 2x\); height = \(x\). Note that each of these expressions must be positive. So we must have \(x > 0\) and \(10 - 2x > 0\) \(\Rightarrow x < 5\), that is, \(0 < x < 5\). So the volume of the box is given by
\[V = (\text{length}) \cdot (\text{width}) \cdot (\text{height}) = (10 - 2x)(10 - 2x)(x) = (10 - 2x)^2(x)\]

a. In order to get a volume equal to 9 cubic feet, we solve \((10 - 2x)^2(x) = 9\).
\[(10 - 2x)^2(x) = 9\]
\[(100 - 40x + 4x^2) x = 9\]
\[100x - 40x^2 + 4x^3 = 9\]
So we need to solve the equation \(4x^3 - 40x^2 + 100x - 9 = 0\).

Graphing \(y_2 = 4x^3 - 40x^2 + 100x - 9\) on a calculator yields the graph

\[\text{The graph indicates that there are three real zeros on the interval [0, 6]. Using the ZERO feature of a graphing calculator, we find that the three roots shown occur at } x = 0.093, x = 4.274 \text{ and } x = 5.632. \text{ But we've already noted that we must have } 0 < x < 5, \text{ so the only practical values for the sides of the square base are } x = 0.093 \text{ feet and } x = 4.274 \text{ feet.}\]

b. Answers will vary.
Section 11.7

1. \[3x + 4 < 8 - x \]
   \[4x < 4 \]
   \[x < 1 \]
   \( \{ x \mid x < 1 \} \) or \((-\infty, 1)\)

2. \[3x - 2y = 6\]
   The graph is a line.
   x-intercept: \[3x - 2(0) = 6\]
   \[3x = 6\]
   \[x = 2\]
   y-intercept: \[3(0) - 2y = 6\]
   \[-2y = 6\]
   \[y = -3\]

3. \[x^2 + y^2 = 9\]
   The graph is a circle. Center: \((0, 0)\); Radius: 3

4. \[y = x^2 + 4\]
   The graph is a parabola.
   x-intercepts: \[0 = x^2 + 4\]
   \[x^2 = -4, \text{ no } x\text{-intercepts}\]
   y-intercept: \[y = 0^2 + 4 = 4\]
   The vertex has x-coordinate:
   \[x = -\frac{b}{2a} = -\frac{0}{2(1)} = 0\,\text{.}\]
   The y-coordinate of the vertex is \(y = 0^2 + 4 = 4\,\text{.}\)

5. True

6. \[y = x^2\]; right; 2

7. dashes; solid

8. half-planes

9. False, see example 7b.

10. unbounded

11. \[x \geq 0\]
   Graph the line \(x = 0\). Use a solid line since the inequality uses \(\geq\). Choose a test point not on the line, such as \((2, 0)\). Since \(2 \geq 0\) is true, shade the side of the line containing \((2, 0)\).

12. \[y \geq 0\]
   Graph the line \(y = 0\). Use a solid line since the inequality uses \(\geq\). Choose a test point not on the line, such as \((0, 2)\). Since \(2 \geq 0\) is true, shade the side of the line containing \((0, 2)\).
13. \[ x \geq 4 \]
Graph the line \( x = 4 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((5, 0)\). Since \( 5 \geq 0 \) is true, shade the side of the line containing \((5, 0)\).

14. \[ y \leq 2 \]
Graph the line \( y = 2 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((5, 0)\). Since \( 0 \leq 2 \) is true, shade the side of the line containing \((5, 0)\).

15. \[ 2x + y \geq 6 \]
Graph the line \( 2x + y = 6 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 2(0) + 0 \geq 6 \) is false, shade the opposite side of the line from \((0, 0)\).

16. \[ 3x + 2y \leq 6 \]
Graph the line \( 3x + 2y = 6 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 3(0) + 2(0) \leq 6 \) is true, shade the side of the line containing \((0, 0)\).

17. \[ x^2 + y^2 > 1 \]
Graph the circle \( x^2 + y^2 = 1 \). Use a dashed line since the inequality uses \( > \). Choose a test point not on the circle, such as \((0, 0)\). Since \( 0^2 + 0^2 > 1 \) is false, shade the opposite side of the circle from \((0, 0)\).

18. \[ x^2 + y^2 \leq 9 \]
Graph the circle \( x^2 + y^2 = 9 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the circle, such as \((0, 0)\). Since \( 0^2 + 0^2 \leq 9 \) is true, shade the same side of the circle as \((0, 0)\).
19. \( y \leq x^2 - 1 \)
   Graph the parabola \( y = x^2 - 1 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the parabola, such as \((0, 0)\). Since \( 0 \leq 0^2 - 1 \) is false, shade the opposite side of the parabola from \((0, 0)\).

20. \( y > x^2 + 2 \)
   Graph the parabola \( y = x^2 + 2 \). Use a dashed line since the inequality uses \( > \). Choose a test point not on the parabola, such as \((0, 0)\). Since \( 0 > 0^2 + 2 \) is false, shade the opposite side of the parabola from \((0, 0)\).

21. \( xy \geq 4 \)
   Graph the hyperbola \( xy = 4 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the hyperbola, such as \((0, 0)\). Since \( 0 \cdot 0 \geq 4 \) is false, shade the opposite side of the hyperbola from \((0, 0)\).

22. \( xy \leq 1 \)
   Graph the hyperbola \( xy = 1 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the hyperbola, such as \((0, 0)\). Since \( 0 \cdot 0 \leq 1 \) is true, shade the same side of the hyperbola as \((0, 0)\).

23. \[ \begin{align*}
   x + y &\leq 2 \\
   2x + y &\geq 4
\end{align*} \]
   Graph the line \( x + y = 2 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 + 0 \leq 2 \) is true, shade the side of the line containing \((0, 0)\). Graph the line \( 2x + y = 4 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 2(0) + 0 \geq 4 \) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution.

24. \[ \begin{align*}
   3x - y &\geq 6 \\
   x + 2y &\leq 2
\end{align*} \]
   Graph the line \( 3x - y = 6 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 3(0) - 0 \geq 6 \) is false, shade the opposite side of the line from \((0, 0)\). Graph the line \( x + 2y = 2 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 + 2(0) \leq 2 \) is true, shade the side of the line containing \((0, 0)\). The overlapping region is the solution.
Section 11.7: Systems of Inequalities

25. \( \begin{align*}
2x - y &\leq 4 \\
3x + 2y &\geq -6
\end{align*} \)

Graph the line \( 2x - y = 4 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 2(0) - 0 \leq 4 \) is true, shade the side of the line containing \((0, 0)\). Graph the line \( 3x + 2y = -6 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 3(0) + 2(0) \geq -6 \) is true, shade the side of the line containing \((0, 0)\). The overlapping region is the solution.

26. \( \begin{align*}
4x - 5y &\leq 0 \\
2x - y &\geq 2
\end{align*} \)

Graph the line \( 4x - 5y = 0 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((2, 0)\). Since \( 4(2) - 5(0) \leq 0 \) is false, shade the opposite side of the line from \((2, 0)\). Graph the line \( 2x - y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 2(0) - 0 \geq 2 \) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution.

27. \( \begin{align*}
2x - 3y &\leq 0 \\
3x + 2y &\leq 6
\end{align*} \)

Graph the line \( 2x - 3y = 0 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 3)\). Since \( 2(0) - 3(3) \leq 0 \) is true, shade the side of the line containing \((0, 3)\). Graph the line \( 3x + 2y = 6 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 3(0) + 2(0) \leq 6 \) is true, shade the side of the line containing \((0, 0)\). The overlapping region is the solution.

28. \( \begin{align*}
4x - y &\geq 2 \\
x + 2y &\geq 2
\end{align*} \)

Graph the line \( 4x - y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 4(0) - 0 \geq 2 \) is false, shade the opposite side of the line from \((0, 0)\). Graph the line \( x + 2y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 + 2(0) \geq 2 \) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution.
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29. \[
\begin{align*}
    x - 2y &\leq 6 \\
    2x - 4y &\geq 0
\end{align*}
\]
Graph the line \( x - 2y = 6 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 - 2(0) \leq 6 \) is true, shade the side of the line containing \((0, 0)\). Graph the line \( 2x - 4y = 0 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 2)\). Since \( 2(0) - 4(2) \geq 0 \) is false, shade the opposite side of the line from \((0, 2)\). The overlapping region is the solution.

30. \[
\begin{align*}
    x + 4y &\leq 8 \\
    x + 4y &\geq 4
\end{align*}
\]
Graph the line \( x + 4y = 8 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 + 4(0) \leq 8 \) is true, shade the side of the line containing \((0, 0)\). Graph the line \( x + 4y = 4 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 + 4(0) \geq 4 \) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution.

31. \[
\begin{align*}
    2x + y &\geq -2 \\
    2x + y &\geq 2
\end{align*}
\]
Graph the line \( 2x + y = -2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 2(0) + 0 \geq -2 \) is true, shade the side of the line containing \((0, 0)\). Graph the line \( 2x + y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 2(0) + 0 \geq 2 \) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution.

32. \[
\begin{align*}
    x - 4y &\leq 4 \\
    x - 4y &\geq 0
\end{align*}
\]
Graph the line \( x - 4y = 4 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 - 4(0) \leq 4 \) is true, shade the side of the line containing \((0, 0)\). Graph the line \( x - 4y = 0 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((1, 0)\). Since \( 1 - 4(0) \geq 0 \) is true, shade the side of the line containing \((1, 0)\). The overlapping region is the solution.
Section 11.7: Systems of Inequalities

33. \[ \begin{cases} 2x + 3y \geq 6 \\ 2x + 3y \leq 0 \end{cases} \]
Graph the line \( 2x + 3y = 6 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 2(0) + 3(0) \geq 6 \) is false, shade the opposite side of the line from \((0, 0)\). Graph the line \( 2x + 3y = 0 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 2)\). Since \( 2(0) + 3(2) \leq 0 \) is false, shade the opposite side of the line from \((0, 2)\). Since the regions do not overlap, the solution is an empty set.

34. \[ \begin{cases} 2x + y \geq 0 \\ 2x + y \geq 2 \end{cases} \]
Graph the line \( 2x + y = 0 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((1, 0)\). Since \( 2(1) + 0 \geq 0 \) is true, shade the side of the line containing \((1, 0)\). Graph the line \( 2x + y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 2(0) + 0 \geq 2 \) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution.

35. \[ \begin{cases} x^2 + y^2 \leq 9 \\ x + y \geq 3 \end{cases} \]
Graph the circle \( x^2 + y^2 = 9 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the circle, such as \((0, 0)\). Since \( 0^2 + 0^2 \leq 9 \) is true, shade the same side of the circle as \((0, 0)\). Graph the line \( x + y = 3 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 + 0 \geq 3 \) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution.

36. \[ \begin{cases} x^2 + y^2 \geq 9 \\ x + y \leq 3 \end{cases} \]
Graph the circle \( x^2 + y^2 = 9 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the circle, such as \((0, 0)\). Since \( 0^2 + 0^2 \geq 9 \) is false, shade the opposite side of the circle as \((0, 0)\). Graph the line \( x + y = 3 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 + 0 \leq 3 \) is true, shade the same side of the line as \((0, 0)\). The overlapping region is the solution.
37. \[ \begin{align*}
    y &\geq x^2 - 4 \\
    y &\leq x - 2
\end{align*} \]
Graph the parabola \( y = x^2 - 4 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the parabola, such as \((0, 0)\). Since \( 0 \geq 0^2 - 4 \) is true, shade the same side of the parabola as \((0, 0)\). Graph the line \( y = x - 2 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 \leq 0 - 2 \) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution.

38. \[ \begin{align*}
    y^2 &\leq x \\
    y &\geq x
\end{align*} \]
Graph the parabola \( y^2 = x \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the parabola, such as \((1, 2)\). Since \( 2^2 \leq 1 \) is false, shade the opposite side of the parabola from \((1, 2)\). Graph the line \( y = x \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((1, 2)\). Since \( 2 \geq 1 \) is true, shade the same side of the line as \((1, 2)\). The overlapping region is the solution.

39. \[ \begin{align*}
    x^2 + y^2 &\leq 16 \\
    y &\geq x^2 - 4
\end{align*} \]
Graph the circle \( x^2 + y^2 = 16 \). Use a solid line since the inequality is not strict. Choose a test point not on the circle, such as \((0, 0)\). Since \( 0^2 + 0^2 \leq 16 \) is true, shade the side of the circle containing \((0, 0)\). Graph the parabola \( y = x^2 - 4 \). Use a solid line since the inequality is not strict. Choose a test point not on the parabola, such as \((0, 0)\). Since \( 0 \geq 0^2 - 4 \) is true, shade the side of the parabola that contains \((0, 0)\). The overlapping region is the solution.

40. \[ \begin{align*}
    x^2 + y^2 &\leq 25 \\
    y &\leq x^2 - 5
\end{align*} \]
Graph the circle \( x^2 + y^2 = 25 \). Use a solid line since the inequality is not strict. Choose a test point not on the circle, such as \((0, 0)\). Since \( 0^2 + 0^2 \leq 25 \) is true, shade the side of the circle containing \((0, 0)\). Graph the parabola \( y = x^2 - 5 \). Use a solid line since the inequality is not strict. Choose a test point not on the parabola, such as \((0, 0)\). Since \( 0 \leq 0^2 - 5 \) is false, shade the side of the parabola opposite that which contains the point \((0, 0)\). The overlapping region is the solution.
41. \[ \begin{cases} xy \geq 4 \\ y \geq x^2 + 1 \end{cases} \]

Graph the hyperbola \( xy = 4 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the parabola, such as (0, 0). Since \( 0 \cdot 0 \geq 4 \) is false, shade the opposite side of the hyperbola from (0, 0). Graph the parabola \( y = x^2 + 1 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the parabola, such as (0, 0). Since \( 0 \geq 0^2 + 1 \) is false, shade the opposite side of the parabola from (0, 0). The overlapping region is the solution.

42. \[ \begin{cases} y + x^2 \leq 1 \\ y \geq x^2 - 1 \end{cases} \]

Graph the parabola \( y + x^2 = 1 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the parabola, such as (0, 0). Since \( 0 + 0^2 \leq 1 \) is true, shade the same side of the parabola as (0, 0). Graph the parabola \( y = x^2 - 1 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the parabola, such as (0, 0). Since \( 0 \geq 0^2 - 1 \) is true, shade the same side of the parabola as (0, 0). The overlapping region is the solution.

43. \[ \begin{cases} x \geq 0 \\ y \geq 0 \\ 2x + y \leq 6 \\ x + 2y \leq 6 \end{cases} \]

Graph \( x \geq 0 \); \( y \geq 0 \). Shaded region is the first quadrant. Graph the line \( 2x + y = 6 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as (0, 0). Since \( 2(0) + 0 \leq 6 \) is true, shade the side of the line containing (0, 0). Graph the line \( x + 2y = 6 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as (0, 0). Since \( 0 + 2(0) \leq 6 \) is true, shade the same side of the line containing (0, 0). The overlapping region is the solution. The graph is bounded. Find the vertices:

The \( x \)-axis and \( y \)-axis intersect at (0, 0). The intersection of \( x + 2y = 6 \) and the \( y \)-axis is (0, 3). The intersection of \( 2x + y = 6 \) and the \( x \)-axis is (3, 0). To find the intersection of \( x + 2y = 6 \) and \( 2x + y = 6 \), solve the system:

\[ \begin{cases} x + 2y = 6 \\ 2x + y = 6 \end{cases} \]

Solve the first equation for \( x \): \( x = 6 - 2y \).

Substitute and solve:

\[ \begin{align*} 2(6-2y) + y &= 6 \\
12 - 4y + y &= 6 \\
12 - 3y &= 6 \\
-3y &= -6 \\
y &= 2 \\
x &= 6 - 2(2) = 2 \end{align*} \]

The point of intersection is (2, 2). The four corner points are (0, 0), (0, 3), (3, 0), and (2, 2).
44. \[
\begin{align*}
\begin{cases}
\quad x \geq 0 \\
\quad y \geq 0 \\
\quad x + y \geq 4 \\
\quad 2x + 3y \geq 6
\end{cases}
\end{align*}
\]
Graph \( x \geq 0 \); \( y \geq 0 \). Shaded region is the first quadrant. Graph the line \( x + y = 4 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \(0 + 0 \geq 4\) is false, shade the opposite side of the line from \((0, 0)\). Graph the line \( 2x + 3y = 6 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \(2(0) + 3(0) \geq 6\) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution. The graph is unbounded.

Find the vertices:
The intersection of \( x + y = 2 \) and the \( y \)-axis is \((0, 4)\). The intersection of \( 2x + y = 4 \) and the \( y \)-axis is \((0, 4)\). The two corner points are \((2, 0)\), and \((0, 4)\).

45. \[
\begin{align*}
\begin{cases}
\quad x \geq 0 \\
\quad y \geq 0 \\
\quad x + y \geq 2 \\
\quad 2x + y \geq 4
\end{cases}
\end{align*}
\]
Graph \( x \geq 0 \); \( y \geq 0 \). Shaded region is the first quadrant. Graph the line \( x + y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \(0 + 0 \geq 2\) is false, shade the opposite side of the line from \((0, 0)\). Graph the line \( 2x + y = 4 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \(2(0) + 0 \geq 4\) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution. The graph is unbounded.

Find the vertices:
The intersection of \( x + y = 2 \) and the \( x \)-axis is \((2, 0)\). The intersection of \( 2x + y = 4 \) and the \( y \)-axis is \((0, 4)\). The two corner points are \((2, 0)\), and \((0, 4)\).
47. \[
\begin{align*}
x & \geq 0 \\
y & \geq 0 \\
x + y & \geq 2 \\
2x + 3y & \leq 12 \\
3x + y & \leq 12
\end{align*}
\]
Graph \(x \geq 0; \ y \geq 0\). Shaded region is the first quadrant. Graph the line \(x + y = 2\). Use a solid line since the inequality uses \(\geq\). Choose a test point not on the line, such as \((0, 0)\). Since \(0 + 0 \geq 2\) is false, shade the opposite side of the line from \((0, 0)\). Graph the line \(2x + 3y = 12\).

Use a solid line since the inequality uses \(\leq\). Choose a test point not on the line, such as \((0, 0)\). Since \(2(0) + 3(0) \leq 12\) is true, shade the side of the line containing \((0, 0)\). The overlapping region is the solution. The graph is bounded.

Find the vertices:
The intersection of \(x + y = 2\) and the \(y\)-axis is \((0, 2)\). The intersection of \(x + y = 2\) and the \(x\)-axis is \((2, 0)\). The intersection of \(2x + 3y = 12\) and the \(y\)-axis is \((0, 4)\). The intersection of \(3x + y = 12\) and the \(x\)-axis is \((4, 0)\).

To find the intersection of \(2x + 3y = 12\) and \(3x + y = 12\), solve the system:
\[
\begin{align*}
2x + 3y &= 12 \\
3x + y &= 12
\end{align*}
\]
Solve the second equation for \(y\): \(y = 12 - 3x\).
Substitute and solve:
\[
\begin{align*}
2x + 3(12 - 3x) &= 12 \\
2x + 36 - 9x &= 12 \\
-7x &= -24
\end{align*}
\]
\[
x = \frac{24}{7}
\]
\[
y = 12 - 3 \left( \frac{24}{7} \right) = 12 - \frac{72}{7} = \frac{12}{7}
\]
The point of intersection is \(\left( \frac{24}{7}, \frac{12}{7} \right)\).
The five corner points are \((0, 2), (0, 4), (2, 0), (4, 0), and \left( \frac{24}{7}, \frac{12}{7} \right)\).

48. \[
\begin{align*}
x & \geq 0 \\
y & \geq 0 \\
x + y & \geq 2 \\
x + y & \leq 8 \\
2x + y & \geq 10
\end{align*}
\]
Graph \(x \geq 0; \ y \geq 0\). Shaded region is the first quadrant. Graph the line \(x + y = 2\). Use a solid line since the inequality uses \(\geq\). Choose a test point not on the line, such as \((0, 0)\). Since \(0 + 0 \geq 2\) is false, shade the opposite side of the line from \((0, 0)\). Graph the line \(2x + y = 10\).

Use a solid line since the inequality uses \(\leq\). Choose a test point not on the line, such as \((0, 0)\). Since \(0 + 0 \leq 8\) is true, shade the side of the line containing \((0, 0)\). The overlapping region is the solution. The graph is bounded.

Find the vertices:
The intersection of \(x + y = 2\) and the \(y\)-axis is \((0, 2)\). The intersection of \(x + y = 2\) and the \(y\)-axis is \((0, 10)\). To find the intersection of \(2x + y = 10\) and \(x + y = 8\), solve the system:
\[
\begin{align*}
2x + y &= 10 \\
x + y &= 8
\end{align*}
\]
Solve the second equation for \(y\): \(y = 8 - x\).
Substitute and solve:
\[
\begin{align*}
2x + 8 - x &= 10 \\
x &= 2
\end{align*}
\]
\[
y = 8 - 2 = 6
\]
The point of intersection is \((2, 6)\).
The five corner points are \((0, 2), (2, 0), (0, 8), (5, 0)\) and \((2, 6)\).
Chapter 11: Systems of Equations and Inequalities

49.
Graph \( x \geq 0; \ y \geq 0 \). Shaded region is the first quadrant. Graph the line \( x + y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0,0)\). Since \(0 + 0 \geq 2\) is false, shade the opposite side of the line from \((0,0)\). Graph the line \( x + y = 8 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0,0)\). Since \(0 + 0 \leq 8\) is true, shade the side of the line containing \((0,0)\). Graph the line \( 2x + y = 10 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0,0)\). Since \(2(0) + 0 \leq 10\) is true, shade the side of the line containing \((0,0)\). The overlapping region is the solution. The graph is bounded.

Find the vertices:
The intersection of \( x + y = 2 \) and the \( y \)-axis is \((0,2)\). The intersection of \( x + y = 2 \) and the \( x \)-axis is \((2,0)\). The intersection of \( x + y = 8 \) and the \( y \)-axis is \((0,8)\). The intersection of \( 2x + y = 10 \) and the \( x \)-axis is \((5,0)\). To find the intersection of \( x + y = 8 \) and \( 2x + y = 10 \), solve the system:

\[
\begin{align*}
x + y &= 8 \\
2x + y &= 10
\end{align*}
\]

Solve the first equation for \( y \): \( y = 8 - x \).
Substitute and solve:
\[
\begin{align*}
2x + 8 - x &= 10 \\
x &= 2 \\
y &= 8 - 2 = 6
\end{align*}
\]
The point of intersection is \((2,6)\).
The five corner points are \((0,2), (0,8), (2,0), (5,0), \) and \((2,6)\).

50.
Graph \( x \geq 0; \ y \geq 0 \). Shaded region is the first quadrant. Graph the line \( x + y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0,0)\). Since \(0 + 0 \geq 2\) is false, shade the opposite side of the line from \((0,0)\). Graph the line \( x + y = 8 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0,0)\). Since \(0 + 0 \leq 8\) is true, shade the side of the line containing \((0,0)\). Graph the line \( x + 2y = 1 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0,0)\). Since \(0 + 2(0) \geq 1\) is false, shade the opposite side of the line from \((0,0)\). The overlapping region is the solution. The graph is bounded.

Find the vertices:
The intersection of \( x + y = 2 \) and the \( y \)-axis is \((0,2)\). The intersection of \( x + y = 2 \) and the \( x \)-axis is \((2,0)\). The intersection of \( x + y = 8 \) and the \( y \)-axis is \((0,8)\). The intersection of \( x + y = 8 \) and the \( x \)-axis is \((8,0)\). The four corner points are \((0,2), (0,8), (2,0), \) and \((8,0)\).
51. \[ \begin{cases} x \geq 0 \\ y \geq 0 \\ x + 2y \geq 1 \\ x + 2y \leq 10 \end{cases} \]

Graph \( x \geq 0; \ y \geq 0 \). Shaded region is the first quadrant. Graph the line \( x + 2y = 1 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \(0 + 2(0) \geq 1\) is false, shade the opposite side of the line from \((0, 0)\). Graph the line \( x + 2y = 10 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \(0 + 2(0) \leq 10\) is true, shade the side of the line containing \((0, 0)\). The overlapping region is the solution. The graph is bounded.

Find the vertices:
The intersection of \( x + 2y = 1 \) and the \( y \)-axis is \((0, 0.5)\). The intersection of \( x + 2y = 1 \) and the \( x \)-axis is \((1, 0)\). The intersection of \( x + 2y = 10 \) and the \( y \)-axis is \((0, 5)\). The intersection of \( x + 2y = 10 \) and the \( x \)-axis is \((10, 0)\). The four corner points are \((0, 0.5), (0, 5), (1, 0), and (10, 0)\).

52. \[ \begin{cases} x \geq 0 \\ y \geq 0 \\ x + 2y \geq 1 \\ x + 2y \leq 10 \\ x + y \geq 2 \\ x + y \leq 8 \end{cases} \]

Graph \( x \geq 0; \ y \geq 0 \). Shaded region is the first quadrant. Graph the line \( x + 2y = 1 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \(0 + 2(0) \geq 1\) is false, shade the opposite side of the line from \((0, 0)\). Graph the line \( x + 2y = 10 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \(0 + 2(0) \leq 10\) is true, shade the side of the line containing \((0, 0)\). The overlapping region is the solution. The graph is bounded.

Find the vertices:
The intersection of \( x + y = 2 \) and the \( y \)-axis is \((0, 2)\). The intersection of \( x + y = 2 \) and the \( x \)-axis is \((2, 0)\). The intersection of \( x + 2y = 10 \) and the \( y \)-axis is \((0, 5)\). The intersection of \( x + y = 8 \) and the \( x \)-axis is \((8, 0)\). To find the intersection of \( x + y = 8 \) and \( x + 2y = 10 \), solve the system:

\[
\begin{aligned}
&x + y = 8 \\
&x + 2y = 10
\end{aligned}
\]

Solve the first equation for \( x \): \( x = 8 - y \).
Substitute and solve:

\[
\begin{aligned}
(8 - y) + 2y &= 10 \\
y &= 2 \\
x &= 8 - 2 = 6
\end{aligned}
\]

The point of intersection is \((6, 2)\).
The five corner points are \((0, 2), (0, 5), (2, 0), (8, 0), and (6, 2)\).

53. The system of linear inequalities is:

\[
\begin{cases}
x \leq 4 \\
x + y \leq 6 \\
x \geq 0 \\
y \geq 0
\end{cases}
\]
54. The system of linear inequalities is:
\[
\begin{align*}
  y &\leq 5 \\
  x + y &\geq 2 \\
  x &\leq 6 \\
  x &\geq 0 \\
  y &\geq 0 
\end{align*}
\]

55. The system of linear inequalities is:
\[
\begin{align*}
  x &\leq 20 \\
  y &\geq 15 \\
  x + y &\leq 50 \\
  x - y &\leq 0 \\
  x &\geq 0 
\end{align*}
\]

56. The system of linear inequalities is:
\[
\begin{align*}
  y &\leq 6 \\
  x &\leq 5 \\
  3x + 4y &\geq 12 \\
  2x - y &\leq 8 \\
  x &\geq 0 
\end{align*}
\]

57. a. Let \( x \) = the amount invested in Treasury bills, and let \( y \) = the amount invested in corporate bonds.
The constraints are:
\( x + y \leq 50,000 \) because the total investment cannot exceed $50,000.
\( x \geq 35,000 \) because the amount invested in Treasury bills must be at least $35,000.
\( y \leq 10,000 \) because the amount invested in corporate bonds must not exceed $10,000.
\( x \geq 0, y \geq 0 \) because a non-negative amount must be invested.
The system is
\[
\begin{align*}
  x + y &\leq 50,000 \\
  x &\geq 35,000 \\
  y &\leq 10,000 \\
  x &\geq 0 \\
  y &\geq 0 
\end{align*}
\]

b. Graph the system.

58. a. Let \( x \) = the # of standard model trucks, and let \( y \) = the # of deluxe model trucks.
The constraints are:
\( x \geq 0, y \geq 0 \) because a non-negative number of trucks must be manufactured.
\( 2x + 3y \leq 80 \) because the total painting hours worked cannot exceed 80.
\( 3x + 4y \leq 120 \) because the total detailing hours worked cannot exceed 120.
The system is
\[
\begin{align*}
  x \geq 0 \\
  y \geq 0 \\
  2x + 3y &\leq 80 \\
  3x + 4y &\leq 120 
\end{align*}
\]

b. Graph the system.

The corner points are \((35,000, 0), (35,000, 10,000), (40,000, 10,000), (50,000, 0)\).
59. a. Let \( x \) = the # of packages of the economy blend, and let \( y \) = the # of packages of the superior blend. The constraints are:
\[
\begin{align*}
x &\geq 0, \quad y \geq 0 \quad \text{because a non-negative # of packages must be produced.} \\
4x + 8y &\leq 75 \cdot 16 \quad \text{because the total amount of “A grade” coffee cannot exceed 75 pounds. (Note: 75 pounds = (75)(16) ounces.)} \\
12x + 8y &\leq 120 \cdot 16 \quad \text{because the total amount of “B grade” coffee cannot exceed 120 pounds. (Note: 120 pounds = (120)(16) ounces.)}
\end{align*}
\]
Simplifying the inequalities, we obtain:
\[
\begin{align*}
x + 2y &\leq 75 - 4 \\
x + 2y &\leq 300
\end{align*}
\]
The system is:
\[
\begin{align*}
x &\geq 0 \\
y &\geq 0 \\
x + 2y &\leq 300 \\
3x + 2y &\leq 480
\end{align*}
\]
b. Graph the system.

60. a. Let \( x \) = the # of lower-priced packages, and let \( y \) = the # of quality packages. The constraints are:
\[
\begin{align*}
x &\geq 0, \quad y \geq 0 \quad \text{because a non-negative # of packages must be produced.} \\
8x + 6y &\leq 120 \cdot 16 \quad \text{because the total amount of peanuts cannot exceed 120 pounds. (Note: 120 pounds = (120)(16) ounces.)} \\
4x + 6y &\leq 90 \cdot 16 \quad \text{because the total amount of cashews cannot exceed 90 pounds. (Note: 90 pounds = (90)(16) ounces.)}
\end{align*}
\]
Simplifying the inequalities, we obtain:
\[
\begin{align*}
x + 2y &\leq 160 \\
3x + 2y &\leq 300
\end{align*}
\]
The corner points are (0, 0), (0, 150), (90, 105), (160, 0).

61. a. Let \( x \) = the # of microwaves, and let \( y \) = the # of printers. The constraints are:
\[
\begin{align*}
x &\geq 0, \quad y \geq 0 \quad \text{because a non-negative # of items must be shipped.} \\
30x + 20y &\leq 1600 \quad \text{because a total cargo weight cannot exceed 1600 pounds.} \\
2x + 3y &\leq 150 \quad \text{because the total cargo volume cannot exceed 150 cubic feet. Note that the inequality } 30x + 20y \leq 1600 \text{ can be simplified: } 3x + 2y \leq 160.
\end{align*}
\]
The system is:
\[
\begin{align*}
3x + 2y &\leq 160 \\
x + 3y &\leq 150 \\
x &\geq 0, \quad y \geq 0
\end{align*}
\]
b. Graph the system.
Chapter 11: Systems of Equations and Inequalities

Section 11.8

1. objective function

2. True

3. \( z = x + y \)

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = x + y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 3)</td>
<td>( z = 0 + 3 = 3 )</td>
</tr>
<tr>
<td>(0, 6)</td>
<td>( z = 0 + 6 = 6 )</td>
</tr>
<tr>
<td>(5, 6)</td>
<td>( z = 5 + 6 = 11 )</td>
</tr>
<tr>
<td>(5, 2)</td>
<td>( z = 5 + 2 = 7 )</td>
</tr>
<tr>
<td>(4, 0)</td>
<td>( z = 4 + 0 = 4 )</td>
</tr>
</tbody>
</table>

The maximum value is 11 at (5, 6), and the minimum value is 3 at (0, 3).

4. \( z = 2x + 3y \)

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 2x + 3y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 3)</td>
<td>( z = 2(0) + 3(3) = 9 )</td>
</tr>
<tr>
<td>(0, 6)</td>
<td>( z = 2(0) + 3(6) = 18 )</td>
</tr>
<tr>
<td>(5, 6)</td>
<td>( z = 2(5) + 3(6) = 28 )</td>
</tr>
<tr>
<td>(5, 2)</td>
<td>( z = 2(5) + 3(2) = 16 )</td>
</tr>
<tr>
<td>(4, 0)</td>
<td>( z = 2(4) + 3(0) = 8 )</td>
</tr>
</tbody>
</table>

The maximum value is 28 at (5, 6), and the minimum value is 8 at (4, 0).

5. \( z = x + 10y \)

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = x + 10y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 3)</td>
<td>( z = 0 + 10(3) = 30 )</td>
</tr>
<tr>
<td>(0, 6)</td>
<td>( z = 0 + 10(6) = 60 )</td>
</tr>
<tr>
<td>(5, 6)</td>
<td>( z = 5 + 10(6) = 65 )</td>
</tr>
<tr>
<td>(5, 2)</td>
<td>( z = 5 + 10(2) = 25 )</td>
</tr>
<tr>
<td>(4, 0)</td>
<td>( z = 4 + 10(0) = 4 )</td>
</tr>
</tbody>
</table>

The maximum value is 65 at (5, 6), and the minimum value is 4 at (4, 0).

6. \( z = 10x + y \)

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 10x + y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 3)</td>
<td>( z = 10(0) + 3 = 3 )</td>
</tr>
<tr>
<td>(0, 6)</td>
<td>( z = 10(0) + 6 = 6 )</td>
</tr>
<tr>
<td>(5, 6)</td>
<td>( z = 10(5) + 6 = 56 )</td>
</tr>
<tr>
<td>(5, 2)</td>
<td>( z = 10(5) + 2 = 52 )</td>
</tr>
<tr>
<td>(4, 0)</td>
<td>( z = 10(4) + 0 = 40 )</td>
</tr>
</tbody>
</table>

The maximum value is 56 at (5, 6), and the minimum value is 3 at (0, 3).

7. \( z = 5x + 7y \)

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 5x + 7y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 3)</td>
<td>( z = 5(0) + 7(3) = 21 )</td>
</tr>
<tr>
<td>(0, 6)</td>
<td>( z = 5(0) + 7(6) = 42 )</td>
</tr>
<tr>
<td>(5, 6)</td>
<td>( z = 5(5) + 7(6) = 67 )</td>
</tr>
<tr>
<td>(5, 2)</td>
<td>( z = 5(5) + 7(2) = 39 )</td>
</tr>
<tr>
<td>(4, 0)</td>
<td>( z = 5(4) + 7(0) = 20 )</td>
</tr>
</tbody>
</table>

The maximum value is 67 at (5, 6), and the minimum value is 20 at (4, 0).

8. \( z = 7x + 5y \)

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 7x + 5y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 3)</td>
<td>( z = 7(0) + 5(3) = 15 )</td>
</tr>
<tr>
<td>(0, 6)</td>
<td>( z = 7(0) + 5(6) = 30 )</td>
</tr>
<tr>
<td>(5, 6)</td>
<td>( z = 7(5) + 5(6) = 65 )</td>
</tr>
<tr>
<td>(5, 2)</td>
<td>( z = 7(5) + 5(2) = 45 )</td>
</tr>
<tr>
<td>(4, 0)</td>
<td>( z = 7(4) + 5(0) = 28 )</td>
</tr>
</tbody>
</table>

The maximum value is 65 at (5, 6), and the minimum value is 15 at (0, 3).

9. Maximize \( z = 2x + y \) subject to \( x \geq 0, y \geq 0, x + y \leq 6, x + y \geq 1 \).

Graph the constraints.

The corner points are (0, 1), (1, 0), (0, 6), (6, 0).

Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 2x + y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 1)</td>
<td>( z = 2(0) + 1 = 1 )</td>
</tr>
<tr>
<td>(0, 6)</td>
<td>( z = 2(0) + 6 = 6 )</td>
</tr>
<tr>
<td>(1, 0)</td>
<td>( z = 2(1) + 0 = 2 )</td>
</tr>
<tr>
<td>(6, 0)</td>
<td>( z = 2(6) + 0 = 12 )</td>
</tr>
</tbody>
</table>

The maximum value is 12 at (6, 0).
10. Maximize \( z = x + 3y \) subject to \( x \geq 0, \ y \geq 0, \ x + y \geq 3, \ x \leq 5, \ y \leq 7 \). Graph the constraints.

The corner points are (0, 3), (3, 0), (0, 7), (5, 0), (5, 7). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = x + 3y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 3)</td>
<td>( z = 0 + 3(3) = 9 )</td>
</tr>
<tr>
<td>(3, 0)</td>
<td>( z = 3 + 3(0) = 3 )</td>
</tr>
<tr>
<td>(0, 7)</td>
<td>( z = 0 + 3(7) = 21 )</td>
</tr>
<tr>
<td>(5, 0)</td>
<td>( z = 5 + 3(0) = 5 )</td>
</tr>
<tr>
<td>(5, 7)</td>
<td>( z = 5 + 3(7) = 26 )</td>
</tr>
</tbody>
</table>

The maximum value is 26 at (5, 7).

11. Minimize \( z = 2x + 5y \) subject to \( x \geq 0, \ y \geq 0, \ x + y \geq 2, \ x \leq 5, \ y \leq 3 \). Graph the constraints.

The corner points are (0, 2), (2, 0), (0, 3), (5, 0), (5, 3). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 2x + 5y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 2)</td>
<td>( z = 2(0) + 5(2) = 10 )</td>
</tr>
<tr>
<td>(0, 3)</td>
<td>( z = 2(0) + 5(3) = 15 )</td>
</tr>
<tr>
<td>(2, 0)</td>
<td>( z = 2(2) + 5(0) = 4 )</td>
</tr>
<tr>
<td>(5, 0)</td>
<td>( z = 2(5) + 5(0) = 10 )</td>
</tr>
<tr>
<td>(5, 3)</td>
<td>( z = 2(5) + 5(3) = 25 )</td>
</tr>
</tbody>
</table>

The minimum value is 4 at (2, 0).

12. Minimize \( z = 3x + 4y \) subject to \( x \geq 0, \ y \geq 0, \ 2x + 3y \geq 6, \ x + y \leq 8 \). Graph the constraints.

The corner points are (0, 2), (3, 0), (0, 8), (8, 0). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 3x + 4y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 2)</td>
<td>( z = 3(0) + 4(2) = 8 )</td>
</tr>
<tr>
<td>(3, 0)</td>
<td>( z = 3(3) + 4(0) = 9 )</td>
</tr>
<tr>
<td>(0, 8)</td>
<td>( z = 3(0) + 4(8) = 32 )</td>
</tr>
<tr>
<td>(8, 0)</td>
<td>( z = 3(8) + 4(0) = 24 )</td>
</tr>
</tbody>
</table>

The minimum value is 8 at (0, 2).

13. Maximize \( z = 3x + 5y \) subject to \( x \geq 0, \ y \geq 0, \ x + y \geq 2, \ 2x + 3y \leq 12, \ 3x + 2y \leq 12 \). Graph the constraints.

To find the intersection of \( 2x + 3y = 12 \) and \( 3x + 2y = 12 \), solve the system:

\[
\begin{align*}
2x + 3y &= 12 \\
3x + 2y &= 12
\end{align*}
\]

Solve the second equation for \( y \): \( y = 6 - \frac{3}{2}x \)

Substitute and solve:
Chapter 11: Systems of Equations and Inequalities

14. Maximize \( z = 5x + 3y \) subject to \( x \geq 0, \ y \geq 0, \ x + y \geq 2, \ x + y \leq 8, \ 2x + y \leq 10 \). Graph the constraints.

To find the intersection of \( x + y = 8 \) and \( 2x + y = 10 \), solve the system:

\[
\begin{align*}
2x + y &= 10 \\
x + y &= 8
\end{align*}
\]

Solve the first equation for \( y \):

\( y = 8 - x \).

Substitute and solve:

\( 2x + (8 - x) = 10 \)

\( x = 2 \)

\( y = 8 - 2 = 6 \)

The point of intersection is \((2, 6)\).

The corner points are \((0, 2), (2, 0), (0, 8), (5, 0), (2, 6)\). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 5x + 3y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 2))</td>
<td>( z = 5(0) + 3(2) = 10 )</td>
</tr>
<tr>
<td>((0, 4))</td>
<td>( z = 5(0) + 3(4) = 16 )</td>
</tr>
<tr>
<td>((2, 0))</td>
<td>( z = 5(2) + 3(0) = 10 )</td>
</tr>
<tr>
<td>((5, 0))</td>
<td>( z = 5(5) + 3(0) = 25 )</td>
</tr>
<tr>
<td>((2, 6))</td>
<td>( z = 5(2) + 3(6) = 28 )</td>
</tr>
</tbody>
</table>

The maximum value is 28 at \((2, 6)\).

15. Minimize \( z = 5x + 4y \) subject to \( x \geq 0, \ y \geq 0, \ x + y \geq 2, \ 2x + 3y \leq 12, \ 3x + y \leq 12 \). Graph the constraints.

To find the intersection of \( 2x + 3y = 12 \) and \( 3x + y = 12 \), solve the system:

\[
\begin{align*}
2x + 3y &= 12 \\
3x + y &= 12
\end{align*}
\]

Solve the second equation for \( y \):

\( y = 12 - 3x \)

Substitute and solve:

\( 2x + 3(12 - 3x) = 12 \)

\( 2x + 36 - 9x = 12 \)

\( -7x = -24 \)

\( x = \frac{24}{7} \)

\( y = 12 - 3\left(\frac{24}{7}\right) = 12 - \frac{72}{7} = \frac{12}{7} \)

The point of intersection is \( \left(\frac{24}{7}, \frac{12}{7}\right) \).

The corner points are \((0, 2), (2, 0), (0, 4), (4, 0), \left(\frac{24}{7}, \frac{12}{7}\right)\). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 5x + 4y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 2))</td>
<td>( z = 5(0) + 4(2) = 8 )</td>
</tr>
<tr>
<td>((0, 4))</td>
<td>( z = 5(0) + 4(4) = 16 )</td>
</tr>
<tr>
<td>((2, 0))</td>
<td>( z = 5(2) + 4(0) = 10 )</td>
</tr>
<tr>
<td>((4, 0))</td>
<td>( z = 5(4) + 4(0) = 20 )</td>
</tr>
<tr>
<td>(\left(\frac{24}{7}, \frac{12}{7}\right))</td>
<td>( z = 5\left(\frac{24}{7}\right) + 4\left(\frac{12}{7}\right) = 24 )</td>
</tr>
</tbody>
</table>

The minimum value is 8 at \((0, 2)\).
16. Minimize \( z = 2x + 3y \) subject to \( x \geq 0 \), 
\( y \geq 0 \), \( x + y \geq 3 \), \( x + y \leq 9 \), \( x + 3y \geq 6 \). 
Graph the constraints.

To find the intersection of \( x + y = 3 \) and \( x + 3y = 6 \), solve the system:

\[
\begin{align*}
\begin{cases}
  x + y &= 3 \\
  x + 3y &= 6
\end{cases}
\end{align*}
\]

Solve the first equation for \( y \): \( y = 3 - x \).

Substitute and solve:

\[
\begin{align*}
  x + 3(3 - x) &= 6 \\
  9 - 3x &= 6 \\
  -3x &= -3 \\
  x &= \frac{3}{2}
\end{align*}
\]

\[
y = 3 - \frac{3}{2} = \frac{3}{2}
\]

The point of intersection is \( \left( \frac{3}{2}, \frac{3}{2} \right) \).

The corner points are \( (0, 3), (6, 0), (0, 9), (9, 0), \)
\( \left( \frac{3}{2}, \frac{3}{2} \right) \). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 2x + 3y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (0, 3) )</td>
<td>( z = 2(0) + 3(3) = 9 )</td>
</tr>
<tr>
<td>( (0, 9) )</td>
<td>( z = 2(0) + 3(9) = 27 )</td>
</tr>
<tr>
<td>( (6, 0) )</td>
<td>( z = 2(6) + 3(0) = 12 )</td>
</tr>
<tr>
<td>( (9, 0) )</td>
<td>( z = 2(9) + 3(0) = 18 )</td>
</tr>
<tr>
<td>( \left( \frac{3}{2}, \frac{3}{2} \right) )</td>
<td>( z = 2 \left( \frac{3}{2} \right) + 3 \left( \frac{3}{2} \right) = \frac{15}{2} )</td>
</tr>
</tbody>
</table>

The minimum value is \( \frac{15}{2} \) at \( \left( \frac{3}{2}, \frac{3}{2} \right) \).

17. Maximize \( z = 5x + 2y \) subject to \( x \geq 0 \), 
\( y \geq 0 \), \( x + y \leq 10 \), \( 2x + y \geq 10 \), \( x + 2y \geq 10 \).
Graph the constraints.

To find the intersection of \( 2x + y = 10 \) and \( x + 2y = 10 \), solve the system:

\[
\begin{align*}
\begin{cases}
  2x + y &= 10 \\
  x + 2y &= 10
\end{cases}
\end{align*}
\]

Solve the first equation for \( y \): \( y = 10 - 2x \).

Substitute and solve:

\[
\begin{align*}
  2x + (10 - 2x) &= 10 \\
  20 - 4x &= 10 \\
  -4x &= -10 \\
  x &= \frac{10}{3}
\end{align*}
\]

\[
y = 10 - 2 \left( \frac{10}{3} \right) = 10 - \frac{20}{3} = \frac{10}{3}
\]

The point of intersection is \( (\frac{10}{3}, \frac{10}{3}) \).

The corner points are \( (0, 10), (10, 0), (10/3, 10/3) \). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 5x + 2y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (0, 10) )</td>
<td>( z = 5(0) + 2(10) = 20 )</td>
</tr>
<tr>
<td>( (10, 0) )</td>
<td>( z = 5(10) + 2(0) = 50 )</td>
</tr>
<tr>
<td>( \left( \frac{10}{3}, \frac{10}{3} \right) )</td>
<td>( z = 5 \left( \frac{10}{3} \right) + 2 \left( \frac{10}{3} \right) = \frac{70}{3} = 23 \frac{1}{3} )</td>
</tr>
</tbody>
</table>

The maximum value is 50 at \( (10, 0) \).
18. Maximize \( z = 2x + 4y \) subject to 
\[ \begin{align*} 
x & \geq 0, \quad y \geq 0, \quad 2x + y \geq 4, \quad x + y \leq 9. 
\end{align*} \]
Graph the constraints.

The corner points are (0, 9), (9, 0), (0, 4), (2, 0). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( z = 2x + 4y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 9)</td>
<td>( z = 2(0) + 4(9) = 36 )</td>
</tr>
<tr>
<td>(9, 0)</td>
<td>( z = 2(9) + 4(0) = 18 )</td>
</tr>
<tr>
<td>(0, 4)</td>
<td>( z = 2(0) + 4(4) = 16 )</td>
</tr>
<tr>
<td>(2, 0)</td>
<td>( z = 2(2) + 4(0) = 4 )</td>
</tr>
</tbody>
</table>

The maximum value is 36 at (0, 9).

19. Let \( x \) be the number of downhill skis produced, and let \( y \) be the number of cross-country skis produced. The total profit is: \( P = 70x + 50y \).

Profit is to be maximized, so this is the objective function. The constraints are:
\[ \begin{align*} 
x & \geq 0, \quad y \geq 0 \quad \text{A positive number of skis must be produced.} 
2x + y & \leq 48 \quad \text{Manufacturing time available.} 
x + y & \leq 32 \quad \text{Finishing time available.} 
\end{align*} \]

Graph the constraints.

To find the intersection of \( x + y = 32 \) and \( 2x + y = 48 \), solve the system:
\[ \begin{align*} 
x + y & = 32 
2x + y & = 48 
\end{align*} \]

Solve the first equation for \( y \): \( y = 32 - x \).

Substitute and solve:
\[ \begin{align*} 
x & = 8 
y & = 32 - 8 = 24 
\end{align*} \]
The point of intersection is (8, 24).
The corner points are (0, 0), (0, 32), (20, 0), (8, 24). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( P = 70x + 50y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>( P = 70(0) + 50(0) = 0 )</td>
</tr>
<tr>
<td>(0, 32)</td>
<td>( P = 70(0) + 50(32) = 1600 )</td>
</tr>
<tr>
<td>(20, 0)</td>
<td>( P = 70(20) + 50(0) = 1400 )</td>
</tr>
<tr>
<td>(8, 24)</td>
<td>( P = 70(8) + 50(24) = 1760 )</td>
</tr>
</tbody>
</table>

The maximum profit is $1760, when 8 downhill skis and 24 cross-country skis are produced.

With the increase of the manufacturing time to 48 hours, we do the following:
The constraints are:
\[ \begin{align*} 
x & \geq 0, \quad y \geq 0 \quad \text{A positive number of skis must be produced.} 
2x + y & \leq 48 \quad \text{Manufacturing time available.} 
x + y & \leq 32 \quad \text{Finishing time available.} 
\end{align*} \]

Graph the constraints.

To find the intersection of \( x + y = 32 \) and \( 2x + y = 48 \), solve the system:
\[ \begin{align*} 
x + y & = 32 
2x + y & = 48 
\end{align*} \]

Solve the first equation for \( y \): \( y = 32 - x \).

Substitute and solve:
\[ \begin{align*} 
x & = 16 
y & = 32 - 16 = 16 
\end{align*} \]
The point of intersection is (16, 16).
The corner points are (0, 0), (0, 32), (24, 0), (16, 16). Evaluate the objective function:
Section 11.8: Linear Programming

The maximum profit is $1920, when 16 downhill skis and 16 cross-country skis are produced.

20. Let $x$ = the number of acres of soybeans planted, and let $y$ = the number of acres of wheat planted. The total profit is: $P = 180x + 100y$. Profit is to be maximized, so this is the objective function. The constraints are:

$x \geq 0$, $y \geq 0$  A non-negative number of acres must be planted.

$x + y \leq 70$  Acres available to plant.

$60x + 30y \leq 1800$  Money available for preparation.

$3x + 4y \leq 120$  Workdays available.

Graph the constraints.

To find the intersection of $60x + 30y = 1800$ and $3x + 4y = 120$, solve the system:

\[
\begin{align*}
60x + 30y &= 1800 \\
3x + 4y &= 120
\end{align*}
\]

Solve the first equation for $y$:

$60x + 30y = 1800$

$2x + y = 60$

$y = 60 - 2x$

Substitute and solve:

$3x + 4(60 - 2x) = 120$

$3x + 240 - 8x = 120$

$-5x = -120$

$x = 24$

$y = 60 - 2(24) = 12$

The point of intersection is (24, 12). The corner points are (0, 0), (0, 30), (30, 0), (24, 12). Evaluate the objective function:

Vertex | Value of $P = 180x + 100y$
--- | ---
(0, 0) | $P = 180(0) + 100(0) = 0$
(0, 30) | $P = 180(0) + 100(30) = 3000$
(30, 0) | $P = 180(30) + 100(0) = 5400$
(24, 12) | $P = 180(24) + 100(12) = 5520$

The maximum profit is $5520, when 24 acres of soybeans and 12 acres of wheat are planted.

With the increase of the preparation costs to $2400, we do the following: The constraints are:

$x \geq 0$, $y \geq 0$  A non-negative number of acres must be planted.

$x + y \leq 70$  Acres available to plant.

$60x + 30y \leq 2400$  Money available for preparation.

$3x + 4y \leq 120$  Workdays available.

Graph the constraints.

The corner points are (0, 0), (0, 30), (40, 0). Evaluate the objective function:

Vertex | Value of $P = 180x + 100y$
--- | ---
(0, 0) | $P = 180(0) + 100(0) = 0$
(0, 30) | $P = 180(0) + 100(30) = 3000$
(40, 0) | $P = 180(40) + 100(0) = 7200$

The maximum profit is $7200, when 40 acres of soybeans and 0 acres of wheat are planted.

21. Let $x$ = the number of rectangular tables rented, and let $y$ = the number of round tables rented. The cost for the tables is: $C = 28x + 52y$. Cost is to be minimized, so this is the objective function. The constraints are:

$x \geq 0$, $y \geq 0$  A non-negative number of tables must be used.

$x + y \leq 35$  Maximum number of tables.

$6x + 10y \geq 250$  Number of guests.

$x \leq 15$  Rectangular tables available.
Chapter 11: Systems of Equations and Inequalities

Graph the constraints.

The corner points are (0, 25), (0, 35), (15, 20), (15, 16). Evaluate the objective function:

Kathleen should rent 15 rectangular tables and 16 round tables in order to minimize the cost. The minimum cost is $1252.00.

22. Let \( x = \) the number of buses rented, and let \( y = \) the number of vans rented. The cost for the vehicles is: \( C = 975x + 350y \). Cost is to be minimized, so this is the objective function. The constraints are:

- \( x \geq 0, \ y \geq 0 \) A non-negative number of buses and vans must be used.
- \( 40x + 8y \geq 320 \) Number of regular seats.
- \( x + 3y \geq 36 \) Number of handicapped seats.

Graph the constraints.

To find the intersection of \( 40x + 8y = 320 \) and \( x + 3y = 36 \), solve the system:

\[
\begin{align*}
40x + 8y &= 320 \\
x + 3y &= 36
\end{align*}
\]

Solve the second equation for \( x \):

\[x = -3y + 36\]

Substitute and solve:

\[
\begin{align*}
40(-3y + 36) + 8y &= 320 \\
-120y + 1440 + 8y &= 320 \\
-112y &= -1120 \\
y &= 10
\end{align*}
\]

\( x = -3(10) + 36 = -30 + 36 = 6 \)

The point of intersection is (6, 10). The corner points are (0, 40), (6, 10), and (36, 0). Evaluate the objective function:

The college should rent 6 buses and 10 vans for a minimum cost of $9350.00.

23. Let \( x = \) the amount invested in junk bonds, and let \( y = \) the amount invested in Treasury bills. The total income is: \( I = 0.09x + 0.07y \). Income is to be maximized, so this is the objective function. The constraints are:

- \( x \geq 0, \ y \geq 0 \) A non-negative amount must be invested.
- \( x + y \leq 20,000 \) Total investment cannot exceed $20,000.
- \( x \leq 12,000 \) Amount invested in junk bonds must not exceed $12,000.
- \( y \geq 8000 \) Amount invested in Treasury bills must be at least $8000.

a. \( y \geq x \) Amount invested in Treasury bills must be equal to or greater than the amount invested in junk bonds.

Graph the constraints.

The college should invest \( 12,000 \) in junk bonds and \( 8000 \) in Treasury bills for a maximum income of $2070.00.
The corner points are (0, 20,000), (0, 8000), (8000, 8000), (10,000, 10,000).

Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of $I = 0.09x + 0.07y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 20000)</td>
<td>$I = 0.09(0) + 0.07(20000)$ = 1400</td>
</tr>
<tr>
<td>(0, 8000)</td>
<td>$I = 0.09(0) + 0.07(8000)$ = 560</td>
</tr>
<tr>
<td>(8000, 8000)</td>
<td>$I = 0.09(8000) + 0.07(8000)$ = 1280</td>
</tr>
<tr>
<td>(10000, 10000)</td>
<td>$I = 0.09(10000) + 0.07(10000)$ = 1600</td>
</tr>
</tbody>
</table>

The maximum income is $1600, when $10,000 is invested in junk bonds and $10,000 is invested in Treasury bills.

b. $y \leq x$ Amount invested in Treasury bills must not exceed the amount invested in junk bonds.

Graph the constraints.

The corner points are (12,000, 8000), (8000, 8000), (10,000, 10,000).

Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of $I = 0.09x + 0.07y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(12000, 8000)</td>
<td>$I = 0.09(12000) + 0.07(8000)$ = 1640</td>
</tr>
<tr>
<td>(8000, 8000)</td>
<td>$I = 0.09(8000) + 0.07(8000)$ = 1280</td>
</tr>
<tr>
<td>(10000, 10000)</td>
<td>$I = 0.09(10000) + 0.07(10000)$ = 1600</td>
</tr>
</tbody>
</table>

The maximum income is $1640, when $12,000 is invested in junk bonds and $8000 is invested in Treasury bills.

24. Let $x$ = the number of hours that machine 1 is operated, and let $y$ = the number of hours that machine 2 is operated. The total cost is: $C = 50x + 30y$. Cost is to be minimized, so this is the objective function.

The constraints are:

$x \geq 0$, $y \geq 0$  A positive number of hours must be used.

$x \leq 10$  Time used on machine 1.

$y \leq 10$  Time used on machine 2.

$60x + 40y \geq 240$  8-inch pliers to be produced.

$70x + 20y \geq 140$  6-inch pliers to be produced.

Graph the constraints.

To find the intersection of $60x + 40y = 240$ and $70x + 20y = 140$, solve the system:

\[
\begin{align*}
60x + 40y & = 240 \\
70x + 20y & = 140
\end{align*}
\]

Divide the first equation by $2$ and add the result to the second equation:

\[
\begin{align*}
30x - 20y & = -120 \\
70x + 20y & = 140
\end{align*}
\]

Divide the first equation by $2$ and add the result to the second equation:

\[
\begin{align*}
40x & = 20 \\
x & = \frac{20}{40} = \frac{1}{2}
\end{align*}
\]

Substitute and solve:

\[
\begin{align*}
60\left(\frac{1}{2}\right) + 40y & = 240 \\
30 + 40y & = 240 \\
40y & = 210 \\
y & = \frac{210}{40} = \frac{21}{4} = \frac{5.25}{4}
\end{align*}
\]

The point of intersection is $\left(\frac{1}{2}, \frac{5.25}{4}\right)$. 

1205

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Chapter 11: Systems of Equations and Inequalities

The corner points are (0, 7), (0, 10), (4, 0), (10, 0), (10, 10), \( \left( \frac{1}{2}, \frac{5}{4} \right) \). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( C = 50x + 30y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 7)</td>
<td>( C = 50(0) + 30(7) = 210 )</td>
</tr>
<tr>
<td>(0, 10)</td>
<td>( C = 50(0) + 30(10) = 300 )</td>
</tr>
<tr>
<td>(4, 0)</td>
<td>( C = 50(4) + 30(0) = 200 )</td>
</tr>
<tr>
<td>(10, 0)</td>
<td>( C = 50(10) + 30(0) = 500 )</td>
</tr>
<tr>
<td>(10, 10)</td>
<td>( C = 50(10) + 30(10) = 800 )</td>
</tr>
<tr>
<td>( \left( \frac{1}{2}, \frac{5}{4} \right) )</td>
<td>( P = 50 \left( \frac{1}{2} \right) + 30 \left( \frac{5}{4} \right) = 182.50 )</td>
</tr>
</tbody>
</table>

The minimum cost is $182.50, when machine 1 is used for \( \frac{1}{2} \) hour and machine 2 is used for \( \frac{5}{4} \) hours.

25. Let \( x \) = the number of pounds of ground beef, and let \( y \) = the number of pounds of ground pork. The total cost is: \( C = 0.75x + 0.45y \). Cost is to be minimized, so this is the objective function. The constraints are:

- \( x \geq 0, \ y \geq 0 \)  A positive number of pounds must be used.
- \( x \leq 200 \)  Only 200 pounds of ground beef are available.
- \( y \geq 50 \)  At least 50 pounds of ground pork must be used.
- \( 0.75x + 0.60y \geq 0.70(x + y) \)  Leanness condition

(Note that the last equation will simplify to \( y \leq \frac{1}{2}x \).) Graph the constraints.

The corner points are (100, 50), (200, 50), (200, 100). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( C = 0.75x + 0.45y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(100, 50)</td>
<td>( C = 0.75(100) + 0.45(50) = 97.50 )</td>
</tr>
<tr>
<td>(200, 50)</td>
<td>( C = 0.75(200) + 0.45(50) = 172.50 )</td>
</tr>
<tr>
<td>(200, 100)</td>
<td>( C = 0.75(200) + 0.45(100) = 195 )</td>
</tr>
</tbody>
</table>

The minimum cost is $97.50, when 100 pounds of ground beef and 50 pounds of ground pork are used.

26. Let \( x \) = the number of gallons of regular, and let \( y \) = the number of gallons of premium. The total profit is: \( P = 0.75x + 0.90y \). Profit is to be maximized, so this is the objective function. The constraints are:

- \( x \geq 0, \ y \geq 0 \)  A positive number of gallons must be used.
- \( y \geq \frac{1}{4}x \)  At least one gallon of premium for every 4 gallons of regular.
- \( 5x + 6y \leq 3000 \)  Daily shipping weight limit.
- \( 24x + 20y \leq 16(725) \)  Available flavoring.
- \( 12x + 20y \leq 16(425) \)  Available milk-fat

(Note: the last two inequalities simplify to \( 6x + 5y \leq 2900 \) and \( 3x + 5y \leq 1700 \).) Graph the constraints.

The corner points are (0, 0), (400, 100), (0, 340). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( P = 0.75x + 0.90y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>( P = 0.75(0) + 0.90(0) = 0 )</td>
</tr>
<tr>
<td>(400, 100)</td>
<td>( P = 0.75(400) + 0.90(100) = 390 )</td>
</tr>
<tr>
<td>(0, 340)</td>
<td>( P = 0.75(0) + 0.90(340) = 306 )</td>
</tr>
</tbody>
</table>

Mom and Pop should produce 400 gallons of regular and 100 gallons of premium ice cream. The maximum profit is $390.00.
27. Let \( x \) = the number of racing skates manufactured, and let \( y \) = the number of figure skates manufactured. The total profit is:
\[ P = 10x + 12y \] Profit is to be maximized, so this is the objective function. The constraints are:
\[ x \geq 0, \quad y \geq 0 \quad \text{A positive number of skates must be manufactured.} \]
\[ 6x + 4y \leq 120 \quad \text{Only 120 hours are available for fabrication.} \]
\[ x + 2y \leq 40 \quad \text{Only 40 hours are available for finishing.} \]
Graph the constraints.

To find the intersection of \( 6x + 4y = 120 \) and \( x + 2y = 40 \), solve the system:
\[
\begin{align*}
6x + 4y &= 120 \\
x + 2y &= 40
\end{align*}
\]
Solve the second equation for \( x \):
\[ x = 40 - 2y \]
Substitute and solve:
\[ 6(40 - 2y) + 4y = 120 \]
\[ 240 - 12y + 4y = 120 \]
\[ -8y = -120 \]
\[ y = 15 \]
\[ x = 40 - 2(15) = 10 \]
The point of intersection is (10, 15).
The corner points are (0, 0), (0, 20), (20, 0), (10, 15). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( P = 10x + 12y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>( P = 10(0) + 12(0) = 0 )</td>
</tr>
<tr>
<td>(0, 20)</td>
<td>( P = 10(0) + 12(20) = 240 )</td>
</tr>
<tr>
<td>(20, 0)</td>
<td>( P = 10(20) + 12(0) = 200 )</td>
</tr>
<tr>
<td>(10, 15)</td>
<td>( P = 10(10) + 12(15) = 280 )</td>
</tr>
</tbody>
</table>

The maximum profit is $280, when 10 racing skates and 15 figure skates are produced.

28. Let \( x \) = the amount placed in the AAA bond. Let \( y \) = the amount placed in a CD. The total return is:
\[ R = 0.08x + 0.04y \] Return is to be maximized, so this is the objective function. The constraints are:
\[ x \geq 0, \quad y \geq 0 \quad \text{A positive amount must be invested in each.} \]
\[ x + y \leq 50,000 \quad \text{Total investment cannot exceed $50,000.} \]
\[ x \leq 20,000 \quad \text{Investment in the AAA bond cannot exceed $20,000.} \]
\[ y \geq 15,000 \quad \text{Investment in the CD must be at least $15,000.} \]
\[ y \geq x \quad \text{Investment in the CD must exceed or equal the investment in the bond.} \]
Graph the constraints.

The corner points are (0, 50,000), (0, 15,000), (15,000, 15,000), (20,000, 20,000), (20,000, 30,000). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( R = 0.08x + 0.04y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 50000)</td>
<td>( R = 0.08(0) + 0.04(50000) = 2000 )</td>
</tr>
<tr>
<td>(0, 15000)</td>
<td>( R = 0.08(0) + 0.04(15000) = 600 )</td>
</tr>
<tr>
<td>(15000, 15000)</td>
<td>( R = 0.08(15000) + 0.04(15000) = 1800 )</td>
</tr>
<tr>
<td>(20000, 20000)</td>
<td>( R = 0.08(20000) + 0.04(20000) = 2400 )</td>
</tr>
<tr>
<td>(20000, 30000)</td>
<td>( R = 0.08(20000) + 0.04(30000) = 2800 )</td>
</tr>
</tbody>
</table>

The maximum return is $2800, when $20,000 is invested in a AAA bond and $30,000 is invested in a CD.
Chapter 11: Systems of Equations and Inequalities

29. Let \( x \) = the number of metal fasteners, and let \( y \) = the number of plastic fasteners. The total cost is: \( C = 9x + 4y \). Cost is to be minimized, so this is the objective function. The constraints are:

- \( x \geq 2, \ y \geq 2 \)  At least 2 of each fastener must be made.
- \( x + y \geq 6 \)  At least 6 fasteners are needed.
- \( 4x + 2y \leq 24 \)  Only 24 hours are available.

Graph the constraints.

The corner points are (2, 4), (2, 8), (4, 2), (5, 2).

Evaluate the objective function:

\[
\begin{array}{c|c|c}
\text{Vertex} & \text{Value of } C = 9x + 4y & \\
(2, 4) & C = 9(2) + 4(4) = 34 & \\
(2, 8) & C = 9(2) + 4(8) = 50 & \\
(4, 2) & C = 9(4) + 4(2) = 44 & \\
(5, 2) & C = 9(5) + 4(2) = 53 & \\
\end{array}
\]

The minimum cost is $34, when 2 metal fasteners and 4 plastic fasteners are ordered.

30. Let \( x \) = the amount of “Gourmet Dog,” and let \( y \) = the amount of “Chow Hound.” The total cost is: \( C = 0.40x + 0.32y \). Cost is to be minimized, so this is the objective function.

The constraints are:

- \( x \geq 0, \ y \geq 0 \)  A non-negative number of cans must be purchased.
- \( 20x + 35y \geq 1175 \)  At least 1175 units of vitamins per month.
- \( 75x + 50y \geq 2375 \)  At least 2375 calories per month.
- \( x + y \leq 60 \)  Storage space for 60 cans.

Graph the constraints.

The corner points are (0, 47.5), (0, 60), (60, 0), (58.75, 0), (15, 25).

Evaluate the objective function:

\[
\begin{array}{c|c|c}
\text{Vertex} & \text{Value of } C = 0.40x + 0.32y & \\
(0, 47.5) & C = 0.40(0) + 0.32(47.5) = 15.20 & \\
(0, 60) & C = 0.40(0) + 0.32(60) = 19.20 & \\
(60, 0) & C = 0.40(60) + 0.32(0) = 24.00 & \\
(58.75, 0) & C = 0.40(58.75) + 0.32(0) = 23.50 & \\
(15, 25) & C = 0.40(15) + 0.32(25) = 14.00 & \\
\end{array}
\]

The minimum cost is $14, when 15 cans of "Gourmet Dog" and 25 cans of “Chow Hound” are purchased.

31. Let \( x \) = the number of first class seats, and let \( y \) = the number of coach seats. Using the hint, the revenue from \( x \) first class seats and \( y \) coach seats is \( Fx + Cy \), where \( F > C > 0 \). Thus, \( R = Fx + Cy \) is the objective function to be maximized. The constraints are:

- \( 8 \leq x \leq 16 \)  Restriction on first class seats.
- \( 80 \leq y \leq 120 \)  Restriction on coach seats.

**a.** \( \frac{x}{y} \leq \frac{1}{12} \)  Ratio of seats.

The constraints are:

- \( 8 \leq x \leq 16 \)
- \( 80 \leq y \leq 120 \)
- \( 12x \leq y \)

Graph the constraints.
The corner points are (8, 96), (8, 120), and (10, 120). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( R = Fx + Cy )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8, 96)</td>
<td>( R = 8F + 96C )</td>
</tr>
<tr>
<td>(8, 120)</td>
<td>( R = 8F + 120C )</td>
</tr>
<tr>
<td>(10, 120)</td>
<td>( R = 10F + 120C )</td>
</tr>
</tbody>
</table>

Since \( C > 0 \), \( 120C > 96C \), so \( 8F + 120C > 8F + 96C \). Since \( F > 0 \), \( 10F > 8F \), so \( 10F + 120C > 8F + 120C \).

Thus, the maximum revenue occurs when the aircraft is configured with 10 first class seats and 120 coach seats.

b. \( \frac{x}{y} \leq \frac{1}{8} \)

The constraints are:

\[
\begin{align*}
8 & \leq x \leq 16 \\
80 & \leq y \leq 120 \\
x & \leq y
\end{align*}
\]

Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of ( R = Fx + Cy )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8, 80)</td>
<td>( R = 8F + 80C )</td>
</tr>
<tr>
<td>(8, 120)</td>
<td>( R = 8F + 120C )</td>
</tr>
<tr>
<td>(15, 120)</td>
<td>( R = 15F + 120C )</td>
</tr>
<tr>
<td>(10, 80)</td>
<td>( R = 10F + 80C )</td>
</tr>
</tbody>
</table>

Since \( F > 0 \) and \( C > 0 \), \( 120C > 96C \), the maximum value of \( R \) occurs at (15, 120).

The maximum revenue occurs when the aircraft is configured with 15 first class seats and 120 coach seats.

c. Answers will vary.

Chapter 11 Review Exercises

1. \[
\begin{align*}
2x - y & = 5 \\
5x + 2y & = 8
\end{align*}
\]

Solve the first equation for \( y \): \( y = 2x - 5 \).

Substitute and solve:

\[
\begin{align*}
5x + 2(2x - 5) & = 8 \\
5x + 4x - 10 & = 8 \\
9x & = 18 \\
x & = 2 \\
y & = 2(2) - 5 = 4 - 5 = -1
\end{align*}
\]

The solution is \( x = 2, \ y = -1 \) or \( (2,-1) \).

2. \[
\begin{align*}
2x + 3y & = 2 \\
7x - y & = 3
\end{align*}
\]

Solve the second equation for \( y \): \( y = 7x - 3 \)

Substitute into the first equation and solve:

\[
\begin{align*}
2x + 3(7x - 3) & = 2 \\
2x + 21x - 9 & = 2 \\
23x & = 11 \\
x & = \frac{11}{23}
\end{align*}
\]

\[
\begin{align*}
y & = 7 \left( \frac{11}{23} \right) - 3 = \frac{77}{23} - \frac{69}{23} = \frac{8}{23}
\end{align*}
\]

The solution is \( x = \frac{11}{23}, \ y = \frac{8}{23} \) or \( \left( \frac{11}{23}, \frac{8}{23} \right) \).

3. \[
\begin{align*}
3x - 4y & = 4 \\
x - 3y & = \frac{1}{2}
\end{align*}
\]

Solve the second equation for \( x \): \( x = 3y + \frac{1}{2} \)

Substitute into the first equation and solve:

\[
\begin{align*}
3 \left( 3y + \frac{1}{2} \right) - 4y & = 4 \\
9y + \frac{3}{2} - 4y & = 4 \\
5y & = \frac{5}{2} \\
y & = \frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
x & = 3 \left( \frac{1}{2} \right) + \frac{1}{2} = 2
\end{align*}
\]

The solution is \( x = 2, \ y = \frac{1}{2} \) or \( \left( 2, \frac{1}{2} \right) \).
4. \[
\begin{align*}
2x + y &= 0 \\
5x - 4y &= -\frac{13}{2}
\end{align*}
\]
Solve the first equation for \( y \): \( y = -2x \)
Substitute into the second equation and solve:
\[
5x - 4(-2x) = -\frac{13}{2}
\]
\[
x + 8x = -\frac{13}{2}
\]
\[
13x = -\frac{13}{2}
\]
\[
x = -\frac{1}{2}
\]
\[
y = -2\left( -\frac{1}{2} \right) = 1
\]
The solution is \( x = -\frac{1}{2}, \ y = 1 \) or \( \left( -\frac{1}{2}, 1 \right) \).

5. \[
\begin{align*}
x - 2y - 4 &= 0 \\
3x + 2y - 4 &= 0
\end{align*}
\]
Solve the first equation for \( x \): \( x = 2y + 4 \)
Substitute into the second equation and solve:
\[
3(2y + 4) + 2y - 4 = 0
\]
\[
6y + 12 + 2y - 4 = 0
\]
\[
8y = -8
\]
\[
y = -1
\]
\[
x = 2(-1) + 4 = 2
\]
The solution is \( x = 2, \ y = -1 \) or \( (2, -1) \).

6. \[
\begin{align*}
x - 3y + 5 &= 0 \\
2x + 3y - 5 &= 0
\end{align*}
\]
Solve the first equation for \( x \):
\[
x = 3y - 5
\]
Substitute into the second equation and solve:
\[
2(3y - 5) + 3y - 5 = 0
\]
\[
6y - 10 + 3y - 5 = 0
\]
\[
9y = 15
\]
\[
y = \frac{5}{3}
\]
\[
x = 3\left( \frac{5}{3} \right) - 5 = 0
\]
The solution is \( x = 0, \ y = \frac{5}{3} \) or \( \left( 0, \frac{5}{3} \right) \).

7. \[
\begin{align*}
y &= 2x - 5 \\
x &= 3y + 4
\end{align*}
\]
Substitute the first equation into the second equation and solve:
\[
x = 3(2x - 5) + 4
\]
\[
x = 6x - 15 + 4
\]
\[
-5x = -11
\]
\[
x = \frac{11}{5}
\]
\[
y = 2\left( \frac{11}{5} \right) - 5 = -\frac{3}{5}
\]
The solution is \( x = \frac{11}{5}, \ y = -\frac{3}{5} \) or \( \left( \frac{11}{5}, -\frac{3}{5} \right) \).

8. \[
\begin{align*}
x &= 5y + 2 \\
y &= 5x + 2
\end{align*}
\]
Substitute the first equation into the second equation and solve:
\[
y = 5(5y + 2) + 2
\]
\[
y = 25y + 10 + 2
\]
\[
-24y = 12
\]
\[
y = -\frac{1}{2}
\]
\[
x = 5\left( -\frac{1}{2} \right) + 2 = -\frac{5}{2} + 2 = -\frac{1}{2}
\]
The solution is \( x = -\frac{1}{2}, \ y = -\frac{1}{2} \) or \( \left( -\frac{1}{2}, -\frac{1}{2} \right) \).

9. \[
\begin{align*}
x - 3y + 4 &= 0 \\
\frac{1}{2}x - \frac{3}{2}y + \frac{4}{3} &= 0
\end{align*}
\]
Multiply each side of the first equation by 3 and each side of the second equation by −6 and add:
\[
\begin{align*}
3x - 9y + 12 &= 0 \\
-3x + 9y - 8 &= 0
\end{align*}
\]
\[
4 = 0
\]
There is no solution to the system. The system is inconsistent.
10. \[
\begin{align*}
  x + \frac{1}{4} y &= 2 \\
  y + 4x + 2 &= 0
\end{align*}
\]
Solve the second equation for \( y \): \( y = -4x - 2 \).
Substitute into the first equation and solve:
\[
\begin{align*}
  x + \frac{1}{4}(-4x - 2) &= 2 \\
  x - x - \frac{1}{2} &= 2 \\
  0 &= \frac{5}{2}
\end{align*}
\]
There is no solution to the system. The system of equations is inconsistent.

11. \[
\begin{align*}
  2x + 3y - 13 &= 0 \\
  3x - 2y &= 0
\end{align*}
\]
Multiply each side of the first equation by 2 and each side of the second equation by 3, and add to eliminate \( y \):
\[
\begin{align*}
  4x + 6y - 26 &= 0 \\
  9x - 6y &= 0
\end{align*}
\]
\[
\begin{align*}
  13x - 26 &= 0 \\
  13x &= 26 \\
  x &= 2
\end{align*}
\]
Substitute and solve for \( y \):
\[
\begin{align*}
  3(2) - 2y &= 0 \\
  -2y &= -6 \\
  y &= 3
\end{align*}
\]
The solution is \( x = 2, \ y = 3 \) or \((2, 3)\).

12. \[
\begin{align*}
  4x + 5y &= 21 \\
  5x + 6y &= 42
\end{align*}
\]
Multiply each side of the first equation by 5 and each side of the second equation by \(-4\) and add to eliminate \( x \):
\[
\begin{align*}
  20x + 25y &= 105 \\
  -20x - 24y &= -168 \\
  y &= -63
\end{align*}
\]
Substitute and solve for \( x \):
\[
\begin{align*}
  4x + 5(-63) &= 21 \\
  4x - 315 &= 21 \\
  4x &= 336 \\
  x &= 84
\end{align*}
\]
The solution is \( x = 84, \ y = -63 \) or \( (84, -63) \).

13. \[
\begin{align*}
  3x - 2y &= 8 \\
  x - \frac{2}{3} y &= 12
\end{align*}
\]
Multiply each side of the second equation by \(-3\) and add to eliminate \( x \):
\[
\begin{align*}
  3x - 2y &= 8 \\
  -3x + 2y &= -36 \\
  0 &= -28
\end{align*}
\]
There is no solution to the system. The system of equations is inconsistent.

14. \[
\begin{align*}
  2x + 5y &= 10 \\
  4x + 10y &= 20
\end{align*}
\]
Multiply each side of the first equation by \(-2\) and add to eliminate \( x \):
\[
\begin{align*}
  -4x - 10y &= -20 \\
  4x + 10y &= 20 \\
  0 &= 0
\end{align*}
\]
The system is dependent.
\[
\begin{align*}
  2x + 5y &= 10 \\
  5y &= -2x + 10 \\
  y &= -\frac{2}{5} x + 2
\end{align*}
\]
The solution is \( y = -\frac{2}{5} x + 2, \ x \) is any real number or \( \{(x, y)|y = -\frac{2}{5} x + 2, \ x \text{ is any real number}\} \).

15. \[
\begin{align*}
  x + 2y - z &= 6 \\
  2x - y + 3z &= -13 \\
  3x - 2y + 3z &= -16
\end{align*}
\]
Multiply each side of the first equation by \(-2\) and add to the second equation to eliminate \( x \):
\[
\begin{align*}
  -2x - 4y + 2z &= -12 \\
  2x - y + 3z &= -13 \\
  -5y + 5z &= -25 \\
  y - z &= 5
\end{align*}
\]
Multiply each side of the first equation by \(-3\) and add to the third equation to eliminate \( x \):
\[
\begin{align*}
  -3x - 6y + 3z &= -18 \\
  3x - 2y + 3z &= -16 \\
  -8y + 6z &= -34
\end{align*}
\]
Multiply each side of the first result by 8 and add to the second result to eliminate \( y \):
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16. 

\[
\begin{align*}
8y - 8z &= 40 \\
-8y + 6z &= -34 \\
-2z &= 6 \\
z &= -3 \\
\end{align*}
\]

Substituting and solving for the other variables:

\[
\begin{align*}
y - (-3) &= 5 \quad \therefore x + 2(2) - (-3) = 6 \\
y &= 2 \quad \therefore x + 4 + 3 = 6 \\
x &= -1 \\
\end{align*}
\]

The solution is \(x = -1, y = 2, z = -3\) or \((-1, 2, -3)\).

17. 

\[
\begin{align*}
2x - 4y + z &= -15 \\
x + 2y - 4z &= 27 \\
5x - 6y - 2z &= -3 \\
\end{align*}
\]

Multiply the first equation by \(-1\) and the second equation by \(2\), and then add to eliminate \(x\):

\[
\begin{align*}
-2x + 4y - z &= 15 \\
2x + 4y - 8z &= 54 \\
8y - 9z &= 69 \\
\end{align*}
\]

Multiply the second equation by \(-5\) and add to the third equation to eliminate \(x\):

\[
\begin{align*}
-5x - 10y + 20z &= -135 \\
5x - 6y - 2z &= -3 \\
\end{align*}
\]

\[
-16y + 18z = -138
\]

Multiply both sides of the first result by \(2\) and add to the second result to eliminate \(y\):

\[
16y - 18z = 138
\]

\[
-16y + 18z = -138
\]

\[
0 = 0
\]

The system is dependent.

\[
-16y + 18z = -138
\]

\[
18z + 138 = 16y
\]

\[
y = \frac{9}{8}z + \frac{69}{8}
\]

Substituting into the second equation and solving for \(x\):

\[
\begin{align*}
x + 2\left(\frac{9}{8}z + \frac{69}{8}\right) - 4z &= 27 \\
x + \frac{9}{4}z + \frac{69}{4} - 4z &= 27 \\
x &= \frac{7}{4}z + \frac{39}{4}
\end{align*}
\]

The solution is \(x = \frac{7}{4}z + \frac{39}{4}, \ y = \frac{9}{8}z + \frac{69}{8}, \ z\) is any real number.

18. 

\[
\begin{align*}
x - 4y + 3z &= 15 \\
-3x + y - 5z &= -5 \\
-7x - 5y - 9z &= 10 \\
\end{align*}
\]

Multiply the first equation by \(3\) and then add the second equation to eliminate \(x\):

\[
\begin{align*}
3x - 12y + 9z &= 45 \\
-3x + y - 5z &= -5 \\
-11y + 4z &= 40 \\
\end{align*}
\]

Multiply the first equation by \(7\) and add to the third equation to eliminate \(x\):

\[
\begin{align*}
7x - 28y + 21z &= 105 \\
-7x - 5y - 9z &= 10 \\
-33y + 12z &= 115 \\
-11y + 4z &= \frac{115}{3}
\end{align*}
\]
Multiply the first result by \(-1\) and adding it to the second result:
\[
\begin{align*}
11y - 4z &= -40 \\
-11y + 4z &= \frac{115}{3} \\
0 &= -\frac{5}{3}
\end{align*}
\]

The system has no solution. The system is inconsistent.

19. \[
\begin{align*}
3x + 2y &= 8 \\
x + 4y &= -1
\end{align*}
\]

20. \[
\begin{align*}
x + 2y + 5z &= -2 \\
5x - 3z &= 8 \\
2x - y &= 0
\end{align*}
\]

21. \[
A + C = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 5 \\ 5 & 2 \end{bmatrix}
= \begin{bmatrix} 4 & -4 \\ 3 & 9 \\ -1 & 2 \end{bmatrix}
\]

22. \[
A - C = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -4 \\ 1 & 5 \\ 5 & 2 \end{bmatrix}
= \begin{bmatrix} -2 & 4 \\ -1 & -1 \\ -1 & -2 \end{bmatrix}
\]

23. \[
6A = 6 \cdot \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 6 & 4 \\ 6 & 0 \end{bmatrix}
= 12 \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix}
\]

24. \[
-4B = -4 \cdot \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix}
= \begin{bmatrix} -4 & 4 & -4(-3) \\ -4 & 1 & -4(-1) \\ -16 & 12 & 0 \end{bmatrix}
\]

25. \[
AB = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix}
= \begin{bmatrix} 1(4) + 0(1) & 1(-3) + 0(1) & 1(0) + 0(-2) \\ 2(4) + 4(1) & 2(-3) + 4(1) & 2(0) + 4(-2) \\ -1(4) + 2(1) & -1(-3) + 2(1) & -1(0) + 2(-2) \end{bmatrix}
= \begin{bmatrix} 4 & -3 & 0 \\ 12 & -2 & -8 \\ -2 & 5 & -4 \end{bmatrix}
\]

26. \[
BA = \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix}
= \begin{bmatrix} 4(1) - 3(2) + 0(-1) & 4(0) - 3(4) + 0(2) \\ 1(1) + 1(2) - 2(-1) & 1(0) + 1(4) - 2(-2) \end{bmatrix}
= \begin{bmatrix} -2 & -12 \\ 5 & 0 \end{bmatrix}
\]

27. \[
CB = \begin{bmatrix} 3 & -4 \\ 1 & 5 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix}
= \begin{bmatrix} 3(4) - 4(1) & 3(-3) - 4(1) & 3(0) - 4(-2) \\ 1(4) + 5(1) & 1(-3) + 5(1) & 1(0) + 5(-2) \\ 5(4) + 2(1) & 5(-3) + 2(1) & 5(0) + 2(-2) \end{bmatrix}
= \begin{bmatrix} 8 & -13 & 8 \\ 9 & 2 & -10 \\ 22 & -13 & -4 \end{bmatrix}
\]

28. \[
BC = \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 3 & -4 \\ 1 & 5 \\ 5 & 2 \end{bmatrix}
= \begin{bmatrix} 4(3) - 3(1) + 0(5) & 4(-4) - 3(5) + 0(2) \\ 1(3) + 1(1) - 2(5) & 1(-4) + 1(5) - 2(-2) \end{bmatrix}
= \begin{bmatrix} 9 & -31 \\ -6 & -3 \end{bmatrix}
\]
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29. \( A = \begin{bmatrix} 4 & 6 \\ 1 & 3 \end{bmatrix} \)

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{pmatrix}
4 & 6 & 1 & 0 \\
1 & 3 & 0 & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 3 & 0 & 1 \\
4 & 6 & 1 & 0
\end{pmatrix}
\text{ (Interchange)}
\]

\( R_1 \text{ and } r_2 \)

\[
\begin{pmatrix}
1 & 3 & 0 & 1 \\
0 & -6 & 1 & -4
\end{pmatrix}
= -4r_1 + r_2
\]

\[
\begin{pmatrix}
1 & 3 & 0 & 1 \\
0 & 1 & -\frac{1}{6} & \frac{2}{3}
\end{pmatrix}
= -r_2
\]

\[
\begin{pmatrix}
1 & 0 & \frac{1}{2} & -1 \\
0 & 1 & -\frac{1}{6} & \frac{2}{3}
\end{pmatrix}
= -3r_2 + r_1
\]

Thus, \( A^{-1} = \begin{pmatrix} \frac{1}{2} & -1 \\ -\frac{1}{6} & \frac{2}{3} \end{pmatrix} \).

30. \( A = \begin{bmatrix} -3 & 2 \\ 1 & -2 \end{bmatrix} \)

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{pmatrix}
-3 & 2 & 1 & 0 \\
1 & -2 & 0 & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & -2 & 0 & 1 \\
-3 & 2 & 1 & 0
\end{pmatrix}
\text{ (Interchange)}
\]

\( r_1 \text{ and } r_2 \)

\[
\begin{pmatrix}
1 & -2 & 0 & 1 \\
0 & -4 & 1 & 3
\end{pmatrix}
= 3r_1 + r_2
\]

\[
\begin{pmatrix}
1 & -2 & 0 & 1 \\
0 & 1 & -\frac{1}{4} & -\frac{3}{4}
\end{pmatrix}
= 2r_2 + r_1
\]

Thus, \( A^{-1} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{3}{4} \end{pmatrix} \).

31. \( A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix} \)

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{pmatrix}
1 & 3 & 3 & 1 & 0 & 0 \\
1 & 2 & 1 & 0 & 1 & 0 \\
1 & -1 & 2 & 0 & 0 & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 3 & 3 & 1 & 0 & 0 \\
0 & -1 & -2 & -1 & 1 & 0 \\
0 & -4 & -1 & 0 & 1 & 0
\end{pmatrix}
\]

\( R_1 = -r_1 + r_2 \)

\( R_3 = -r_1 + r_3 \)

\[
\begin{pmatrix}
1 & 3 & 3 & 1 & 0 & 0 \\
0 & 1 & 2 & 1 & -1 & 0 \\
0 & -4 & -1 & 0 & 1 & 0
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 3 & 3 & 1 & 0 & 0 \\
0 & 1 & 2 & 1 & -1 & 0 \\
0 & 0 & 7 & 3 & -4 & 1
\end{pmatrix}
\]

\( R_1 = -3r_2 + r_1 \)

\( R_3 = 4r_2 + r_3 \)

\[
\begin{pmatrix}
1 & 0 & 3 & -2 & 3 & 0 \\
0 & 1 & 2 & 1 & -1 & 0 \\
0 & 0 & 1 & \frac{3}{7} & -\frac{4}{7} & \frac{1}{7}
\end{pmatrix}
\]

\( R_1 = 3r_2 + r_1 \)

\( R_2 = -2r_3 + r_2 \)

Thus, \( A^{-1} = \begin{pmatrix} -\frac{5}{7} & \frac{9}{7} & \frac{3}{7} \\ \frac{3}{7} & -\frac{4}{7} & \frac{1}{7} \end{pmatrix} \).

32. \( A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix} \)

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{pmatrix}
3 & 1 & 2 & 1 & 0 & 0 \\
3 & 2 & -1 & 0 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 1 & 1 & 0 & 0 & 1 \\
3 & 2 & -1 & 0 & 1 & 0 \\
3 & 1 & 2 & 1 & 0 & 0
\end{pmatrix}
\text{ (Interchange)}
\]

\( r_1 \text{ and } r_3 \)

\[
\begin{pmatrix}
1 & 1 & 1 & 0 & 0 & 1 \\
0 & -1 & -4 & 0 & 1 & -3 \\
0 & -2 & -1 & 1 & 0 & -3
\end{pmatrix}
= -3r_1 + r_2
\]

\[
\begin{pmatrix}
1 & 1 & 1 & 0 & 0 & 1 \\
0 & 1 & 4 & 0 & -1 & 3 \\
0 & -2 & -1 & 1 & 0 & -3
\end{pmatrix}
= -r_2
\]
Chapter 11 Review Exercises

33. \( A = \begin{bmatrix} 4 & -8 \\ -1 & 2 \end{bmatrix} \)

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix} 4 & -8 & | & 1 & 0 \\ -1 & 2 & | & 0 & 1 \end{bmatrix}
\]

\[
\rightarrow \begin{bmatrix} -1 & 2 & 0 & 1 \\ 4 & -8 & 1 & 0 \end{bmatrix} \quad \text{(Interchange) \( r_1 \) and \( r_2 \)}
\]

\[
\rightarrow \begin{bmatrix} -1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix} \quad \text{(} R_2 = 4r_1 + r_2 \text{)}
\]

\[
\rightarrow \begin{bmatrix} 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 4 \end{bmatrix} \quad \text{(} R_1 = -r_1 \text{)}
\]

There is no inverse because there is no way to obtain the identity on the left side. The matrix is singular.

34. \( A = \begin{bmatrix} -3 & 1 \\ -6 & 2 \end{bmatrix} \)

Augment the matrix with the identity and use row operations to find the inverse:

\[
\begin{bmatrix} -3 & 1 & | & 1 & 0 \\ -6 & 2 & | & 0 & 1 \end{bmatrix}
\]

\[
\rightarrow \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & 0 \\ -6 & 2 & 0 & 1 \end{bmatrix} \quad \text{(} R_1 = -\frac{1}{3}r_1 \text{)}
\]

\[
\rightarrow \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix} \quad \text{(} R_2 = 6r_1 + r_2 \text{)}
\]

There is no inverse because there is no way to obtain the identity on the left side. The matrix is singular.

35. \( \begin{cases} 3x - 2y = 1 \\ 10x + 10y = 5 \end{cases} \)

Write the augmented matrix:

\[
\begin{bmatrix} 3 & -2 & | & 1 \\ 10 & 10 & | & 5 \end{bmatrix}
\]

\[
\rightarrow \begin{bmatrix} 3 & -2 & | & 1 \\ 1 & 16 & 2 \end{bmatrix} \quad \text{(Interchange) \( r_1 \) and \( r_2 \)}
\]

\[
\rightarrow \begin{bmatrix} 1 & 16 & 2 \\ 0 & -50 & -5 \end{bmatrix} \quad \text{(} R_2 = -3r_1 + r_2 \text{)}
\]

\[
\rightarrow \begin{bmatrix} 1 & 16 & 2 \\ 0 & 1 & \frac{1}{10} \end{bmatrix} \quad \text{(} R_2 = -\frac{1}{50}r_2 \text{)}
\]

\[
\rightarrow \begin{bmatrix} 1 & 0 & \frac{2}{5} \\ 0 & 1 & \frac{1}{10} \end{bmatrix} \quad \text{(} R_1 = -16r_2 + r_1 \text{)}
\]

The solution is \( x = \frac{2}{5}, y = \frac{1}{10} \) or \( \left( \frac{2}{5}, \frac{1}{10} \right) \).

36. \( \begin{cases} 3x + 2y = 6 \\ x - y = \frac{1}{2} \end{cases} \)

Write the augmented matrix:

\[
\begin{bmatrix} 3 & 2 & | & 6 \\ 1 & -1 & | & \frac{1}{2} \end{bmatrix}
\]

\[
\rightarrow \begin{bmatrix} 1 & -1 & -\frac{1}{2} \\ 3 & 2 & 6 \end{bmatrix} \quad \text{(Interchange) \( r_1 \) and \( r_2 \)}
\]

\[
\rightarrow \begin{bmatrix} 1 & -1 & -\frac{1}{2} \\ 0 & 5 & 15 \end{bmatrix} \quad \text{(} R_2 = -3r_1 + r_2 \text{)}
\]

\[
\rightarrow \begin{bmatrix} 1 & -1 & -\frac{1}{2} \\ 0 & 1 & \frac{3}{2} \end{bmatrix} \quad \text{(} R_2 = \frac{1}{3}r_2 \text{)}
\]

\[
\rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & \frac{3}{2} \end{bmatrix} \quad \text{(} R_1 = r_2 + r_1 \text{)}
\]

The solution is \( x = 1, y = \frac{3}{2} \) or \( \left( 1, \frac{3}{2} \right) \).
Chapter 11: Systems of Equations and Inequalities

37. \[
\begin{align*}
5x - 6y - 3z &= 6 \\
4x - 7y - 2z &= -3 \\
3x + y - 7z &= 1
\end{align*}
\]

Write the augmented matrix:
\[
\begin{bmatrix}
5 & -6 & -3 & 6 \\
4 & -7 & -2 & -3 \\
3 & 1 & -7 & 1
\end{bmatrix}
\]

\[
\begin{align*}
R_1 &= -r_2 + r_1 \\
R_2 &= -4r_1 + r_2 \\
R_3 &= -3r_1 + r_3
\end{align*}
\]

There is no solution; the system is inconsistent.

38. \[
\begin{align*}
x + 2z &= 1 \\
2x + 3y &= -3 \\
4x - 3y - 4z &= 3
\end{align*}
\]

Write the augmented matrix:
\[
\begin{bmatrix}
1 & 0 & -2 & 1 \\
2 & 3 & 0 & -3 \\
4 & -3 & -4 & 3
\end{bmatrix}
\]

The solution is \( \left( \frac{13}{3}, \frac{13}{3}, \frac{13}{3} \right) \) or \( \left( 9, \frac{13}{3}, \frac{13}{3} \right) \).

39. \[
\begin{align*}
x - 2z &= 1 \\
2x + 3y &= -3 \\
4x - 3y - 4z &= 3
\end{align*}
\]

Write the augmented matrix:
\[
\begin{bmatrix}
1 & 0 & -2 & 1 \\
0 & 3 & 4 & -5 \\
0 & -3 & 4 & -1
\end{bmatrix}
\]

The solution is \( x = -\frac{1}{2}, y = -\frac{2}{3}, z = -\frac{3}{4} \) or \( \left( -\frac{1}{2}, -\frac{2}{3}, -\frac{3}{4} \right) \).


Chapter 11 Review Exercises

40. \[
\begin{align*}
\begin{cases}
x + 2y - z &= 2 \\
2x - 2y + z &= -1 \\
6x + 4y + 3z &= 5
\end{cases}
\end{align*}
\]
Write the augmented matrix:

\[
\begin{bmatrix}
1 & 2 & -1 & | & 2 \\
2 & -2 & 1 & | & -1 \\
6 & 4 & 3 & | & 5
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & 2 & -1 & | & 2 \\
0 & -6 & 3 & | & -5 \\
0 & -8 & 9 & | & -7
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & 2 & -1 & | & 2 \\
0 & 1 & -\frac{1}{2} & | & \frac{5}{6} \\
0 & 0 & 2 & | & \frac{1}{6}
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & | & \frac{1}{3} \\
0 & 1 & -\frac{1}{2} & | & \frac{5}{6} \\
0 & 0 & 1 & | & -\frac{1}{15}
\end{bmatrix}
\]

The solution is \(x = \frac{1}{3},\ y = \frac{4}{5},\ z = -\frac{1}{15}\) or

\[
\begin{bmatrix}
\frac{1}{3} \\
\frac{4}{5} \\
\frac{1}{15}
\end{bmatrix}
\]

41. \[
\begin{align*}
\begin{cases}
x - y + z &= 0 \\
x - y - 5z &= 6 \\
2x - 2y + z &= 1
\end{cases}
\end{align*}
\]
Write the augmented matrix:

\[
\begin{bmatrix}
1 & -1 & 1 & | & 0 \\
1 & -1 & -5 & | & 6 \\
2 & -2 & 1 & | & 1
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & -1 & 1 & | & 0 \\
0 & 0 & -6 & | & 6 \\
0 & 0 & -1 & | & 1
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & -1 & 1 & | & 0 \\
0 & 0 & 1 & | & -1 \\
0 & 0 & -1 & | & 1
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & | & -\frac{1}{6} \\
0 & 0 & 1 & | & -1 \\
0 & 0 & 1 & | & 1
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & | & -\frac{1}{6} \\
0 & 0 & 1 & | & -1 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & 0 & 0 & | & -\frac{1}{6} \\
0 & 0 & 1 & | & -1 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\]

The system is dependent.

\[
\begin{bmatrix}
x = y + 1 \\
z = -1
\end{bmatrix}
\]

The solution is \(x = y + 1,\ z = -1,\ y\) is any real number or \(\{ (x, y, z) | x = y + 1,\ z = -1,\ y\ is\ any\ real\ number \}\).

42. \[
\begin{align*}
\begin{cases}
4x - 3y + 5z &= 0 \\
2x + 4y - 3z &= 0 \\
6x + 2y + z &= 0
\end{cases}
\end{align*}
\]
Write the augmented matrix:

\[
\begin{bmatrix}
4 & -3 & 5 & | & 0 \\
2 & 4 & -3 & | & 0 \\
6 & 2 & 1 & | & 0
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & -\frac{3}{4} & \frac{5}{4} & | & 0 \\
0 & \frac{11}{2} & -\frac{11}{2} & | & 0 \\
0 & \frac{13}{2} & -\frac{13}{2} & | & 0
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & -\frac{3}{4} & \frac{5}{4} & | & 0 \\
0 & 1 & -1 & | & 0 \\
0 & 0 & 1 & | & 0
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & 0 & \frac{1}{2} & | & 0 \\
0 & 1 & -1 & | & 0 \\
0 & 0 & 1 & | & 0
\end{bmatrix}
\]

\[
\rightarrow
\begin{bmatrix}
1 & 0 & \frac{1}{2} & | & 0 \\
0 & 1 & -1 & | & 0 \\
0 & 0 & 1 & | & 0
\end{bmatrix}
\]

The solution is \(x = -\frac{1}{2}z,\ y = z,\ z\) is any real number or \(\{ (x, y, z) | x = -\frac{1}{2}z,\ y = z,\ z\ is\ any\ real\ number \}\).
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43. Write the augmented matrix:
\[
\begin{pmatrix}
1 & -1 & -1 & 1 \\
2 & 1 & -1 & 2 \\
1 & -2 & -2 & -3 \\
3 & -4 & 1 & 5 & -3
\end{pmatrix}
\]
\[
\begin{pmatrix}
1 & -1 & -1 & -1 & 1 \\
0 & 3 & 1 & 4 & 1 \\
0 & -1 & -1 & -2 & -1 \\
0 & -1 & 4 & 8 & -6
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & -1 & -1 & -1 \\
0 & 3 & 1 & 4 \\
0 & -1 & -1 & -2 \\
0 & -1 & 4 & 8
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & -1 & -1 & -1 \\
0 & 3 & 1 & 4 \\
0 & -1 & 4 & 8
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & -1 & -1 & -1 \\
0 & 3 & 1 & 4 \\
0 & -1 & 4 & 8
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 2 \\
0 & 0 & -2 & -2 \\
0 & 0 & 5 & 10
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 2 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 2
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 2 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

The solution is \( x = 4, \ y = 2, \ z = 3, \ t = -2 \) or \((4, 2, 3, -2)\).

44. Write the augmented matrix:
\[
\begin{pmatrix}
x - y + z - t &= 1 \\
2x + y - z + 2t &= 3 \\
x - 2y - 2z - 3t &= 0 \\
3x - 4y + z + 5t &= -3
\end{pmatrix}
\]
\[
\begin{pmatrix}
x - 3y + 3z - t &= 4 \\
x + 2y - z &= -3 \\
x + 3z + 2t &= 3 \\
x + y + 5z &= 6
\end{pmatrix}
\]
\[
\begin{pmatrix}
1 & -3 & 3 & -1 & 4 \\
1 & 2 & -1 & 0 & -3 \\
1 & 3 & 2 & 3 \\
1 & 1 & 5 & 0 & 6
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & -3 & 3 & -1 \\
0 & 5 & -4 & 1 & -7 \\
0 & 3 & 0 & 3 & -1 \\
0 & 4 & 2 & 1 & 2
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & -3 & 3 & -1 \\
0 & 5 & -4 & 1 & -7 \\
0 & 4 & 2 & 1 & 2 \\
0 & 1 & -\frac{4}{5} & \frac{1}{5} & -\frac{7}{5}
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & -\frac{1}{5} & -\frac{2}{5} & \frac{1}{5} & -\frac{7}{5} \\
0 & 1 & -\frac{4}{5} & \frac{1}{5} & -\frac{7}{5} \\
0 & 0 & \frac{12}{5} & \frac{12}{5} & 16 \frac{5}{3} \\
0 & 0 & \frac{26}{5} & \frac{1}{5} & \frac{38}{5}
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & 0 & \frac{3}{5} & -\frac{2}{5} & -\frac{1}{5} \\
0 & 1 & -\frac{4}{5} & \frac{1}{5} & -\frac{7}{5} \\
0 & 0 & \frac{1}{5} & \frac{1}{5} & \frac{38}{5} \\
0 & 0 & \frac{26}{5} & \frac{1}{5} & \frac{38}{5}
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & 0 & 0 & -1 & -1 \\
0 & 1 & 0 & 1 & -\frac{1}{3} \\
0 & 0 & 1 & 1 & \frac{4}{3} \\
0 & 0 & 0 & -5 & \frac{1}{3}
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & 0 & 0 & 1 & -1 \\
0 & 1 & 0 & 1 & -\frac{1}{3} \\
0 & 0 & 1 & 1 & \frac{4}{3} \\
0 & 0 & 0 & 1 & -\frac{1}{3}
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & 0 & 0 & 0 & -\frac{17}{15} \\
0 & 1 & 0 & 0 & -\frac{1}{5} \\
0 & 0 & 1 & 0 & \frac{22}{15} \\
0 & 0 & 0 & 1 & -\frac{4}{15}
\end{pmatrix}
\]
→
\[
\begin{pmatrix}
1 & 0 & 0 & 0 & -\frac{17}{15} \\
0 & 1 & 0 & 0 & -\frac{1}{5} \\
0 & 0 & 1 & 0 & \frac{22}{15} \\
0 & 0 & 0 & 1 & -\frac{4}{15}
\end{pmatrix}
\]

The solution is \( x = -\frac{17}{15}, \ y = -\frac{1}{5}, \ z = \frac{22}{15}, \ t = -\frac{2}{15} \) or \((-\frac{17}{15}, -\frac{1}{5}, \frac{22}{15}, -\frac{2}{15})\).
45. \[
\begin{bmatrix}
3 & 4 \\
1 & 3
\end{bmatrix} = 3(3) - 4(1) = 9 - 4 = 5
\]

46. \[
\begin{bmatrix}
-4 & 0 \\
1 & 3
\end{bmatrix} = -4(3) - 1(0) = -12 - 0 = -12
\]

47. \[
\begin{bmatrix}
1 & 4 & 0 \\
-1 & 2 & 6 \\
4 & 1 & 3
\end{bmatrix} = 1(2) - 4(-1) + 0(-6) = 1 + 4 = 5
\]

48. \[
\begin{bmatrix}
2 & 3 & 10 \\
0 & 1 & 5 \\
-1 & 2 & 3
\end{bmatrix} = 2(1) - 3(-3) + 10(0) = 2(1) + 9 = 11
\]

49. \[
\begin{bmatrix}
2 & 1 & -3 \\
5 & 0 & 1 \\
2 & 6 & 0
\end{bmatrix} = 2(0) - 1(-2) - 3(30) = 2 + 60 = 62
\]

50. \[
\begin{bmatrix}
-2 & 1 & 0 \\
1 & 2 & 3 \\
-1 & 4 & 2
\end{bmatrix} = -2(4) - 1(2) + 0(4) = -2(-8) + 0(5) + 0(6) = 16 + 0 = 16
\]

51. \[
\begin{align*}
\begin{cases}
x - 2y &= 4 \\ 3x + 2y &= 4
\end{cases}
\end{align*}
\]

Set up and evaluate the determinants to use Cramer’s Rule:
\[
D = \begin{vmatrix}
1 & -2 \\
3 & 2
\end{vmatrix} = 1(2) - 3(-2) = 2 + 6 = 8
\]
\[
D_x = \begin{vmatrix}
4 & -2 \\
4 & 2
\end{vmatrix} = 4(2) - 4(-2) = 8 + 8 = 16
\]
\[
D_y = \begin{vmatrix}
1 & 4 \\
3 & 4
\end{vmatrix} = 1(4) - 3(4) = 4 - 12 = -8
\]

The solution is \( x = \frac{D_x}{D} = \frac{16}{8} = 2 \), \( y = \frac{D_y}{D} = \frac{-8}{8} = -1 \) or \((2, -1)\).

52. \[
\begin{align*}
\begin{cases}
x - 3y &= -5 \\ 2x + 3y &= 5
\end{cases}
\end{align*}
\]

Set up and evaluate the determinants to use Cramer’s Rule:
\[
D = \begin{vmatrix}
1 & -3 \\
2 & 3
\end{vmatrix} = 1(3) - 2(-3) = 3 + 6 = 9
\]
\[
D_x = \begin{vmatrix}
-5 & -3 \\
5 & 3
\end{vmatrix} = -5(3) - 5(-3) = -15 + 15 = 0
\]
\[
D_y = \begin{vmatrix}
1 & -5 \\
2 & 5
\end{vmatrix} = 1(5) - 2(-5) = 5 + 10 = 15
\]

The solution is \( x = \frac{D_x}{D} = \frac{0}{9} = 0 \), \( y = \frac{D_y}{D} = \frac{15}{9} = \frac{5}{3} \) or \(\left(0, \frac{5}{3}\right)\).

53. \[
\begin{align*}
\begin{cases}
2x + 3y &= -13 \\ 3x - 2y &= 0
\end{cases}
\end{align*}
\]

Write the system in standard form:
\[
\begin{align*}
\begin{cases}
2x + 3y &= -13 \\ 3x - 2y &= 0
\end{cases}
\end{align*}
\]

Set up and evaluate the determinants to use Cramer’s Rule:
\[
D = \begin{vmatrix}
2 & 3 \\
3 & -2
\end{vmatrix} = 2(-2) - 3(3) = -4 - 9 = -13
\]
\[
D_x = \begin{vmatrix}
13 & 3 \\
0 & -2
\end{vmatrix} = 13(-2) - 3(0) = -26 - 0 = -26
\]
\[
D_y = \begin{vmatrix}
2 & 13 \\
3 & 0
\end{vmatrix} = 2(0) - 13(3) = 0 - 39 = -39
\]

The solution is \( x = \frac{D_x}{D} = \frac{-26}{-13} = 2 \), \( y = \frac{D_y}{D} = \frac{-39}{-13} = 3 \) or \((2, 3)\).
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54. \[ \begin{align*} 3x - 4y - 12 &= 0 \\ 5x + 2y + 6 &= 0 \end{align*} \]

Write the system in standard form:

\[ \begin{align*} 3x - 4y &= 12 \\ 5x + 2y &= -6 \end{align*} \]

Set up and evaluate the determinants to use Cramer's Rule:

\[ D = \begin{vmatrix} 3 & -4 \\ 5 & 2 \end{vmatrix} = 6 + 20 = 26 \]

\[ D_x = \begin{vmatrix} 12 & -4 \\ -6 & 2 \end{vmatrix} = 24 - 24 = 0 \]

\[ D_y = \begin{vmatrix} 3 & 12 \\ 5 & -6 \end{vmatrix} = -18 - 60 = -78 \]

The solution is \( x = \frac{D_x}{D} = \frac{0}{26} = 0 \), \( y = \frac{D_y}{D} = \frac{-78}{26} = -3 \) or \((0, -3)\).

55. \[ \begin{align*} x + 2y - z &= 6 \\ 2x - y + 3z &= -13 \\ 3x - 2y + 3z &= -16 \end{align*} \]

Set up and evaluate the determinants to use Cramer's Rule:

\[ D = \begin{vmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 3 \end{vmatrix} = 28 + 15 - 11 = 32 \]

\[ D_x = \begin{vmatrix} 6 & 2 & -1 \\ -13 & -1 & 3 \\ -16 & -2 & 3 \end{vmatrix} = 6(-3+6) - 2(-3+6) + (1)(-4+3) = 10 \]

\[ D_y = \begin{vmatrix} 1 & 2 & -1 \\ -2 & 3 & 3 \\ -2 & 3 & -2 \end{vmatrix} = 2(-2-3) + (1)(-4+3) = 10 \]

\[ D_z = \begin{vmatrix} 1 & 2 & -1 \\ -13 & -1 & 3 \\ -16 & -2 & 3 \end{vmatrix} = 6(-3+6) - 2(-3+6) + (1)(-4+3) = 10 \]

\[ x = \frac{D_x}{D} = \frac{10}{32} = \frac{5}{16}, \quad y = \frac{D_y}{D} = \frac{10}{32} = \frac{5}{16}, \quad z = \frac{D_z}{D} = \frac{10}{32} = \frac{5}{16} \]

56. \[ \begin{align*} 2x + 3y - z &= 8 \\ 3x - y - 9z &= 9 \end{align*} \]

Set up and evaluate the determinants to use Cramer’s Rule:

\[ D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 1 \\ 3 & -1 & 9 \end{vmatrix} = 1 \]

\[ D_x = \begin{vmatrix} 3 & -1 & 1 \\ -1 & -1 & 1 \\ 8 & -1 & 1 \end{vmatrix} = 8 \]

\[ D_y = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 9 \\ 1 & 3 & -9 \end{vmatrix} = 1 \]

\[ D_z = \begin{vmatrix} 1 & 2 & -1 \\ -1 & -1 & 1 \\ 9 & -1 & -9 \end{vmatrix} = 8 \]

\[ x = \frac{D_x}{D} = \frac{8}{1} = 8, \quad y = \frac{D_y}{D} = \frac{1}{1} = 1, \quad z = \frac{D_z}{D} = \frac{8}{1} = 8 \]

The solution is \( x = 8, \quad y = 1, \quad z = 8 \) or \((8, 1, 8)\).

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Chapter 11 Review Exercises

60. Find the partial fraction decomposition:

\[
\frac{x}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}
\]

Multiply both sides by \((x+2)(x-3)\).

\[x = A(x-3) + B(x+2)\]

Let \(x = -2\), then \(-2 = A(-2-3) + B(-2+2)\)

\[-5A = -2\]

\[A = \frac{2}{5}\]

Let \(x = 3\), then \(3 = A(3-3) + B(3+2)\)

\[5B = 3\]

\[B = \frac{3}{5}\]

\[
\frac{x}{(x+2)(x-3)} = \frac{2}{5} + \frac{3}{x-3}
\]

61. Find the partial fraction decomposition:

\[
\frac{x-4}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}
\]

Multiply both sides by \(x^2(x-1)\)

\[x-4 = Ax(x-1) + B(x-1) + Cx^2\]

Let \(x = 1\), then

\[1-4 = A(1)(1-1) + B(1-1) + C(1)^2\]

\[-3 = C\]

Let \(x = 0\), then

\[0-4 = A(0)(0-1) + B(0-1) + C(0)^2\]

\[-4 = -B\]

\[B = 4\]

Let \(x = 2\), then

\[2-4 = A(2)(2-1) + B(2-1) + C(2)^2\]

\[-2 = 2A + B + 4C\]

\[2A = -2 - 4(-3)\]

\[2A = 6\]

\[A = 3\]

\[
\frac{x-4}{x^2(x-1)} = \frac{3}{x} + \frac{4}{x^2} + \frac{-3}{x-1}
\]

57. Let \(\begin{vmatrix} x & y \\ a & b \end{vmatrix} = 8\).

Then \(\frac{2x}{2a} \cdot \frac{y}{b} = 2(8) = 16\) by Theorem (14).

The value of the determinant is multiplied by \(k\) when the elements of a column are multiplied by \(k\).

58. Let \(\begin{vmatrix} x & y \\ a & b \end{vmatrix} = 8\).

Then \(\frac{y}{b} \cdot \frac{x}{a} = -8\) by Theorem (11). The value of the determinant changes sign when any 2 columns are interchanged.

59. Find the partial fraction decomposition:

\[
x(x-4) = x(x-4) \left(\frac{6}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}\right)
\]

\[6 = A(x-4) + Bx\]

Let \(x = 4\), then \(6 = A(4-4) + B(4)\)

\[4B = 6\]

\[B = \frac{3}{2}\]

Let \(x = 0\), then \(6 = A(0-4) + B(0)\)

\[-4A = 6\]

\[A = -\frac{3}{2}\]

\[
\frac{6}{x(x-4)} = \frac{\frac{3}{2}}{x} + \frac{\frac{3}{2}}{x-4}
\]
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62. Find the partial fraction decomposition:
\[
\frac{2x - 6}{(x - 2)^2(x - 1)} = \frac{A}{x - 2} + \frac{B}{(x - 2)^2} + \frac{C}{x - 1}
\]
Multiply both sides by \((x - 2)^2(x - 1)\).
\[
2x - 6 = A(x - 2)(x - 1) + B(x - 1) + C(x - 2)^2
\]
Let \(x = 1\), then
\[
2(1) - 6 = A(1 - 2)(1 - 1) + B(1 - 1) + C(1 - 2)^2
\]
\[-4 = C \]
\[
C = -4
\]
Let \(x = 2\), then
\[
2(2) - 6 = A(2 - 2)(2 - 1) + B(2 - 1) + C(2 - 2)^2
\]
\[-2 = B \]
Let \(x = 0\), then
\[
2(0) - 6 = A(0 - 2)(0 - 1) + B(0 - 1) + C(0 - 2)^2
\]
\[-6 = 2A - B + 4C \]
\[-6 = 2A - (-2) + 4(-4) \]
\[2A = 8 \]
\[
A = 4
\]
\[
\frac{2x - 6}{(x - 2)^2(x - 1)} = \frac{4}{x - 2} + \frac{-2}{(x - 2)^2} + \frac{-4}{x - 1}
\]

63. Find the partial fraction decomposition:
\[
\frac{x}{(x^2 + 9)(x + 1)} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 9}
\]
Multiply both sides by \((x + 1)(x^2 + 9)\).
\[
x = A(x^2 + 9) + (Bx + C)(x + 1)
\]
Let \(x = -1\), then
\[-1 = A((-1)^2 + 9) + (B(-1) + C)(-1 + 1) \]
\[-1 = A(10) + (-B + C)(0) \]
\[-1 = 10A \]
\[
A = -\frac{1}{10}
\]
Let \(x = 0\), then
\[0 = A(0^2 + 9) + (B(0) + C)(0 + 1) \]
\[0 = 9A + C \]
\[
0 = 9\left(-\frac{1}{10}\right) + C \]
\[
C = \frac{9}{10}
\]
Let \(x = 1\), then
\[3 = A(1^2 + 1) + (B(1) + C)(1 - 1) \]
\[3 = 2A - B - C \]
\[
3 = 2\left(B\left(\frac{6}{5}\right) - B - \frac{3}{5}\right)
\]
\[
B = -\frac{6}{5}
\]
\[
\frac{3x}{(x - 2)(x^2 + 1)} = \frac{\frac{6}{5} - \frac{6}{5}x + \frac{3}{5}}{x - 2} + \frac{\frac{9}{5}}{x^2 + 1}
\]

Let \(x = 1\), then
\[1 = A\left(1^2 + 9\right) + (B(1) + C)(1 - 1) \]
\[1 = A(10) + (B + C)(2) \]
\[1 = 10A + 2B + 2C \]
\[
1 = 10\left(-\frac{1}{10}\right) + 2B + 2\left(\frac{9}{10}\right) \]
\[
1 = -1 + 2B + \frac{9}{5}
\]
\[
2B = \frac{1}{5}
\]
\[
B = \frac{1}{10}
\]
64. Find the partial fraction decomposition:
\[
\frac{3x}{(x - 2)(x^2 + 1)} = \frac{A}{x - 2} + \frac{Bx + C}{x^2 + 1}
\]
Multiply both sides by \((x - 2)(x^2 + 1)\).
\[
3x = A(x^2 + 1) + (Bx + C)(x - 2)
\]
Let \(x = 2\), then
\[3(2) = A((2)^2 + 1) + (B(2) + C)(2 - 2) \]
\[6 = 5A \]
\[
A = \frac{6}{5}
\]
Let \(x = 0\), then
\[3(0) = A(0^2 + 1) + (B(0) + C)(0 - 2) \]
\[0 = A - 2C \]
\[
0 = \frac{6}{5} - 2C \]
\[
2C = \frac{6}{5} \]
\[
C = \frac{3}{5}
\]
Let \(x = 1\), then
\[3(1) = A(1^2 + 1) + (B(1) + C)(1 - 2) \]
\[3 = 2A - B - C \]
\[
3 = 2\left(B\left(\frac{6}{5}\right) - B - \frac{3}{5}\right)
\]
\[
B = -\frac{6}{5}
\]
\[
\frac{3x}{(x - 2)(x^2 + 1)} = \frac{\frac{6}{5} - \frac{6}{5}x + \frac{3}{5}}{x - 2} + \frac{\frac{9}{5}}{x^2 + 1}
\]
65. Find the partial fraction decomposition:
\[
\frac{x^3}{(x^2 + 4)^2} = \frac{Ax + B}{x^2 + 4} + \frac{Cx + D}{(x^2 + 4)^2}
\]
Multiply both sides by \((x^2 + 4)^2\).
\[
x^3 = (Ax + B)(x^2 + 4) + Cx + D
\]
\[
x^3 = Ax^3 + Bx^2 + 4Ax + 4B + Cx + D
\]
\[
x^3 = Ax^3 + Bx^2 + (4A + C)x + 4B + D
\]
\[A = 1; \quad B = 0\]
\[4A + C = 0\]
\[4(1) + C = 0\]
\[C = -4\]
\[4B + D = 0\]
\[4(0) + D = 0\]
\[D = 0\]

\[
\frac{x^3}{(x^2 + 4)^2} = \frac{x}{x^2 + 4} + \frac{-4x}{(x^2 + 4)^2}
\]

66. Find the partial fraction decomposition:
\[
\frac{x^3 + 1}{(x^2 + 16)^2} = \frac{Ax + B}{x^2 + 16} + \frac{Cx + D}{(x^2 + 16)^2}
\]
Multiply both sides by \((x^2 + 16)^2\).
\[
x^3 + 1 = (Ax + B)(x^2 + 16) + Cx + D
\]
\[
x^3 + 1 = Ax^3 + Bx^2 + 16Ax + 16B + Cx + D
\]
\[
x^3 + 1 = Ax^3 + Bx^2 + (16A + C)x + 16B + D
\]
\[A = 1; \quad B = 0\]
\[16A + C = 0\]
\[16(1) + C = 0\]
\[C = -16\]
\[16B + D = 1\]
\[16(0) + D = 1\]
\[D = 1\]

\[
\frac{x^3 + 1}{(x^2 + 16)^2} = \frac{x}{x^2 + 16} + \frac{-16x + 1}{(x^2 + 16)^2}
\]

67. Find the partial fraction decomposition:
\[
\frac{x^2}{(x^2 + 1)(x^2 - 1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx + D}{x^2 + 1}
\]
Multiply both sides by \((x-1)(x+1)(x^2 + 1)\).
\[
x^2 = A(x+1)(x^2 + 1) + B(x-1)(x^2 + 1) + (Cx + D)(x-1)(x+1)
\]
Let \(x = 1\), then
\[
1^2 = A(1+1)(1^2 + 1) + B(1-1)(1^2 + 1) + (C(1) + D)(1-1)(1+1)
\]
\[1 = 4A\]
\[A = \frac{1}{4}\]
Let \(x = -1\), then
\[
(-1)^2 = A(-1+1)((-1)^2 + 1) + B(-1-1)((-1)^2 + 1) + (C(-1) + D)(-1-1)(-1+1)
\]
\[1 = -4B\]
\[B = -\frac{1}{4}\]
Let \(x = 0\), then
\[
0^2 = A(0+1)(0^2 + 1) + B(0-1)(0^2 + 1) + (C(0) + D)(0-1)(0+1)
\]
\[0 = A - B - D\]
\[0 = \frac{1}{4} - \left(-\frac{1}{4}\right) - D\]
\[D = \frac{1}{2}\]
Let \(x = 2\), then
\[
2^2 = A(2+1)(2^2 + 1) + B(2-1)(2^2 + 1) + (C(2) + D)(2-1)(2+1)
\]
\[4 = 15A + 5B + 6C + 3D\]
\[4 = 15 \left(\frac{1}{4}\right) + 5 \left(-\frac{1}{4}\right) + 6C + 3 \left(\frac{1}{2}\right)\]
\[6C = 4 - \frac{15}{4} + \frac{5}{4} - 3\]
\[6C = 0\]
\[C = 0\]
\[
\frac{x^2}{(x^2 + 1)(x^2 - 1)} = \frac{1}{4} \cdot \frac{1}{x-1} + \frac{1}{4} \cdot \frac{1}{x+1} + \frac{1}{2} \cdot \frac{1}{x^2 + 1}
\]
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68. Find the partial fraction decomposition:

\[
\frac{4}{(x^2 + 4)(x^2 - 1)} = \frac{A}{x - 1} + \frac{B}{x + 1} + \frac{Cx + D}{x^2 + 4}
\]

Multiply both sides by \((x^2 + 4)(x - 1)(x + 1)\).

\[4 = A(x + 1)(x^2 + 4) + B(x - 1)(x^2 + 4) + (Cx + D)(x - 1)(x + 1)\]

Let \(x = 1\), then

\[4 = A(1 + 1)(1^2 + 4) + B(1 - 1)(1^2 + 4) + (C(1) + D)(1 - 1)(1 + 1)\]

\[4 = 10A\]

\[A = \frac{4}{10} = \frac{2}{5}\]

Let \(x = -1\), then

\[4 = A(-1 + 1)((-1)^2 + 4) + B(-1 - 1)((-1)^2 + 4) + (C(-1) + D)(-1 - 1)(-1 + 1)\]

\[4 = -10B\]

\[B = \frac{4}{10} = -\frac{2}{5}\]

Let \(x = 0\), then

\[4 = A(0 + 1)(0^2 + 4) + B(0 - 1)(0^2 + 4) + (C(0) + D)(0 - 1)(0 + 1)\]

\[4 = 4A - 4B - D\]

\[4 = 4\left(\frac{2}{5}\right) - 4\left(-\frac{2}{5}\right) - D\]

\[4 = \frac{8}{5} + \frac{8}{5} - D\]

\[D = -\frac{4}{5}\]

Let \(x = 2\), then

\[4 = A(2 + 1)(2^2 + 4) + B(2 - 1)(2^2 + 4) + (C(2) + D)(2 - 1)(2 + 1)\]

\[4 = 24A + 8B + 6C + 3D\]

\[4 = 24\left(\frac{2}{5}\right) + 8\left(-\frac{2}{5}\right) + 6C + 3\left(-\frac{4}{5}\right)\]

\[4 = \frac{48}{5} - \frac{16}{5} + 6C - \frac{12}{5}\]

\[6C = 0\]

\[C = 0\]

\[
\frac{4}{(x^2 + 4)(x^2 - 1)} = \frac{\frac{2}{5}}{x - 1} + \frac{-\frac{2}{5}}{x + 1} + \frac{-\frac{4}{5}}{x^2 + 4}
\]

69. Solve the first equation for \(y\), substitute into the second equation and solve:

\[
\begin{align*}
2x + y^2 &= 0 \\
x^2 + y^2 &= 5
\end{align*}
\]

\[x^2 + (-2x - 3)^2 = 5 \rightarrow x^2 + 4x^2 + 12x + 9 = 5\]

\[5x^2 + 12x + 4 = 0 \rightarrow (5x + 2)(x + 2) = 0\]

\[\Rightarrow x = -\frac{2}{5} \quad \text{or} \quad x = -2\]

\[y = -\frac{11}{5} \quad \text{and} \quad y = 1\]

Solutions: \(\left(-\frac{2}{5}, -\frac{11}{5}\right), (-2, 1)\).

70. Add the equations to eliminate \(y\), and solve:

\[
\begin{align*}
2x - y^2 &= 16 \\
x^2 + 2x &= 8
\end{align*}
\]

\[x^2 + 2x - 8 = 0 \rightarrow (x + 4)(x - 2) = 0\]

\[x = -4 \quad \text{or} \quad x = 2\]

If \(x = -4\):

\((-4)^2 + y^2 = 16 \rightarrow y^2 = 0 \rightarrow y = 0\)

If \(x = 2\):

\[(2)^2 + y^2 = 16 \rightarrow y^2 = 12 \rightarrow y = \pm 2\sqrt{3}\]

Solutions: \((-4, 0), (2, 2\sqrt{3}), (2, -2\sqrt{3})\).

71. Multiply each side of the second equation by 2 and add the equations to eliminate \(xy\):

\[
\begin{align*}
2xy + y^2 &= 10 \quad \rightarrow \quad 2xy + y^2 = 10 \\
-xy + 3y^2 &= 2 \quad \rightarrow \quad -2xy + 6y^2 = 4
\end{align*}
\]

\[7y^2 = 14\]

\[y^2 = 2\]

\[y = \pm \sqrt{2}\]

If \(y = \sqrt{2}\):

\[2x\sqrt{2} + (\sqrt{2})^2 = 10 \rightarrow 2\sqrt{2}x = 8 \rightarrow x = 2\sqrt{2}\]

If \(y = -\sqrt{2}\):

\[2x(-\sqrt{2}) + (-\sqrt{2})^2 = 10 \rightarrow -2\sqrt{2}x = 8 \rightarrow x = -2\sqrt{2}\]

Solutions: \((2\sqrt{2}, \sqrt{2}), (2\sqrt{2}, -\sqrt{2})\)
72. Multiply each side of the first equation by –2 and add the equations to eliminate y:
\[
\begin{align*}
3x^2 - y^2 &= 1 \\
7x^2 - 2y^2 &= 5
\end{align*}
\]
\[
\begin{align*}
-6x^2 + 2y^2 &= -2 \\
7x^2 - 2y^2 &= 5
\end{align*}
\]
So, \[x^2 = 3\]
\[x = \pm \sqrt{3}\]
If \(x = \sqrt{3}\):
\[3(\sqrt{3})^2 - y^2 = 1 \rightarrow - y^2 = -8\]
\[\rightarrow y = \pm \sqrt{8} = \pm 2\sqrt{2}\]
If \(x = -\sqrt{3}\):
\[3(-\sqrt{3})^2 - y^2 = 1 \rightarrow - y^2 = -8\]
\[\rightarrow y = \pm \sqrt{8} = \pm 2\sqrt{2}\]
Solutions:
\((-\sqrt{3}, -2\sqrt{2}), (-\sqrt{3}, 2\sqrt{2}), (\sqrt{3}, -2\sqrt{2}), (\sqrt{3}, 2\sqrt{2})\)

73. Substitute into the second equation into the first equation and solve:
\[
\begin{align*}
x^2 + y^2 &= 6y \\
3y + y^2 &= 6y
\end{align*}
\]
\[y^2 - 3y = 0\]
\[y(y - 3) = 0 \rightarrow y = 0 \text{ or } y = 3\]
If \(y = 0\):
\[x^2 = 3(0) \rightarrow x^2 = 0 \rightarrow x = 0\]
If \(y = 3\):
\[x^2 = 3(3) \rightarrow x^2 = 9 \rightarrow x = \pm 3\]
Solutions: \((0, 0), (-3, 3), (3, 3)\).

74. Multiply each side of the second equation by –1 and add the equations to eliminate y:
\[
\begin{align*}
2x^2 + y^2 &= 9 \\
-2x^2 + y^2 &= 9
\end{align*}
\]
\[2x^2 + y^2 = 9\]
\[-2x^2 - y^2 = -9\]
\[\Rightarrow x^2 = 0 \Rightarrow x = 0\]
If \(x = 0\):
\[0^2 + y^2 = 9 \rightarrow y^2 = 9 \rightarrow y = \pm 3\]
Solutions: \((0, 3), (0, -3)\)

75. Factor the second equation, solve for \(x\), substitute into the first equation and solve:
\[
\begin{align*}
x^2 + 4xy + 5y^2 &= 8 \\
x^2 + 3xy + 2y^2 &= 0
\end{align*}
\]
\[(x + 2y)(x + y) = 0 \rightarrow x = -2y \text{ or } x = -y\]
Substitute \(x = -2y\) and solve:
\[3x^2 + 4xy + 5y^2 = 8\]
\[3(-2y)^2 + 4(-2y)y + 5y^2 = 8\]
\[12y^2 - 8y^2 + 5y^2 = 8\]
\[9y^2 = 8\]
\[y^2 = \frac{8}{9} \Rightarrow y = \pm \frac{2\sqrt{2}}{3}\]
Substitute \(x = -y\) and solve:
\[3x^2 + 4xy + 5y^2 = 8\]
\[3(-y)^2 + 4(-y)y + 5y^2 = 8\]
\[3y^2 - 4y^2 + 5y^2 = 8\]
\[4y^2 = 8\]
\[y^2 = 2 \Rightarrow y = \pm \sqrt{2}\]
If \(y = \frac{2\sqrt{2}}{3}\):
\[x = -2\left(\frac{2\sqrt{2}}{3}\right) = -\frac{4\sqrt{2}}{3}\]
If \(y = -\frac{2\sqrt{2}}{3}\):
\[x = -2\left(-\frac{2\sqrt{2}}{3}\right) = \frac{4\sqrt{2}}{3}\]
If \(y = \sqrt{2}\):
\[x = -\sqrt{2}\]
If \(y = -\sqrt{2}\):
\[x = \sqrt{2}\]
Solutions:
\((-\frac{4\sqrt{2}}{3}, \frac{2\sqrt{2}}{3}), (\frac{4\sqrt{2}}{3}, -\frac{2\sqrt{2}}{3}), (-\sqrt{2}, \sqrt{2})\),
\[(\sqrt{2}, -\sqrt{2})\]

76. Multiply each side of the first equation by 2 and each side of the second equation by 3 and add to eliminate the constant:
\[
\begin{align*}
3x^2 + 2xy - 2y^2 &= 6 \\
x^2 - 2y^2 &= -4
\end{align*}
\]
\[6x^2 + 4xy - 4y^2 = 12\]
\[3xy - 6y^2 = -12\]
\[6x^2 + 7xy - 10y^2 = 0\]
Chapter 11: Systems of Equations and Inequalities

(6x - 5y)(x + 2y) = 0
x = \frac{5}{6} y \text{ or } x = -2y
Substitute \( x = \frac{5}{6} y \) and solve:
\[ \frac{5}{6} y \cdot y - 2y^2 = -4 \]
\[ -\frac{7}{6} y^2 = -4 \rightarrow y^2 = \frac{24}{7} \rightarrow y = \pm \frac{2\sqrt{42}}{7} \]
Substitute \( x = -2y \) and solve:
\[ -2y \cdot y - 2y^2 = -4 \]
\[ -4y^2 = -4 \]
\[ y^2 = 1 \]
\[ y = \pm 1 \]
If \( y = \frac{2\sqrt{42}}{7} \):
\[ x = \frac{5}{6} \cdot \frac{2\sqrt{42}}{7} = \frac{5\sqrt{42}}{21} \]
If \( y = -\frac{2\sqrt{42}}{7} \):
\[ x = \frac{5}{6} \cdot -\frac{2\sqrt{42}}{7} = -\frac{5\sqrt{42}}{21} \]
If \( y = 1 \): \( x = -2 \); if \( y = -1 \): \( x = 2 \)
Solutions:
\[ \left( \frac{5\sqrt{42}}{21}, \frac{2\sqrt{42}}{7} \right), \left( -\frac{5\sqrt{42}}{21}, -\frac{2\sqrt{42}}{7} \right), (-2,1), (2,-1) \]

77. \[ \begin{cases} x^2 - 3x + y^2 + y = -2 \\ x^2 - x + y + 1 = 0 \end{cases} \]

Multiply each side of the second equation by \(-y \) and add the equations to eliminate \( y \):
\[ x^2 - 3x + y^2 + y = -2 \]
\[ -x^2 + x - y^2 - y = 0 \]
\[ -2x = -2 \]
\[ x = 1 \]
If \( x = 1 \): \( 1^2 - 3(1) + y^2 + y = -2 \)
\[ y^2 + y = 0 \]
\[ y(1+y) = 0 \]
\[ y = 0 \text{ or } y = -1 \]
Note that \( y \neq 0 \) because that would cause division by zero in the original system.
Solution: \( (1, -1) \)

78. Multiply each side of the second equation by \(-x \) and add the equations to eliminate \( x \):
\[ \begin{cases} x^2 + x + y^2 = y + 2 \\ x = 1 \end{cases} \]
\[ \rightarrow \begin{cases} x^2 + x + y^2 = y + 2 \\ x + 1 = \frac{2 - y}{x} \rightarrow \frac{-x^2 - x}{x} = y - 2 \end{cases} \]
\[ y^2 = 2y \]
\[ y^2 - 2y = 0 \]
\[ y(y-2) = 0 \]
\[ y = 0 \text{ or } y = 2 \]
If \( y = 0 \):
\[ x^2 + x + 0^2 = 0 + 2 \rightarrow x^2 + x - 2 = 0 \]
\[ \rightarrow (x-1)(x+2) = 0 \rightarrow x = 1 \text{ or } x = -2 \]
If \( y = 2 \):
\[ x^2 + x + 2^2 = 2 + 2 \rightarrow x^2 + x = 0 \]
\[ \rightarrow x(x+1) = 0 \rightarrow x = 0 \text{ or } x = -1 \]
Note that \( x \neq 0 \) because that would cause division by zero in the original system.
Solutions: \( (1, 0), (-2, 0), (-1, 2) \)

79. \( 3x + 4y \leq 12 \)
Graph the line \( 3x + 4y = 12 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \( (0, 0) \). Since \( 3(0) + 4(0) \leq 12 \) is true, shade the side of the line containing \( (0, 0) \).

80. \( 2x - 3y \geq 6 \)
Graph the line \( 2x - 3y = 6 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \( (0, 0) \). Since \( 2(0) - 3(0) \geq 6 \) is false, shade the side of the line opposite \( (0, 0) \).
81.  \( y \leq x^2 \)

Graph the parabola \( y = x^2 \). Use a solid curve since the inequality uses \( \leq \). Choose a test point not on the parabola, such as (0, 1). Since \( 0 \leq 1^2 \) is false, shade the opposite side of the parabola from (0, 1).

82.  \( x \geq y^2 \)

Graph the circle \( x = y^2 \). Use a solid curve since the inequality uses \( \geq \). Choose a test point not on the parabola, such as (1, 0). Since \( 1 \geq 0^2 \) is true, shade the same side of the parabola as (1, 0).

83. \[
\begin{align*}
-2x + y &\leq 2 \\
x + y &\geq 2
\end{align*}
\]

Graph the line \(-2x + y = 2\). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as (0, 0). Since \(-2(0) + 0 \leq 2 \) is true, shade the side of the line containing (0, 0). Graph the line \( x + y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as (0, 0). Since \( 0 + 0 \geq 2 \) is false, shade the opposite side of the line from (0, 0). The overlapping region is the solution.

The graph is unbounded. Find the vertices:
To find the intersection of \( x + y = 2 \) and \(-2x + y = 2 \), solve the system:
\[
\begin{align*}
x + y &= 2 \\
-2x + y &= 2
\end{align*}
\]
Solve the first equation for \( x \): \( x = 2 - y \).
Substitute and solve:
\[
\begin{align*}
-2(2 - y) + y &= 2 \\
-4 + 2y + y &= 2
\end{align*}
\]
\[
\begin{align*}
3y &= 6 \\
y &= 2
\end{align*}
\]
\[
x = 2 - 2 = 0
\]
The point of intersection is (0, 2).
The corner point is (0, 2).

84. \[
\begin{align*}
x - 2y &\leq 6 \\
2x + y &\geq 2
\end{align*}
\]

Graph the line \( x - 2y = 6 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as (0, 0). Since \( 0 - 2(0) \leq 6 \) is true, shade the side of the line containing (0, 0). Graph the line \( 2x + y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as (0, 0). Since \( 2(0) + 0 \geq 2 \) is false, shade the opposite side of the line from (0, 0). The overlapping region is the solution.

The graph is unbounded. Find the vertices:  To find the intersection of \( x - 2y = 6 \) and \( 2x + y = 2 \), solve the system:
\[
\begin{align*}
x - 2y &= 6 \\
2x + y &= 2
\end{align*}
\]
Solve the first equation for \( x \): \( x = 2y + 6 \).
Substitute and solve:
\[
\begin{align*}
2(2y + 6) + y &= 2 \\
4y + 12 + y &= 2
\end{align*}
\]
\[
\begin{align*}
5y &= -10 \\
y &= -2
\end{align*}
\]
\[
x = 2(-2) + 6 = 2
\]
The point of intersection is \( (2, -2) \).
The corner point is \( (2, -2) \).
Chapter 11: Systems of Equations and Inequalities

85. \[
\begin{align*}
&x \geq 0 \\
y &\geq 0 \\
x + y &\leq 4 \\
2x + 3y &\leq 6
\end{align*}
\]  
Graph \( x \geq 0, y \geq 0 \). Shaded region is the first quadrant. Graph the line \( x + y = 4 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 + 0 \leq 4 \) is true, shade the side of the line containing \((0, 0)\).  
Graph the line \( 2x + 3y = 6 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 2(0) + 3(0) \leq 6 \) is true, shade the side of the line containing \((0, 0)\).

The overlapping region is the solution. The graph is bounded. Find the vertices: The intersection of \( x + y = 4 \) and the \( y \)-axis is \((0, 4)\). The intersection of \( x + y = 4 \) and the \( x \)-axis is \((4, 0)\). The four corner points are \((0, 4), (4, 0), (4, 0), (0, 4)\).

86. \[
\begin{align*}
&x \geq 0 \\
y &\geq 0 \\
3x + y &\leq 6 \\
2x + y &\geq 2
\end{align*}
\]  
Graph \( x \geq 0, y \geq 0 \). Shaded region is the first quadrant. Graph the line \( 3x + y = 6 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 3(0) + 0 \geq 6 \) is false, shade the opposite side of the line from \((0, 0)\).  
Graph the line \( 2x + y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 2(0) + 0 \geq 2 \) is false, shade the opposite side of the line from \((0, 0)\).

The overlapping region is the solution. The graph is bounded. Find the vertices: The intersection of \( 3x + y = 6 \) and the \( y \)-axis is \((0, 6)\). The intersection of \( 3x + y = 6 \) and the \( x \)-axis is \((2, 0)\). The two corner points are \((0, 6), (2, 0)\).
88. Graph the system of inequalities:
\[
\begin{align*}
&x \geq 0 \\
y \geq 0 \\
&3x + y \leq 9 \\
&2x + 3y \geq 6
\end{align*}
\]
Graph \( x \geq 0 \); \( y \geq 0 \). Shaded region is the first quadrant. Graph the line \( 3x + y = 9 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 3(0) + 0 \leq 9 \) is true, shade the side of the line containing \((0, 0)\).

Graph the line \( 2x + 3y = 6 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 2(0) + 3(0) \geq 6 \) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution.

89. Graph the system of inequalities:
\[
\begin{align*}
&x^2 + y^2 \leq 16 \\
x + y \geq 2
\end{align*}
\]
Graph the circle \( x^2 + y^2 = 16 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the circle, such as \((0, 0)\). Since \( 0^2 + 0^2 \leq 16 \) is true, shade the same side of the circle as \((0, 0)\).

Graph the line \( x + y = 2 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 + 0 \geq 2 \) is false, shade the opposite side of the line from \((0, 0)\). The overlapping region is the solution.

90. Graph the system of inequalities:
\[
\begin{align*}
y^2 &\leq x - 1 \\
x - y &\leq 3
\end{align*}
\]
Graph the parabola \( y^2 = x - 1 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the parabola, such as \((0, 0)\). Since \( 0^2 \leq 0 - 1 \) is false, shade the opposite side of the parabola from \((0, 0)\).

Graph the line \( x - y = 3 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the line, such as \((0, 0)\). Since \( 0 - 0 \leq 3 \) is true, shade the same side of the line as \((0, 0)\). The overlapping region is the solution.

91. Graph the system of inequalities:
\[
\begin{align*}
y &\leq x^2 \\
x y &\leq 4
\end{align*}
\]
Graph the parabola \( y = x^2 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the parabola, such as \((1, 2)\). Since \( 2 \leq 1^2 \) is false, shade the opposite side of the parabola from \((1, 2)\).

Graph the hyperbola \( xy = 4 \). Use a solid line since the inequality uses \( \leq \). Choose a test point not on the hyperbola, such as \((1, 2)\). Since \( 1 \cdot 2 \leq 4 \) is true, shade the same side of the hyperbola as \((1, 2)\). The overlapping region is the solution.
92. Graph the system of inequalities:
\[
\begin{align*}
 x^2 + y^2 &\geq 1 \\
 x^2 + y^2 &\leq 4 \\
\end{align*}
\]
Graph the circle $x^2 + y^2 = 1$. Use a solid line since the inequality uses $\geq$. Choose a test point not on the circle, such as $(0, 0)$. Since $0^2 + 0^2 \geq 1$ is false, shade the opposite side of the circle from $(0, 0)$.
Graph the circle $x^2 + y^2 = 4$. Use a solid line since the inequality uses $\leq$. Choose a test point not on the circle, such as $(0, 0)$. Since $0^2 + 0^2 \leq 4$ is true, shade the same side of the circle as $(0, 0)$. The overlapping region is the solution.

93. Maximize $z = 3x + 4y$ subject to $x \geq 0$, $y \geq 0$, $3x + 2y \geq 6$, $x + y \leq 8$. Graph the constraints.

The corner points are $(0, 3), (2, 0), (0, 8), (8, 0)$. Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of $z = 3x + 4y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0, 3)$</td>
<td>$z = 3(0) + 4(3) = 12$</td>
</tr>
<tr>
<td>$(0, 8)$</td>
<td>$z = 3(0) + 4(8) = 32$</td>
</tr>
<tr>
<td>$(2, 0)$</td>
<td>$z = 3(2) + 4(0) = 6$</td>
</tr>
<tr>
<td>$(8, 0)$</td>
<td>$z = 3(8) + 4(0) = 24$</td>
</tr>
</tbody>
</table>

The maximum value is 32 at $(0, 8)$.

94. Maximize $z = 2x + 4y$ subject to $x \geq 0$, $y \geq 0$, $x + y \leq 6$, $x \geq 2$. Graph the constraints.

The corner points are $(2, 4), (2, 0), (6, 0)$.
Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of $z = 2x + 4y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(2, 4)$</td>
<td>$z = 2(2) + 4(4) = 20$</td>
</tr>
<tr>
<td>$(2, 0)$</td>
<td>$z = 2(2) + 4(0) = 4$</td>
</tr>
<tr>
<td>$(6, 0)$</td>
<td>$z = 2(6) + 4(0) = 12$</td>
</tr>
</tbody>
</table>

The maximum value is 20 at $(2, 4)$.

95. Minimize $z = 3x + 5y$ subject to $x \geq 0$, $y \geq 0$, $x + y \geq 1$, $3x + 2y \leq 12$, $x + 3y \leq 12$.
Graph the constraints.

To find the intersection of $3x + 2y = 12$ and $x + 3y = 12$, solve the system:
\[
\begin{align*}
 3x + 2y &= 12 \\
  x + 3y &= 12 \\
\end{align*}
\]
Solve the second equation for $x$: $x = 12 - 3y$
Substitute and solve:

\[3(12 - 3y) + 2y = 12\]
\[36 - 9y + 2y = 12\]
\[-7y = -24\]
\[y = \frac{24}{7}\]
\[x = 12 - 3\left(\frac{24}{7}\right) = 12 - \frac{72}{7} = \frac{12}{7}\]

The point of intersection is \(\left(\frac{12}{7}, \frac{24}{7}\right)\).

The corner points are \((0, 1), (1, 0), (0, 4), (4, 0), \left(\frac{12}{7}, \frac{24}{7}\right)\).

Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of (z = 3x + 5y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 1)</td>
<td>(z = 3(0) + 5(1) = 5)</td>
</tr>
<tr>
<td>(0, 4)</td>
<td>(z = 3(0) + 5(4) = 20)</td>
</tr>
<tr>
<td>(1, 0)</td>
<td>(z = 3(1) + 5(0) = 3)</td>
</tr>
<tr>
<td>(4, 0)</td>
<td>(z = 3(4) + 5(0) = 12)</td>
</tr>
<tr>
<td>(\left(\frac{12}{7}, \frac{24}{7}\right))</td>
<td>(z = 3\left(\frac{12}{7}\right) + 5\left(\frac{24}{7}\right) = \frac{156}{7})</td>
</tr>
</tbody>
</table>

The minimum value is 3 at \((1, 0)\).

96. Minimize \(z = 3x + y\) subject to \(x \geq 0, y \geq 0, x \leq 8, y \leq 6, 2x + y \geq 4\). Graph the constraints.

The corner points are \((0, 4), (2, 0), (0, 6), (8, 0), (8, 6)\). Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of (z = 3x + y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 6)</td>
<td>(z = 3(0) + 6 = 6)</td>
</tr>
<tr>
<td>(0, 4)</td>
<td>(z = 3(0) + 4 = 4)</td>
</tr>
<tr>
<td>(2, 0)</td>
<td>(z = 3(2) + 0 = 6)</td>
</tr>
<tr>
<td>(8, 0)</td>
<td>(z = 3(8) + 0 = 24)</td>
</tr>
<tr>
<td>(8, 6)</td>
<td>(z = 3(8) + 6 = 30)</td>
</tr>
</tbody>
</table>

The minimum value is 4 at \((0, 4)\).

97. \(2x + 5y = 5\)
\(4x + 10y = A\)

Multiply each side of the first equation by \(-2\) and eliminate \(x\):
\[
\begin{align*}
-4x - 10y &= -10 \\
4x + 10y &= A
\end{align*}
\]
\[0 = A - 10\]

If there are to be infinitely many solutions, the result of elimination should be \(0 = 0\). Therefore, \(A - 10 = 0\) or \(A = 10\).

98. \(2x + 5y = 5\)
\(4x + 10y = A\)

Multiply each side of the first equation by \(-2\) and eliminate \(x\):
\[
\begin{align*}
-4x - 10y &= -10 \\
4x + 10y &= A
\end{align*}
\]
\[0 = A - 10\]

If the system is to be inconsistent, the result of elimination should be \(0 = \text{any number except 0}\). Therefore, \(A - 10 \neq 0\) or \(A \neq 10\).

99. \(y = ax^2 + bx + c\)

At \((0, 1)\) the equation becomes:
\(1 = a(0)^2 + b(0) + c\)
\(c = 1\)

At \((1, 0)\) the equation becomes:
\(0 = a(1)^2 + b(1) + c\)
\(0 = a + b + c\)
\(a + b + c = 0\)

At \((-2, 1)\) the equation becomes:
\(1 = a(-2)^2 + b(-2) + c\)
\(1 = 4a - 2b + c\)
\(4a - 2b + c = 1\)

The system of equations is:
\[
\begin{align*}
a + b + c &= 0 \\
4a - 2b + c &= 1 \\
c &= 1
\end{align*}
\]

Substitute \(c = 1\) into the first and second equations and simplify:
\[
\begin{align*}
a + b + 1 &= 0 \\
4a - 2b + 1 &= 1 \\
a + b &= -1 \\
4a - 2b &= 0
\end{align*}
\]
\(a = -b - 1\)
Chapter 11: Systems of Equations and Inequalities

Solve the first equation for \( a \), substitute into the second equation and solve:

\[
\begin{align*}
4(-b - 1) - 2b &= 0 \\
-4b - 4 - 2b &= 0 \\
-6b &= 4 \\
b &= -\frac{2}{3} \\
\end{align*}
\]

\[
a = -\frac{2}{3} - 1 = -\frac{5}{3}
\]

The quadratic function is \( y = -\frac{1}{3}x^2 - \frac{2}{3}x + 1 \).

100. \( x^2 + y^2 + Dx + Ey + F = 0 \)

At (0, 1) the equation becomes:

\[
0^2 + 1^2 + D(0) + E(1) + F = 0 \\
E + F = -1
\]

At (1, 0) the equation becomes:

\[
1^2 + 0^2 + D(1) + E(0) + F = 0 \\
D + F = -1
\]

At (-2, 1) the equation becomes:

\[
(-2)^2 + 1^2 + D(-2) + E(1) + F = 0 \\
-2D + E + F = -5
\]

The system of equations is:

\[
\begin{align*}
E + F &= -1 \\
D + F &= -1 \\
-2D + E + F &= -5
\end{align*}
\]

Substitute \( E + F = -1 \) into the third equation and solve for \( D \):

\[
-2D + (-1) = -5 \\
-2D = -4
\]

\[
D = 2
\]

Substitute and solve:

\[
2 + F = -1 \\
E + (-3) = -1
\]

\[
F = -3 \\
E = 2
\]

The equation of the circle is \( x^2 + y^2 + 2x + 2y - 3 = 0 \).

101. Let \( x \) = the number of pounds of coffee that costs \$6.00 per pound, and let \( y \) = the number of pounds of coffee that costs \$9.00 per pound. Then \( x + y = 100 \) represents the total amount of coffee in the blend. The value of the blend will be represented by the equation:

\[
6x + 9y = 6.90(100) .
\]

Solve the system of equations:

\[
\begin{align*}
x + y &= 100 \\
6x + 9y &= 690
\end{align*}
\]

Solve the first equation for \( y \): \( y = 100 - x \).

Solve by substitution:

\[
6x + 9(100 - x) = 690
\]

\[
6x + 900 - 9x = 690 \\
-3x = -210 \\
x = 70 \\
y = 100 - 70 = 30
\]

The blend is made up of 70 pounds of the \$6.00-per-pound coffee and 30 pounds of the \$9.00-per-pound coffee.

102. Let \( x \) = the number of acres of corn, and let \( y \) = the number of acres of soybeans. Then \( x + y = 1000 \) represents the total acreage on the farm. The total cost will be represented by the equation: \( 65x + 45y = 54,325 \). Solve the system of equations:

\[
\begin{align*}
65x + 45y &= 54,325 \\
x + y &= 1000
\end{align*}
\]

Solve the first equation for \( y \): \( y = 1000 - x \).

Solve by substitution:

\[
65x + 45(1000 - x) = 54,325
\]

\[
65x + 45,000 - 45x = 54,325 \\
20x = 9325 \\
x = 466.25
\]

\[
y = 1000 - 466.25 = 533.75
\]

Corn should be planted on 466.25 acres and soybeans should be planted on 533.75 acres.
103. Let \( x \) = the number of small boxes, let \( y \) = the number of medium boxes, and let \( z \) = the number of large boxes.

Oatmeal raisin equation: \( x + 2y + 2z = 15 \)
Chocolate chip equation: \( x + y + 2z = 10 \)
Shortbread equation: \( y + 3z = 11 \)

Multiply each side of the second equation by \(-1\) and add to the first equation to eliminate \( x \):

\[
\begin{align*}
x + 2y + 2z &= 15 \\
-x - y - 2z &= -10 \\
y + 3z &= 11 \\
\end{align*}
\]

Substituting and solving for the other variables:

\[
\begin{align*}
x + 2y + 2z &= 15 \\
3z &= 6 \\
x &= 1 \\
y &= 5 \\
z &= 2
\end{align*}
\]

Thus, 1 small box, 5 medium boxes, and 2 large boxes of cookies should be purchased.

104. a. Let \( x \) = the number of lower-priced packages, and let \( y \) = the number of quality packages.

Peanut inequality: \( 8x + 6y \leq 120 \)  
Cashew inequality: \( 4x + 6y \leq 72 \)

The system of inequalities is:

\[
\begin{align*}
x &\geq 0 \\
y &\geq 0 \\
4x + 3y &\leq 960 \\
2x + 3y &\leq 576
\end{align*}
\]

b. Graphing:

To find the intersection of \( 2x + 3y = 576 \) and \( 4x + 3y = 960 \), solve the system:

\[
\begin{align*}
4x + 3y &= 960 \\
2x + 3y &= 576
\end{align*}
\]

Subtract the second equation from the first:

\[
\begin{align*}
4x + 3y &= 960 \\
2x &= 384 \\
x &= 192
\end{align*}
\]

Substitute and solve:

\[
\begin{align*}
2(192) + 3y &= 576 \\
3y &= 192 \\
y &= 64
\end{align*}
\]

The corner points are \((0, 0), (0, 192), (240, 0), \) and \((192, 64)\).

105. Let \( x \) = the speed of the boat in still water, and let \( y \) = the speed of the river current.  The distance from Chiritza to the Flotel Orellana is 100 kilometers.

<table>
<thead>
<tr>
<th></th>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>trip downstream</td>
<td>( x + y )</td>
<td>( 5/2 )</td>
<td>100</td>
</tr>
<tr>
<td>trip downstream</td>
<td>( x - y )</td>
<td>3</td>
<td>100</td>
</tr>
</tbody>
</table>

The system of equations is:

\[
\begin{align*}
\frac{5}{2}(x + y) &= 100 \\
3(x - y) &= 100
\end{align*}
\]

Multiply both sides of the first equation by 6, multiply both sides of the second equation by 5, and add the results:

\[
\begin{align*}
15x + 15y &= 600 \\
15x - 15y &= 500
\end{align*}
\]

\[
\begin{align*}
30x &= 1100 \\
x &= \frac{1100}{30} = \frac{110}{3}
\end{align*}
\]

\[
\begin{align*}
3 \left( \frac{110}{3} \right) - 3y &= 100 \\
110 - 3y &= 100 \\
10 &= 3y \\
y &= \frac{10}{3}
\end{align*}
\]

The speed of the boat is \( \frac{110}{3} \) \( \approx \) 36.67 km/hr; the speed of the current is \( \frac{10}{3} \) \( \approx \) 3.33 km/hr.
106. Let $x$ = the speed of the jet stream, and let $d$ = the distance from Chicago to Ft. Lauderdale. The jet stream flows from Chicago to Ft. Lauderdale because the time is shorter in that direction.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicago to Ft. Lauderdale</td>
<td>$475 + x$</td>
<td>$5/2$</td>
</tr>
<tr>
<td>Ft. Lauderdale to Chicago</td>
<td>$475 - x$</td>
<td>$17/6$</td>
</tr>
</tbody>
</table>

The system of equations is:

\[
\begin{align*}
(475 + y)(5/2) &= d \\
(475 - y)(17/6) &= d
\end{align*}
\]

Simplifying the system, we obtain:

\[
\begin{align*}
2d - 5x &= 2375 \\
6d + 17x &= 8075
\end{align*}
\]

Multiply the first equation by $-3$ and add the result to the second equation:

\[
\begin{align*}
-6d + 15x &= -7125 \\
6d + 17x &= 8075
\end{align*}
\]

\[
x = \frac{950}{32} = \frac{475}{16}
\]

The speed of the jet stream is approximately $475/16 = 29.69$ miles per hour.

107. Let $x$ = the number of hours for Bruce to do the job alone, let $y$ = the number of hours for Bryce to do the job alone, and let $z$ = the number of hours for Marty to do the job alone. Then $1/x$ represents the fraction of the job that Bruce does in one hour.\n
\[1/y\] represents the fraction of the job that Bryce does in one hour.\n
\[1/z\] represents the fraction of the job that Marty does in one hour.\n
The equation representing Bruce and Bryce working together is:

\[
\frac{1}{x} + \frac{1}{y} = \frac{1}{4/3} = \frac{3}{4} = 0.75
\]

The equation representing Bryce and Marty working together is:

\[
\frac{1}{y} + \frac{1}{z} = \frac{1}{8/5} = \frac{5}{8} = 0.625
\]

The equation representing Bruce and Marty working together is:

\[
\frac{1}{x} + \frac{1}{z} = \frac{1}{8/3} = \frac{3}{8} = 0.375
\]

Solve the system of equations:

\[
\begin{align*}
x = x^{-1}, & y = y^{-1}, & w = z^{-1} \\
\end{align*}
\]

\[
\begin{align*}
\frac{1}{x} + \frac{1}{y} &= 0.75 \\
\frac{1}{y} + \frac{1}{z} &= 0.625 \\
\frac{1}{x} + \frac{1}{z} &= 0.375
\end{align*}
\]

Let $u = x^{-1}$, $v = y^{-1}$, $w = z^{-1}$

\[
\begin{align*}
u + v &= 0.75 \\
v + w &= 0.625 \\
u + w &= 0.375
\end{align*}
\]

Solve the first equation for $u$: $u = 0.75 - v$.

Solve the second equation for $w$: $w = 0.625 - v$.

Substitute into the third equation and solve:

\[
\begin{align*}
0.75 - v + (0.625 - v) &= 0.375 \\
-2v &= -1 \\
v &= 0.5 \\
u &= 0.75 - 0.5 = 0.25 \\
w &= 0.625 - 0.5 = 0.125
\end{align*}
\]

Solve for $x$, $y$, and $z$:

\[x = 4, \; y = 2, \; z = 8\] (reciprocals)

Bruce can do the job in 4 hours, Bryce in 2 hours, and Marty in 8 hours.

108. Let $x$ = the number of dancing girls produced, and let $y$ = the number of mermaids produced. The total profit is: $P = 25x + 30y$. Profit is to be maximized, so this is the objective function. The constraints are:

\[
\begin{align*}
x &\geq 0, \; y &\geq 0 \quad \text{A non-negative number of figurines must be produced.} \\
3x + 3y &\leq 90 \quad 90 \text{ hours are available for molding.} \\
6x + 4y &\leq 120 \quad 120 \text{ hours are available for painting.} \\
2x + 3y &\leq 60 \quad 60 \text{ hours are available for glazing.}
\end{align*}
\]

Graph the constraints.
To find the intersection of $6x + 4y = 120$ and $2x + 3y = 60$, solve the system:

\[
\begin{align*}
6x + 4y &= 120 \\
2x + 3y &= 60
\end{align*}
\]

Multiply the second equation by 3, and subtract from the first equation:

\[
\begin{align*}
6x + 4y &= 120 \\
-6x - 9y &= -180 \\
y &= 12
\end{align*}
\]

Substitute and solve:

\[
2x + 3(12) = 60 \\
x = 24
\]

\[
x = 12
\]

The point of intersection is (12, 12). The corner points are (0, 0), (0, 20), (20, 0), (12, 12).

Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of $P = 25x + 30y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0)</td>
<td>$P = 25(0) + 30(0) = 0$</td>
</tr>
<tr>
<td>(0, 20)</td>
<td>$P = 25(0) + 30(20) = 600$</td>
</tr>
<tr>
<td>(20, 0)</td>
<td>$P = 25(20) + 30(0) = 500$</td>
</tr>
<tr>
<td>(12, 12)</td>
<td>$P = 25(12) + 30(12) = 660$</td>
</tr>
</tbody>
</table>

The maximum profit is $660, when 12 dancing girl and 12 mermaid figurines are produced each day. To determine the excess, evaluate each constraint at $x = 12$ and $y = 12$:

Molding: $3x + 3y = 3(12) + 3(12) = 36 + 36 = 72$

Painting: $6x + 4y = 6(12) + 4(12) = 72 + 48 = 120$

Glazing: $2x + 3y = 2(12) + 3(12) = 24 + 36 = 60$

Painting and glazing are at their capacity. Molding has 18 more hours available, since only 72 of the 90 hours are used.

109. Let $x$ = the number of gasoline engines produced each week, and let $y$ = the number of diesel engines produced each week. The total cost is: $C = 450x + 550y$. Cost is to be minimized; thus, this is the objective function. The constraints are:

- $20 \leq x \leq 60$ number of gasoline engines needed and capacity each week.
- $15 \leq y \leq 40$ number of diesel engines needed and capacity each week.
- $x + y \geq 50$ number of engines produced to prevent layoffs.

Graph the constraints.

The corner points are (20, 30), (20, 40), (35, 15), (60, 15), (60, 40)

Evaluate the objective function:

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Value of $C = 450x + 550y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(20, 30)</td>
<td>$C = 450(20) + 550(30) = 25,500$</td>
</tr>
<tr>
<td>(20, 40)</td>
<td>$C = 450(35) + 550(40) = 31,000$</td>
</tr>
<tr>
<td>(35, 15)</td>
<td>$C = 450(35) + 550(15) = 24,000$</td>
</tr>
<tr>
<td>(60, 15)</td>
<td>$C = 450(60) + 550(15) = 35,250$</td>
</tr>
<tr>
<td>(60, 40)</td>
<td>$C = 450(60) + 550(40) = 49,000$</td>
</tr>
</tbody>
</table>

The minimum cost is $24,000, when 35 gasoline engines and 15 diesel engines are produced. The excess capacity is 15 gasoline engines, since only 20 gasoline engines had to be delivered.

110. Answers will vary.

Chapter 11 Test

1. \[
\begin{align*}
-2x + y &= -7 \\
4x + 3y &= 9
\end{align*}
\]

**Substitution:**

We solve the first equation for $y$, obtaining $y = 2x - 7$.

Next we substitute this result for $y$ in the second equation and solve for $x$.

\[
\begin{align*}
4x + 3(2x - 7) &= 9 \\
4x + 6x - 21 &= 9 \\
10x &= 30 \\
x &= \frac{30}{10} = 3
\end{align*}
\]

We can now obtain the value for $y$ by letting $x = 3$ in our substitution for $y$. 

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Chapter 11: Systems of Equations and Inequalities

$$y = 2x - 7$$
$$y = 2(3) - 7 = 6 - 7 = -1$$
The solution of the system is \( x = 3, y = -1 \) or \((3, -1)\).

**Elimination:**
Multiply each side of the first equation by 2 so that the coefficients of \( x \) in the two equations are negatives of each other. The result is the equivalent system

\[
\begin{align*}
-4x + 2y &= -14 \\
4x + 3y &= 9
\end{align*}
\]

We can replace the second equation of this system by the sum of the two equations. The result is the equivalent system

\[
\begin{align*}
-4x + 2y &= -14 \\
5y &= -5
\end{align*}
\]

Now we solve the second equation for \( y \).

\[
y = \frac{-5}{5} = -1
\]

We back-substitute this value for \( y \) into the original first equation and solve for \( x \).

\[
-2x + y = -7
\]

\[
-2x + (-1) = -7
\]

\[
-2x = -6
\]

\[
x = \frac{-6}{-2} = 3
\]

The solution of the system is \( x = 3, y = -1 \) or \((3, -1)\).

2. \[
\begin{align*}
\frac{1}{3}x - 2y &= 1 \\
5x - 30y &= 18
\end{align*}
\]
We choose to use the method of elimination and multiply the first equation by \(-15\) to obtain the equivalent system

\[
\begin{align*}
-5x + 30y &= -15 \\
5x - 30y &= 18
\end{align*}
\]

We replace the second equation by the sum of the two equations to obtain the equivalent system

\[
\begin{align*}
-5x + 30y &= -15 \\
0 &= 3
\end{align*}
\]

The second equation is a contradiction and has no solution. This means that the system itself has no solution and is therefore inconsistent.

3. \[
\begin{align*}
x - y + 2z &= 5 \\
x + 4y - z &= -2 \\
5x + 2y + 3z &= 8
\end{align*}
\]
We use the method of elimination and begin by eliminating the variable \( y \) from equation (1).

Multiply each side of equation (1) by 4 and add the result to equation (2). This result becomes our new equation (2).

\[
\begin{align*}
x - y + 2z &= 5 \\
4x - 4y + 8z &= 20 \\
3x + 4y - z &= -2 \\
7x + 7z &= 18
\end{align*}
\]

We now eliminate the variable \( y \) from equation (3) by multiplying each side of equation (1) by 2 and adding the result to equation (3). The result becomes our new equation (3).

\[
\begin{align*}
x - y + 2z &= 5 \\
2x - 2y + 4z &= 10 \\
5x + 2y + 3z &= 8 \\
7z &= 18
\end{align*}
\]

Our (equivalent) system now looks like

\[
\begin{align*}
x - y + 2z &= 5 \\
7x + 7z &= 18 \\
7x + 7z &= 18
\end{align*}
\]

We now have the equivalent system

\[
\begin{align*}
x - y + 2z &= 5 \\
7x + 7z &= 18 \\
0 &= 0
\end{align*}
\]

This is equivalent to a system of two equations with three variables. Since one of the equations contains three variables and one contains only two variables, the system will be dependent. There are infinitely many solutions.

We solve equation (2) for \( x \) and determine that

\[
x = -z + \frac{18}{7}
\]

We substitute this expression into equation (1) to obtain \( y \) in terms of \( z \).
Chapter 11 Test

11. Test

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\[
x - y + 2z = 5 \\
(\frac{-z + 18}{7}) - y + 2z = 5 \\
-z + \frac{18}{7} - y + 2z = 5 \\
y + z = \frac{17}{7} \\
y = z - \frac{17}{7} \\
\]

The solution is \( x = -z + \frac{18}{7}, \ y = z - \frac{17}{7}, \ z \) is any real number or \( \{(x, y, z) \mid x = -z + \frac{18}{7}, \ y = z - \frac{17}{7}, \ z \text{ is any real number}\} \).

4. \[
\begin{align*}
3x + 2y - 8z &= -3 \quad (1) \\
-x - \frac{2}{3}y + z &= 1 \quad (2) \\
6x - 3y + 15z &= 8 \quad (3)
\end{align*}
\]

We start by clearing the fraction in equation (2) by multiplying both sides of the equation by 3.

\[
\begin{align*}
3x + 2y - 8z &= -3 \quad (1) \\
-3x - 2y + 3z &= 3 \quad (2) \\
6x - 3y + 15z &= 8 \quad (3)
\end{align*}
\]

We use the method of elimination and begin by eliminating the variable \( x \) from equation (2). The coefficients on \( x \) in equations (1) and (2) are negatives of each other so we simply add the two equations together. This result becomes our new equation (2).

\[
\begin{align*}
3x + 2y - 8z &= -3 \\
-3x - 2y + 3z &= 3 \\
\hline
-5z &= 0 \quad (2)
\end{align*}
\]

We now eliminate the variable \( x \) from equation (3) by multiplying each side of equation (1) by \( -2 \) and adding the result to equation (3). The result becomes our new equation (3).

\[
\begin{align*}
3x + 2y - 8z &= -3 \\
-6x - 4y + 16z &= 6 \\
\hline
6x - 3y + 15z &= 8 \\
-7y + 31z &= 14 \quad (3)
\end{align*}
\]

Our (equivalent) system now looks like

\[
\begin{align*}
3x + 2y - 8z &= -3 \quad (1) \\
-5z &= 0 \quad (2) \\
-7y + 31z &= 14 \quad (3)
\end{align*}
\]

We solve equation (2) for \( z \) by dividing both sides of the equation by \( -5 \).

\[
-5z = 0 \\
z = 0
\]

Back-substitute \( z = 0 \) into equation (3) and solve for \( y \).

\[
\begin{align*}
-7y + 31(0) &= 14 \\
-7y &= 14 \\
y &= -2
\end{align*}
\]

Finally, back-substitute \( y = -2 \) and \( z = 0 \) into equation (1) and solve for \( x \).

\[
\begin{align*}
3x + 2(-2) - 8(0) &= -3 \\
3x - 4 &= -3 \\
3x &= 1 \\
x &= \frac{1}{3}
\end{align*}
\]

The solution of the original system is \( x = \frac{1}{3}, \ y = -2, \ z = 0 \) or \( \left( \frac{1}{3}, -2, 0 \right) \).

5. \[
\begin{align*}
4x - 5y + z &= 0 \\
-2x - y + 6 &= -19 \\
x + 5y - 5z &= 10
\end{align*}
\]

We first check the equations to make sure that all variable terms are on the left side of the equation and the constants are on the right side. If a variable is missing, we put it in with a coefficient of 0. Our system can be rewritten as

\[
\begin{align*}
4x - 5y + z &= 0 \\
-2x - y + 0z &= -19 \\
x + 5y - 5z &= 10
\end{align*}
\]

The augmented matrix is

\[
\begin{bmatrix}
4 & -5 & 1 & 0 \\
-2 & -1 & 0 & -25 \\
1 & 5 & -5 & 10
\end{bmatrix}
\]

6. The matrix has three rows and represents a system with three equations. The three columns to the left of the vertical bar indicate that the system has three variables. We can let \( x, y, \) and \( z \) denote these variables. The column to the right of the vertical bar represents the constants on the right side of the equations. The system is

\[
\begin{align*}
3x + 2y + 4z &= -6 \\
x + 0y + 8z &= 2 \\
-2x + y + 3z &= -11
\end{align*}
\]

The solution of the original system is \( x = \frac{1}{3}, \ y = -2, \ z = 0 \) or \( \left( \frac{1}{3}, -2, 0 \right) \).
Chapter 11: Systems of Equations and Inequalities

7. \[ 2A + C = \begin{bmatrix} 2 & -2 \\ 0 & -8 \\ 6 & -1 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 1 & -3 \\ 1 & -11 \\ 8 & 12 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & -5 \\ 6 & 1 \\ 12 & 18 \end{bmatrix} \]

8. \[ A - 3C = \begin{bmatrix} 1 & -1 \\ 0 & -4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ 12 & 18 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 46 \\ 3 & 12 \\ 2 & 24 \end{bmatrix} \]

9. Here we are taking the product of a \( 3 \times 2 \) matrix and a \( 2 \times 3 \) matrix. Since the number of columns in the first matrix is the same as the number of rows in the second matrix (2 in both cases), the operation can be performed and will result in a \( 3 \times 3 \) matrix.

\[
CB = \begin{bmatrix} 4 & 6 \\ 1 & -3 \\ 1 & 8 \\ 0 & 3 \\ 1 & 24 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 5 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 \cdot 1 + 6 \cdot 0 & 4 \cdot (-2) + 6 \cdot 3 & 4 \cdot 5 + 6 \cdot 1 \\ 1 \cdot 1 + (-3) \cdot 0 & 1 \cdot (-2) + (-3) \cdot 3 & 1 \cdot 5 + (-3) \cdot 1 \\ (-1) \cdot 1 + 8 \cdot 0 & (-1) \cdot (-2) + 8 \cdot 3 & (-1) \cdot 5 + 8 \cdot 1 \end{bmatrix} = \begin{bmatrix} 4 & 10 & 26 \\ 1 & -11 & 2 \\ -1 & 26 & 3 \end{bmatrix}
\]

10. Here we are taking the product of a \( 2 \times 3 \) matrix and a \( 3 \times 2 \) matrix. Since the number of columns in the first matrix is the same as the number of rows in the second matrix (3 in both cases), the operation can be performed and will result in a \( 2 \times 2 \) matrix.

\[
BA = \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + (-2) \cdot 0 + 5 \cdot 3 & 1 \cdot (-1) + (-2) \cdot (-4) + 5 \cdot 2 \\ 0 \cdot 1 + 3 \cdot 0 + 1 \cdot 3 & 0 \cdot (-1) + 3 \cdot (-4) + 1 \cdot 2 \end{bmatrix} = \begin{bmatrix} 16 & 17 \\ 3 & -10 \end{bmatrix}
\]

11. We first form the matrix

\[
\begin{bmatrix} A | I_2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}
\]

Next we use row operations to transform \( \begin{bmatrix} A | I_2 \end{bmatrix} \) into reduced row echelon form.

\[
\begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & \frac{1}{3} \\ 0 & \frac{1}{3} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{3} \\ 0 & 1 \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

Therefore, \( A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \).

12. We first form the matrix

\[
\begin{bmatrix} B | I_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & 5 \\ 2 & 3 \end{bmatrix}
\]

Next we use row operations to transform \( \begin{bmatrix} B | I_3 \end{bmatrix} \) into reduced row echelon form.
Chapter 11 Test

13. \[ \begin{align*}
6x + 3y &= 12 \\
2x - y &= -2
\end{align*} \]

We start by writing the augmented matrix for the system.
\[
\begin{bmatrix}
6 & 3 & 12 \\
2 & -1 & -2
\end{bmatrix}
\]

Next we use row operations to transform the augmented matrix into row echelon form.
\[
\begin{bmatrix}
6 & 3 & 12 \\
2 & -1 & -2
\end{bmatrix} \rightarrow 
\begin{bmatrix}
1 & -\frac{1}{2} & -1 \\
0 & 3 & 12
\end{bmatrix} (R_1 = r_2)
\]

\[
\begin{bmatrix}
1 & -\frac{1}{2} & -1 \\
0 & 3 & 12
\end{bmatrix} \rightarrow 
\begin{bmatrix}
1 & -\frac{1}{2} & -1 \\
0 & 6 & 18
\end{bmatrix} (R_1 = \frac{1}{2} r_1)
\]

\[
\begin{bmatrix}
1 & -\frac{1}{2} & -1 \\
0 & 6 & 18
\end{bmatrix} \rightarrow 
\begin{bmatrix}
1 & -\frac{1}{2} & -1 \\
0 & 1 & 3
\end{bmatrix} (R_2 = \frac{1}{6} r_2)
\]

\[
\begin{bmatrix}
1 & 0 & \frac{1}{2} \\
0 & 1 & 3
\end{bmatrix} (R_2 = \frac{1}{2} r_2 + r_1)
\]

Thus, \[ B^{-1} = \begin{bmatrix}
3 & 3 & -4 \\
-2 & -2 & 3 \\
-4 & -5 & 7
\end{bmatrix} \]

The solution of the system is \[ x = \frac{1}{2}, \ y = 3 \] or \[ \left( \frac{1}{2}, 3 \right) \]

14. \[ \begin{align*}
x + \frac{1}{4} y &= 7 \\
8x + 2y &= 56
\end{align*} \]

We start by writing the augmented matrix for the system.
\[
\begin{bmatrix}
1 & \frac{1}{4} & 7 \\
8 & 2 & 56
\end{bmatrix}
\]

Next we use row operations to transform the augmented matrix into row echelon form.
\[
\begin{bmatrix}
1 & \frac{1}{4} & 7 \\
8 & 2 & 56
\end{bmatrix} \rightarrow 
\begin{bmatrix}
1 & \frac{1}{4} & 7 \\
0 & 0 & 0
\end{bmatrix} (R_2 = -8r_1 + r_2)
\]

The augmented matrix is now in row echelon form. Because the bottom row consists entirely of 0’s, the system actually consists of one equation in two variables. The system is dependent and therefore has an infinite number of solutions. Any ordered pair satisfying the equation \[ x + \frac{1}{4} y = 7, \] or \[ y = -4x + 28, \] is a solution to the system.

15. \[ \begin{align*}
x + 2y + 4z &= -3 \\
2x + 7y + 15z &= -12 \\
4x + 7y + 13z &= -10
\end{align*} \]

We start by writing the augmented matrix for the system.
\[
\begin{bmatrix}
1 & 2 & 4 & -3 \\
2 & 7 & 15 & -12 \\
4 & 7 & 13 & -10
\end{bmatrix}
\]

Next we use row operations to transform the augmented matrix into row echelon form.
Chapter 11: Systems of Equations and Inequalities

16. \[
\begin{align*}
2x + 2y - 3z &= 5 \\
x - y + 2z &= 8 \\
3x + 5y - 8z &= -2
\end{align*}
\]

We start by writing the augmented matrix for the system.

\[
\begin{bmatrix}
2 & 2 & -3 & 5 \\
1 & -1 & 2 & 8 \\
3 & 5 & -8 & -2
\end{bmatrix}
\]

Next we use row operations to transform the augmented matrix into row echelon form.

The matrix is now in row echelon form. The last row represents the equation \( z = 0 \). Using \( z = 0 \) we back-substitute into the equation \( y + 3z = -2 \) (from the second row) and obtain

\[
y + 3(0) = -2
\]

\[
y = -2
\]

Using \( y = -2 \) and \( z = 0 \), we back-substitute into the equation \( x + 2y + 4z = -3 \) (from the first row) and obtain

\[
x + 2(-2) + 4(0) = -3
\]

\[
x = 1
\]

The solution is \( x = 1 \), \( y = -2 \), \( z = 0 \) or (1, -2, 0).

17. \[
\begin{bmatrix}
-2 & 5 \\
3 & 7
\end{bmatrix}
\]

The last row represents the equation \( 0 = -4 \) which is a contradiction. Therefore, the system has no solution and is inconsistent.

18. \[
\begin{bmatrix}
2 & -4 & 6 \\
1 & 4 & 0 \\
-1 & 2 & -4
\end{bmatrix}
\]

The determinant \( D \) of the coefficients of the variables is

\[
D = \begin{vmatrix}
2 & 4 & 0 \\
-4 & 0 & 1 \\
-4 & -1 & 4
\end{vmatrix}
\]

\[
= 2[4(-4) - 2(0)] + 4[1(-4) - (-1)(0)] + 6[1(2) - (-1)(4)]
\]

\[
= 2(-16) + 4(-4) + 6(6)
\]

\[
= -32 - 16 + 36
\]

\[
= -12
\]

19. \[
\begin{align*}
4x + 3y &= -23 \\
3x - 5y &= 19
\end{align*}
\]

The determinant \( D \) of the coefficients of the variables is

\[
D = \begin{vmatrix}
4 & 3 \\
3 & -5
\end{vmatrix}
\]

\[
= (4)(-5) - (3)(3) = -20 - 9 = -29
\]

Since \( D \neq 0 \), Cramer’s Rule can be applied.

\[
D_x = \begin{vmatrix}
-23 & 3 \\
19 & -5
\end{vmatrix}
\]

\[
= (-23)(-5) - (3)(19) = 58
\]
Chapter 11 Test

11. \( D_y = \begin{vmatrix} 4 & 15 & 2 \\ 5 & 18 & 2 \end{vmatrix} = \begin{vmatrix} 4 & 15 & 2 \\ 5 & 18 & 2 \end{vmatrix} = 4(-30+54) - 15(-4+15) + 2(-36+75) = 4(24) - 15(11) + 2(39) = -9 \)

\( D_z = \begin{vmatrix} 4 & 15 & 2 \\ 5 & 18 & 2 \end{vmatrix} = \begin{vmatrix} 4 & 15 & 2 \\ 5 & 18 & 2 \end{vmatrix} = 4(-30+54) - 15(-4+15) + 2(-36+75) = 4(24) - 15(11) + 2(39) = -9 \)

20. \[
\begin{align*}
4x - 3y + 2z &= 15 \\
-2x + y - 3z &= -15 \\
5x - 5y + 2z &= 18
\end{align*}
\]
The determinant \( D \) of the coefficients of the variables is
\[
D = \begin{vmatrix} 4 & -3 & 2 \\ -2 & 1 & -3 \\ 5 & -5 & 2 \end{vmatrix} = 4\begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & -3 \\ 5 & -5 \end{vmatrix} + 2\begin{vmatrix} 1 & -3 \\ 5 & -5 \end{vmatrix} = 4(-15) + 3(11) + 2(5) = -52 + 33 + 10 = -9
\]
Since \( D \neq 0 \), Cramer’s Rule can be applied.
\[
D_x = \begin{vmatrix} 4 & 15 & 2 \\ -2 & 1 & -3 \\ 5 & -5 & 2 \end{vmatrix} = 4\begin{vmatrix} 1 & -3 \\ 5 & -5 \end{vmatrix} - 3\begin{vmatrix} 1 & -3 \\ 5 & -5 \end{vmatrix} + 2\begin{vmatrix} 1 & -3 \\ 5 & -5 \end{vmatrix} = 4(-15) + 3(11) + 2(5) = -52 + 33 + 10 = -9
\]
\[
D_y = \begin{vmatrix} 4 & 15 & 2 \\ -2 & 1 & -3 \\ 5 & -5 & 2 \end{vmatrix} = 4\begin{vmatrix} 1 & -3 \\ 5 & -5 \end{vmatrix} - 2\begin{vmatrix} 1 & -3 \\ 5 & -5 \end{vmatrix} + 1\begin{vmatrix} 1 & -3 \\ 5 & -5 \end{vmatrix} = 4(-15) + 3(11) + 2(5) = -52 + 33 + 10 = -9
\]
\[
D_z = \begin{vmatrix} 4 & 15 & 2 \\ -2 & 1 & -3 \\ 5 & -5 & 2 \end{vmatrix} = 4\begin{vmatrix} 1 & -3 \\ 5 & -5 \end{vmatrix} - 3\begin{vmatrix} 1 & -3 \\ 5 & -5 \end{vmatrix} + 2\begin{vmatrix} 1 & -3 \\ 5 & -5 \end{vmatrix} = 4(-15) + 3(11) + 2(5) = -52 + 33 + 10 = -9
\]
The solution of the system is \( x = -2 \), \( y = -5 \) or \((-2,-5)\).

21. \[
\begin{align*}
3x^2 + y^2 &= 12 \\
y^2 &= 9x
\end{align*}
\]
Substitute \( 9x \) for \( y^2 \) into the first equation and solve for \( x \):
\[
3x^2 + (9x) = 12
\]
\[
3x^2 + 9x - 12 = 0
\]
\[
x^2 + 3x - 4 = 0
\]
\[
(x-1)(x+4) = 0
\]
x = 1 or \( x = -4 \)
Back substitute these values into the second equation to determine \( y \):
\[
x = 1: \ y^2 = 9(1) = 9 \\
y = \pm 3
\]
\[
x = -4: \ y^2 = 9(-4) = -36 \\
y = \pm \sqrt{-36} = -6
\]
The solutions of the system are \((1, -3)\) and \((1, 3)\).
Chapter 11: Systems of Equations and Inequalities

22. \[ \begin{align*}
2y^2 - 3x^2 &= 5 \\
y - x &= 1 \\
\Rightarrow y &= x + 1
\end{align*} \]
Substitute \( x + 1 \) for \( y \) into the first equation and solve for \( x \):
\[ \begin{align*}
2(x + 1)^2 - 3x^2 &= 5 \\
2(x^2 + 2x + 1) - 3x^2 &= 5 \\
2x^2 + 4x + 2 - 3x^2 &= 5 \\
-x^2 + 4x - 3 &= 0 \\
x^2 - 4x + 3 &= 0 \\
(x - 1)(x - 3) &= 0 \\
x = 1 \quad \text{or} \quad x = 3
\end{align*} \]
Back substitute these values into the second equation to determine \( y \):
\[ \begin{align*}
x = 1: & \quad y = 1 + 1 = 2 \\
x = 3: & \quad y = 3 + 1 = 4
\end{align*} \]
The solutions of the system are \((1, 2)\) and \((3, 4)\).

23. \[ \begin{align*}
x^2 + y^2 &\leq 100 \\
4x - 3y &\geq 0
\end{align*} \]
Graph the circle \( x^2 + y^2 = 100 \). Use a solid curve since the inequality uses \( \leq \). Choose a test point not on the circle, such as \((0, 0)\). Since \( 0^2 + 0^2 \leq 100 \) is true, shade the same side of the circle as \((0, 0)\); that is, inside the circle.

Graph the line \( 4x - 3y = 0 \). Use a solid line since the inequality uses \( \geq \). Choose a test point not on the line, such as \((0, 1)\). Since \( 4(0) - 3(1) \geq 0 \) is false, shade the opposite side of the line from \((0, 1)\). The overlapping region is the solution.

24. \[ \frac{3x + 7}{(x + 3)^2} \]
The denominator contains the repeated linear factor \( x + 3 \). Thus, the partial fraction decomposition takes on the form
\[ \frac{3x + 7}{(x + 3)^2} = \frac{A}{x + 3} + \frac{B}{(x + 3)^2} \]
Clear the fractions by multiplying both sides by \( (x + 3)^2 \). The result is the identity
\[ 3x + 7 = A(x + 3) + B \]
or
\[ 3x + 7 = Ax + (3A + B) \]
We equate coefficients of like powers of \( x \) to obtain the system
\[ \begin{align*}
3 &= A \\
7 &= 3A + B
\end{align*} \]
Therefore, we have \( A = 3 \). Substituting this result into the second equation gives
\[ 7 = 3A + B \]
\[ 7 = 3(3) + B \]
\[ -2 = B \]
Thus, the partial fraction decomposition is
\[ \frac{3x + 7}{(x + 3)^2} = \frac{3}{x + 3} + \frac{-2}{(x + 3)^2} \].

25. \[ \frac{4x^2 - 3}{x(x^2 + 3)^2} \]
The denominator contains the linear factor \( x \) and the repeated irreducible quadratic factor \( x^2 + 3 \). The partial fraction decomposition takes on the form
\[ \frac{4x^2 - 3}{x(x^2 + 3)^2} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3} + \frac{Dx + E}{(x^2 + 3)^2} \]
We clear the fractions by multiplying both sides by \( x(x^2 + 3)^2 \) to obtain the identity
\[ 4x^2 - 3 = A(x^2 + 3)^2 + (Bx + C)(x^2 + 3) + x(Dx + E) \]
Collecting like terms yields
\[ 4x^2 - 3 = (A + B)x^4 + Cx^3 + (6A + 3B + D)x^2 + (3C + E)x + (9A) \]
Equating coefficients, we obtain the system
From the last equation we get \( A = -\frac{1}{3} \).

Substituting this value into the first equation gives \( B = \frac{1}{3} \). From the second equation, we know \( C = 0 \). Substituting this value into the fourth equation yields \( E = 0 \).

Substituting \( A = -\frac{1}{3} \) and \( B = \frac{1}{3} \) into the third equation gives us

\[
6\left(-\frac{1}{3}\right) + 3\left(\frac{1}{3}\right) + D = 4
\]

\[-2 + 1 + D = 4
\]

\[D = 5
\]

Therefore, the partial fraction decomposition is

\[
\frac{4x^2 - 3}{x(x^2 + 3)^2} = \frac{-1}{3x} + \frac{1}{3(x^2 + 3)} + \frac{5x}{(x^2 + 3)^2}
\]

26. \[
x \geq 0
\]
\[
y \geq 0
\]
\[
x + 2y \geq 8
\]
\[
2x - 3y \geq 2
\]

The inequalities \( x \geq 0 \) and \( y \geq 0 \) require that the graph be in quadrant I.

\[
x + 2y \geq 8
\]

\[
y \geq -\frac{1}{2}x + 4
\]

Test the point \((0,0)\).

\[
x + 2y \geq 8
\]

\[
0 + 2(0) \geq 8
\]

\[
0 \geq 8 \quad \text{false}
\]

The point \((0,0)\) is not a solution. Thus, the graph of the inequality \( x + 2y \geq 8 \) includes the half-plane above the line \( y = -\frac{1}{2}x + 4 \). Because the inequality is non-strict, the line is also part of the graph of the solution.

27. The objective function is \( z = 5x + 8y \). We seek the largest value of \( z \) that can occur if \( x \) and \( y \) are solutions of the system of linear inequalities

\[
\begin{align*}
2x + y & \leq 8 \\
x - 3y & \leq -3 \\
2x + y & = 8 \\
x - 3y & = -3
\end{align*}
\]

The graph of this system (the feasible points) is shown as the shaded region in the figure below. The corner points of the feasible region are \((0,1), (3,2), \text{and} (0,8)\).
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Corner point, \((x, y)\) | Value of obj. function, \(z\)
---|---
\((0,1)\) | \(z = 5(0) + 8(1) = 8\)
\((3,2)\) | \(z = 5(3) + 8(2) = 31\)
\((0,8)\) | \(z = 5(0) + 8(8) = 64\)

From the table, we can see that the maximum value of \(z\) is 64, and it occurs at the point \((0,8)\).

28. Let \(j\) = unit price for flare jeans, \(c\) = unit price for camisoles, and \(t\) = unit price for t-shirts. The given information yields a system of equations with each of the three women yielding an equation.

\[
\begin{align*}
2j + 2c + 4t &= 90 \quad \text{(Megan)} \\
j + 3t &= 42.5 \quad \text{(Paige)} \\
j + 3c + 2t &= 62 \quad \text{(Kara)}
\end{align*}
\]

We can solve this system by using matrices.

\[
\begin{bmatrix}
2 & 2 & 4 & 90 \\
1 & 0 & 3 & 42.5 \\
1 & 3 & 2 & 62
\end{bmatrix}
= \begin{bmatrix}
1 & 1 & 2 & 45 \\
0 & -1 & 1 & -2.5 \\
0 & 2 & 0 & 17
\end{bmatrix}
= \begin{bmatrix}
1 & 1 & 2 & 45 \\
0 & 2 & 0 & 17 \\
0 & 1 & -1 & 2.5
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 3 & 42.5 \\
0 & 0 & 2 & 12 \\
0 & 1 & -1 & 2.5
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 3 & 42.5 \\
0 & 0 & 2 & 12 \\
0 & 0 & 1 & 6
\end{bmatrix}
\]

The last row represents the equation \(z = 6\). Substituting this result into \(y - z = 2.5\) (from the second row) gives

\[
\begin{align*}
y - 2.5 &= z \\
y - z &= 2.5
\end{align*}
\]

Substituting \(z = 6\) into \(x + 3z = 42.5\) (from the first row) gives

\[
\begin{align*}
x + 3z &= 42.5 \\
x + 3(6) &= 42.5 \\
x &= 24.5
\end{align*}
\]

Thus, flare jeans cost $24.50, camisoles cost $8.50, and t-shirts cost $6.00.

Chapter 11 Cumulative Review

1. \(2x^2 - x = 0\)

\[
x(2x-1) = 0
\]

\[
x = 0 \quad \text{or} \quad 2x-1 = 0
\]

\[
x = 1
\]

\[
x = \frac{1}{2}
\]

The solution set is \(\left\{0, \frac{1}{2}\right\}\).

2. \(\sqrt{3x+1} = 4\)

\[
\left(\sqrt{3x+1}\right)^2 = 4^2
\]

\[
3x+1 = 16
\]

\[
x = 5
\]

Check:

\[
\sqrt{3\cdot 5+1} = 4
\]

\[
\sqrt{16} = 4
\]

\[
x = 4
\]

The solution set is \(\{5\}\).

3. \(2x^3 - 3x^2 - 8x - 3 = 0\)

The graph of \(Y_1 = 2x^3 - 3x^2 - 8x - 3\) appears to have an \(x\)-intercept at \(x = 3\).

Using synthetic division:

\[
\begin{array}{c|cccc}
3 & 2 & -3 & -8 & -3 \\
0 & 6 & 9 & 3 \\
\hline
2 & 3 & 1 & 0
\end{array}
\]

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Therefore, \(2x^3 - 3x^2 - 8x - 3 = 0\)

\[(x - 3)(2x^2 + 3x + 1) = 0\]

\[(x - 3)(2x + 1)(x + 1) = 0\]

\[x = 3 \text{ or } x = -\frac{1}{2} \text{ or } x = -1\]

The solution set is \(\{-1, -\frac{1}{2}, 3\}\).

4. \(3^x = 9^{x+1}\)

\[3^x = (3^2)^{x+1}\]

\[3^x = 3^{2x+2}\]

\[x = 2x + 2\]

\[x = -2\]

The solution set is \(\{-2\}\).

5. \(\log_3(x - 1) + \log_3(2x + 1) = 2\)

\[\log_3[(x - 1)(2x + 1)] = 2\]

\[(x - 1)(2x + 1) = 3^2\]

\[2x^2 - x - 1 = 9\]

\[2x^2 - x - 10 = 0\]

\[(2x - 5)(x + 2) = 0\]

\[x = \frac{5}{2} \text{ or } x = -2\]

Since \(x = -2\) makes the original logarithms undefined, the solution set is \(\left\{\frac{5}{2}\right\}\).

6. \(3^x = e\)

\[\ln(3^x) = \ln e\]

\[x \ln 3 = 1\]

\[x = \frac{1}{\ln 3} = 0.910\]

The solution set is \(\left\{\frac{1}{\ln 3} = 0.910\right\}\).

7. \(g(x) = \frac{2x^3}{x^2 + 1}\)

\[g(-x) = \frac{2(-x)^3}{(-x)^2 + 1} = -\frac{2x^3}{x^2 + 1} = -g(x)\]

Thus, \(g\) is an odd function and its graph is symmetric with respect to the origin.

8. \(x^2 + y^2 - 2x + 4y - 11 = 0\)

\[x^2 - 2x + y^2 + 4y = 11\]

\[(x^2 - 2x + 1) + (y^2 + 4y + 4) = 11 + 1 + 4\]

\[(x - 1)^2 + (y + 2)^2 = 16\]

Center: \((1, -2)\); Radius: 4

9. \(f(x) = 3^{x-2} + 1\)

Using the graph of \(y = 3^x\), shift the graph horizontally 2 units to the right, then shift the graph vertically upward 1 unit.

Domain: \((\infty, \infty)\) Range: \((1, \infty)\)

Horizontal Asymptote: \(y = 1\)

10. \(f(x) = \frac{5}{x + 2}\)

\[y = \frac{5}{x + 2}\]

\[x = \frac{5}{y + 2}\]

Inverse

\[x(y + 2) = 5\]

\[xy + 2x = 5\]

\[xy = 5 - 2x\]

\[y = \frac{5 - 2x}{x} = \frac{5}{x} - 2\]

Thus, \(f^{-1}(x) = \frac{5}{x} - 2\)

Domain of \(f\) = \{\(x \mid x \neq -2\}\}

Range of \(f\) = \{\(y \mid y \neq 0\}\}

Domain of \(f^{-1}\) = \{\(x \mid x \neq 0\}\}

Range of \(f^{-1}\) = \{\(y \mid y \neq -2\)\}. 

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11. a. \( y = 3x + 6 \)
   The graph is a line.
   \( x \)-intercept:
   \[ 0 = 3x + 6 \]
   \[ 3x = -6 \]
   \[ x = -2 \]
   \( y \)-intercept:
   \[ y = 3(0) + 6 \]
   \[ y = 6 \]

b. \( x^2 + y^2 = 4 \)
   The graph is a circle with center \((0, 0)\) and radius 2.

c. \( y = x^3 \)

d. \( y = \frac{1}{x} \)

e. \( y = \sqrt{x} \)

f. \( y = e^x \)

g. \( y = \ln x \)
h. \(2x^2 + 5y^2 = 1\)
The graph is an ellipse.
\[
\frac{x^2}{\frac{1}{2}} + \frac{y^2}{\frac{5}{3}} = 1
\]
\[
\left(\frac{x}{\sqrt{\frac{1}{2}}}\right)^2 + \left(\frac{y}{\sqrt{\frac{5}{3}}}\right)^2 = 1
\]

i. \(x^2 - 3y^2 = 1\)
The graph is a hyperbola
\[
\frac{x^2}{1} - \frac{y^2}{\frac{1}{3}} = 1
\]
\[
\left(\frac{x}{1}\right)^2 - \left(\frac{y}{\sqrt{\frac{1}{3}}}\right)^2 = 1
\]

j. \(x^2 - 2x - 4y + 1 = 0\)
\[
x^2 - 2x + 1 = 4y
\]
\[
y = \frac{1}{4}(x-1)^2
\]

12. \(f(x) = x^3 - 3x + 5\)

a. Let \(Y_1 = x^3 - 3x + 5\).

The zero of \(f\) is approximately \(-2.28\).

b. \(f\) has a local maximum of 7 at \(x = -1\) and a local minimum of 3 at \(x = 1\).

c. \(f\) is increasing on the intervals \((-\infty, -1)\) and \((1, \infty)\).
Chapter 11 Projects

Project I – Internet Based Project

1. \[
\begin{array}{ccc}
80\% &=& 0.80 \\
18\% &=& 0.18 \\
2\% &=& 0.02 \\
40\% &=& 0.40 \\
50\% &=& 0.50 \\
10\% &=& 0.10 \\
20\% &=& 0.20 \\
60\% &=& 0.60 \\
20\% &=& 0.20 \\
\end{array}
\]

2. \[
\begin{bmatrix}
0.80 & 0.18 & 0.02 \\
0.40 & 0.50 & 0.10 \\
0.20 & 0.60 & 0.20 \\
\end{bmatrix}
\]

3. \[
\begin{array}{ccc}
0.80 + 0.18 + 0.02 &=& 1.00 \\
0.40 + 0.50 + 0.10 &=& 1.00 \\
0.20 + 0.60 + 0.20 &=& 1.00 \\
\end{array}
\]

The sum of each row is 1 (or 100%). These represent the three possibilities of educational achievement for a parent of a child, unless someone does not attend school at all. Since these are rounded percents, chances are the other possibilities are negligible.

4. \[
p^2 = \begin{bmatrix}
0.8 & 0.18 & 0.02 \\
0.4 & 0.5 & 0.1 \\
0.2 & 0.6 & 0.2 \\
\end{bmatrix}
= \begin{bmatrix}
0.716 & 0.246 & 0.038 \\
0.54 & 0.382 & 0.078 \\
0.44 & 0.456 & 0.104 \\
\end{bmatrix}
\]

Grandchild of a college graduate is a college graduate: entry (1, 1): 0.716. The probability is 71.6%.

5. Grandchild of a high school graduate finishes college: entry (2, 1): 0.54. The probability is 54%.

6. Grandchildren \( \rightarrow k = 2 \).

\[
v^{(2)} = v^{(0)} p^2
= \begin{bmatrix}
0.288 & 0.569 & 0.143 \\
0.54 & 0.382 & 0.078 \\
0.44 & 0.456 & 0.104 \\
\end{bmatrix}
\]

College: = 57.6%
High School: = 35%
Elementary: = 7%

7. The matrix totally stops changing at

\[
p^{30} = \begin{bmatrix}
0.64885496 & 0.29770992 & 0.05343511 \\
0.64885496 & 0.29770992 & 0.05343511 \\
0.64885496 & 0.29770992 & 0.05343511 \\
\end{bmatrix}
\]

Chapter 11: Systems of Equations and Inequalities

Project II

a. \( 2 \times 2 \times 2 \times 2 = 16 \) codewords.

b. \( v = uG \)

\( u \) will be the matrix representing all of the 4-digit information bit sequences.

\[
u = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 \\
\end{bmatrix}
\]

(Remember, this is mod two. That means that you only write down the remainder when dividing by 2.)
Project III

a. \( A^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \end{bmatrix} \)

b. \( B = (A^T A)^{-1} A^T Y \)
\[
B = \begin{bmatrix} -2.357 \\ 2.0357 \end{bmatrix}
\]

c. \( y = 2.0357x - 2.357 \)

d. \( y = 2.0357x - 2.357 \)

Project IV

Answers will vary.